1.25

Digitized by the Internet Archive
in 2021 with funding from
Kahle/Austin Foundation

https://archive.org/details/electriccurentsg0000unse

International Library of Technology
383B

Electric Currents
Generation, Measurement, and Use

316 Illustrations

Prepared Under Supervision of
FRANCIS H. DOANE, A. M. B.
DIRECTOR, ELECTRICAL SCHOOLS, INTERNATIONAL CORRESPONDENCE SCHOOLS

ELECTRICITY AND MAGNETISM
THEORY OF DIRECT-CURRENT GENERATORS AND MOTORS
DIRECT-CURRENT GENERATORS
DIRECT-CURRENT MOTORS
DIRECT-CURRENT MACHINERY OPERATION
RESISTANCE MEASUREMENTS
DIRECT-CURRENT MEASURING INSTRUMENTS
ALTERNATING CURRENTS

Published by
INTERNATIONAL TEXTBOOK COMPANY
SCRANTON, PA.
1928

Electricity and Magnetism, Parts 1 and 2: Copyright, 1915, by International Textbook Company.

Theory of Direct-Current Generators and Motors: Copyright, 1915, by International Textbook Company.

Direct-Current Generators: Copyright, 1915, by International Textbook Company.

Direct-Current Motors: Copyright, 1915, by International Textbook Company.

Direct-Current Machinery Operation: Copyright, 1917, by International Textbook Company.

Resistance Measurements: Copyright, 1914, by International Textbook Company.

Direct-Current Measuring Instruments: Copyright, 1914, by International Textbook Company.

Alternating Currents, Parts 1 and 2: Copyright, 1920, 1914, by International Textbook Company.

Copyright in Great Britain

All rights reserved

Printed in U. S. A.

Press of
International Textbook Company
Scranton, Pa.
383B

93190

PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools. The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts. The manuscripts are then carefully edited to make them suitable for correspondence instruction. The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students. Necessary technical expressions are clearly explained when introduced.

The great majority of our students wish to prepare themselves for advancement in their vocations or to qualify for more congenial occupations. Usually they are employed and able to devote only a few hours a day to study. Therefore every effort must be made to give them practical and accurate information in clear and concise form and to make this information include all of the essentials but none of the nonessentials. To make the text clear, illustrations are used freely. These illustrations are especially made by our own Illustrating Department in order to adapt them fully to the requirements of the text.

In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed.

INTERNATIONAL TEXTBOOK COMPANY

CONTENTS

Note.—This volume is made up of a number of separate parts, or sections, as indicated by their titles, and the page numbers of each usually begin with 1. In this list of contents the titles of the parts are given in the order in which they appear in the book, and under each title is a full synopsis of the subjects treated.

ELECTRICITY AND MAGNETISM, PART 1

Pages

Introduction 1

Electrodynamics 2–24

Terms and Expressions 2– 3

Generation of Electromotive Force 4– 6

Electrical Units 7–14

Quantity of electricity; Current; Resistance; Electromotive force; Electrical work; Electric power; Heat, work, and power units compared.

Electric Circuits 15–24

Circuit terms; Series grouping; Parallel grouping; Three-wire circuits; Ohm's law.

Electrostatics 25–30

Electric Charges 25–28

Condensers 29–30

ELECTRICITY AND MAGNETISM, PART 2

Permanent Magnetism 1– 9

Magnets and Magnetic Properties 1– 6

Magnetic Circuits 7– 9

Electromagnetism 10–19

Magnetic Effect of Electric Current 10–13

Electromagnetic Solenoid 14–19

General theory; Electromagnets.

Electric Calculations 20–28

ELECTRICITY AND MAGNETISM, PART 2

(*Continued*)

Pages
Cross-Sectional Areas of Electric Conductors......... 20–23
Resistance of Conductors......................... 24–28
Dielectric Circuits 29–35
Electric Condensers.............................. 29–31
Dielectric Strength 32–35

THEORY OF DIRECT-CURRENT GENERATORS AND MOTORS

Electromagnetic Induction 1– 8
Fundamental Principles 1
Direction of Electromotive Force.................. 2– 4
Self-Induction 5
Mutual Induction 6– 7
Non-Inductive Circuit............................ 8
Electric Generator 9–23
Elementary Theory 9–15
Parts of Magnetic Fields......................... 16
Classification by Methods of Field Excitation........ 17–23
Electric Motor 24–30
Theory of Motor Action........................... 24–30
Development and direction of turning force; Counter-electromotive force; Purpose of starting resistance; Commutating-pole motors.

DIRECT-CURRENT GENERATORS

Classification 1– 5
Construction 6–27
General Arrangement 6
Armatures 6–23
Cores, commutators, and brushes; Armature windings.
Field Magnets 24–27
Design Features Determining Output............... 28–35
Dimensions 28
Commutation 29–35
Armature reaction; Compensating pole-face winding; Unipolar generator.
Adaptation to Service Conditions.................. 36–56

DIRECT–CURRENT GENERATORS
(*Continued*)

Pages
Three-Wire Systems 36–50
Three-wire generators; Three-wire balancers.

Motor Generators and Dynamotors 51–53
Electroplating Generators 54
Constant-Current Generators 54–56
Direct-Current Voltage Regulation 57–61

DIRECT–CURRENT MOTORS

Classification 1–15
Sizes of Motors 1
Field Winding 2– 4
Speed 5
Enclosure 6– 9
Mechanical Characteristics 10–13
Service Conditions 14–15
Direct-Current Motor Control 16–37
Torque 16
Motor Starting 17–26
Motor-Speed Regulation 27–29
Speed-Regulating Devices 30–37

DIRECT–CURRENT MACHINERY OPERATION

Motors and Generators 1–23
Introduction 1– 2
Preliminary investigation; Safe operating temperatures.

Operation of Direct-Current Motors 3– 8
Shunt motors; Compound motors; Series motors.

Operation of Direct-Current Generators 9–23
Preliminary inspection; Shunt generators; Compound generators; Series generators; Field circuits of direct-current generators; Edison three-wire system; Tirrill regulators with direct-current generators; Routine care and inspection.

Care and Maintenance 24–47
General Instructions 24–26
Installation; Bearings.

DIRECT-CURRENT MACHINERY OPERATION
(*Continued*)

Pages

Troubles of Direct-Current Machines 27–37

Poor commutation; Excessive heating; Noisy operation; Low voltage of generator; Failure of motor to operate; Failure of generator to operate.

Dangers of Physical Injury 38–40

Dangers from mechanical sources; Danger from electrical sources.

Tests for Faults in Electrical Circuits 41–47

Testing devices; Continuity test; Test for crosses and grounds; Differential test; Tests for defects in direct-current armatures.

RESISTANCE MEASUREMENTS

Galvanometer 1– 7

Elementary type; Kelvin galvanometer; D'Arsonval galvanometer; Methods of reading deflections; Ayrton galvanometer shunt.

Conductor Resistance 8–17
Wheatstone Bridge 8–13
Slide-Wire Bridge 14–15
Ohmmeter 16–17
Insulation Resistance 18–21
Megger 18–21

DIRECT-CURRENT MEASURING INSTRUMENTS

Description of Measuring Instruments 1–27
Moving-Iron Instruments 1– 3
Instruments of D'Arsonval Type 4–13

General discussion; Ammeters and shunts; Voltmeters.

Electrodynamometer Instruments 14–16

Wattmeters; Voltmeters and ammeters.

Electrostatic Instruments 17–18
Hot-Wire Instruments 19–21
Recording Instruments 22–27

Direct-acting recorder; Relay recorder.

Special Forms and Application of Instruments 28–32
Direct-Reading Ohmmeter 28–29

DIRECT-CURRENT MEASURING INSTRUMENTS
(*Continued*)

Pages
Electric Tachometer 30
Electric Pyrometers 31–32
Use of Instruments for Measurements 33–38
Measurements With Ammeters and Voltmeters 32–34
Measurements With a Voltmeter 35–37
Measurement of Power 38
Calibration of Instruments 39–42

ALTERNATING CURRENTS, PART 1

General Theory 1–67
Representation and Definitions 1–10
Generation of an alternating electromotive force; Cycle, frequency, and alternation; Space-degrees and time-degrees.
Values and Phase Relations 11–22
Values used in practice; Phase relations.
Calculations With Wave Forms 23–25
Self-Induction 26–40
Effects of self-induction; Inductance of a circuit.
Capacity 41–49
Alternating-Current Conductors and Circuits 50–53
Vectorial Representations 54–67

ALTERNATING CURRENTS, PART 2

Single-Phase Circuits 1–24
Series Circuits 1– 9
Parallel Circuits 10–17
Power in Two-Wire Circuits 18–24
Theoretical considerations; Calculation of power.
Polyphase Circuits 25–64
Fundamental Principles 25–34
Two-Phase Circuits 35–44
Three-Phase Circuits 45–61
Calculation of Power 62–64

ELECTRICITY AND MAGNETISM

(PART 1)

INTRODUCTION

1. Manifestations of Electricity.—If a glass rod is rubbed with a piece of dry silk cloth, both the rod and the cloth, at the parts rubbed, will attract small bits of paper and string brought near them. If a stick of sealing wax is rubbed with a piece of dry flannel cloth, the surfaces rubbed together have the same property. A cat's fur rubbed in cold weather sparks with perceptible noise. During a thunder storm, lightning flashes across the clouds or between the clouds and the earth. All these are manifestations of electricity.

2. The name *electricity* comes from the Greek word *electron*, meaning *amber*, a substance that the ancients found would attract small particles of matter after it had been rubbed. Substances having this property are said to be *electrified*, or *charged*. **Electricity** may be defined as the cause of all electrical phenomena, or manifestations, though its exact nature is unknown. Much is known, however, as to its manifestations and its use, and the study of electricity applies chiefly to its uses and to what may be expected from it under different conditions.

3. Classes of Electricity.—Electricity at rest is called **static electricity**; this is the kind produced by rubbing glass with silk and wax with flannel. The name given the study of static electricity is **electrostatics.**

Electricity in motion is called **dynamic electricity**, and the name given to its study is **electrodynamics.** Substances

COPYRIGHTED BY INTERNATIONAL TEXTBOOK COMPANY. ALL RIGHTS RESERVED

through which electricity flows readily are **conductors**, and those offering very high resistance to its flow are **non-conductors**, or **insulators**. Electricity moving through a conductor is **electric current.**

If an iron rod is grasped in one hand and a wooden rod of the same length in the other, and the free ends of these rods are held in a fire, the iron rod will become too hot to hold comfortably before the wooden rod has warmed. This is because iron conducts heat better than does wood. So it is with different substances in respect to electricity; some conduct it better than others.

ELECTRODYNAMICS

TERMS AND EXPRESSIONS

4. Electricity in motion is of chief interest and its study will be taken up first. Dynamic electricity is of two general classes, *direct-current electricity* and *alternating-current electricity*. Direct-current electricity flows continuously in one direction, and is therefore frequently called *continuous current*. Its strength may change, but its direction of flow does not change. Alternating-current electricity reverses its direction of flow many times per second. Alternating current will be fully discussed in future Sections. The statements in this Section apply particularly to the action of direct current, though some of these statements are true also of alternating current.

5. The **electric potential** of any part of a body refers to its electrical condition, or charge, compared with some other part, and a **difference of electric potential** between two points connected by a conductor causes flow of electricity, or current, from the point of high potential to the point of low potential. Difference of electric potential is *electric pressure*, and it may be compared with water pressure. If there is pressure in a water system and outlets are opened, water flows; but if the outlets are closed no water can flow. Likewise, electric pressure may exist with or without flow of electricity, according to whether a conducting path is complete. Difference of electric potential is therefore that which causes or tends to cause flow of electricity, or electric current.

6. **Electromotive force** (abbreviated E. M. F.) is, strictly speaking, the force that establishes a difference of electric potential; but for practical purposes electromotive force and difference of electric potential may be considered as the same thing. Either causes or tends to cause electric current; the strength of current, or the rate of flow of electricity, depends on the strength of the electromotive force.

7. An **electric circuit** is a conducting path through which electric current will flow when the path is complete between points of different potentials. A complete path between such points is called a *closed circuit;* an incomplete path is called an *open circuit.*

8. The **direction of current** in an electric circuit is assumed to be from a condition called *positive*, or *high*, *potential* to a condition called *negative*, or *low*, *potential*. The signs + and − are used to indicate these two conditions. Thus electricity is said to flow from + to −.

9. **Electric resistance** is opposition to flow of electricity. Non-conductors have high resistance and conductors have low resistance. No known substance is wholly without resistance. The strength of current in a circuit depends not only on the electromotive force acting in the circuit, but also inversely on the resistance of the circuit; that is, the current increases with increased electromotive force, but decreases with increased resistance.

10. In general, the resistance of a long circuit or conductor is greater than that of a short one of the same size and material, and the resistance of a large body is less than that of a small one of the same material. Stated mathematically, *the resistance of a circuit or conductor depends directly on its length and inversely on its cross-sectional area*, or size if cut square across. The resistances of different materials are widely different, some

having very much higher resistance than others. The resistance of any given material is not the same at all temperatures.

11. Good conductors of electricity are made of materials having low resistance. Among such materials, arranged in order with the best conducting material first, are silver, copper, gold, aluminum, zinc, brass, phosphor-bronze, platinum, tin, nickel, lead, German silver, steel, iron, mercury, carbon, and water. Silver and gold are too expensive to be generally used for electric conductors. Copper, being plentiful and comparatively cheap, is in very general use, and aluminum is also much employed for long transmission lines.

12. An **insulator** is a non-conductor of such high resistance that practically no electricity can get through it. Among the best known insulating materials are glass, porcelain, rubber, mica, ebonite, dry paraffined wood, paper, vulcanized fiber, asbestos, pure asphalt, air, and oils. Insulators are used to support conductors and to keep the electricity confined to the circuits intended for it. For example, telegraph, telephone, and electric-light wires on poles are fastened to glass or porcelain insulators.

GENERATION OF ELECTROMOTIVE FORCE

13. The methods most commonly used to establish electromotive force are: (1) by chemical action in *primary cells;* (2) by heat action in *thermoelectric couples;* and (3) by magnetic induction in *dynamos*, or *electric generators*. Electric generators are used in power stations for electric-light and electric-railway work. They will be described in later Sections.

14. A **primary cell,** also called a *voltaic*, or *galvanic, cell,* is a combination of materials in which chemical action establishes electromotive force as soon as the materials are properly combined. Fig. 1 shows a simple primary cell consisting of a copper plate C and a zinc plate Z partly immersed in liquid in a glass jar A. Under such conditions, electromotive force is established between the plates, and if they were electrically connected outside the liquid, as would be the case if the conductors shown attached to the plates were joined together, electricity would flow from copper to zinc outside the liquid and from zinc to copper through the liquid. When electricity flows through such a combination, some of the water in the liquid is separated, or decomposed, into two gases, hydrogen and oxygen. The hydrogen gathers on the surface of the copper plate, where it appears to be small air bubbles. The oxygen combines chemically with some of the zinc, and the zinc plate therefore gradually wastes away.

15. The two solid materials in the simple cell, Fig. 1, are called **electrodes**, or *elements;* taken together, they form the **voltaic couple.** The liquid is called the **electrolyte.** The electrode from which electricity enters the electrolyte is the **anode,** and the electrode toward which electricity flows in the electrolyte is the **cathode;** thus, the zinc plate is the anode and the copper plate, the cathode.

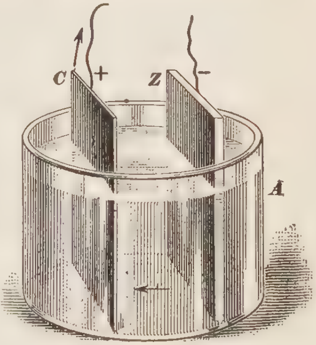


FIG. 1

The plates, the liquid between them, and the conductors connecting them outside the cell form a closed circuit. The part of this circuit inside the cell is the **internal circuit,** and the part outside the cell is the **external circuit.** Electricity flows from the anode to the cathode in the internal circuit and from the cathode to the anode in the external circuit. The flow continues in this direction until the cathode of a simple primary cell becomes covered with hydrogen gas. Such a flow of electricity is properly called continuous current, because its direction is unvarying and its flow steady. The hydrogen gas gradually gathers in small bubbles on the cathode, making the resistance of the circuit higher and the current less. When the cathode is completely covered the current ceases and the cell is said to be **polarized.**

16. Many different electrolytes are used in primary cells. The simplest are salt water or water with a little acid added.

The composition depends largely on the service for which the cell is intended. Electrolytes in commercial cells usually contain some substance that absorbs hydrogen or prevents it from gathering on the cathode. Such a substance is called a **depolarizer.**

17. A **primary battery** is a group of primary cells so connected electrically that either their electromotive forces or their currents are added. The conductors attached to the plates are the *cell terminals*, and these terminals can be joined in many ways. Primary batteries are used in telegraph, telephone, and signal work.

18. When two dissimilar metals are joined and their joint heated or cooled, electromotive force is established. The name *thermoelectromotive force* is applied, the word *thermo* being derived from a Greek word meaning heat; the current resulting when this force acts in a closed circuit is called *thermoelectric current.* Such a combination of metals is called a **thermoelectric couple** and several such couples together form a **thermoelectric pile.**

In order to establish thermoelectromotive force, the temperature of the couple or the pile must differ from that of the rest of the circuit of which it forms a part. If higher, the force acts in one direction; if lower, the force acts in the other direction.

19. The chief use of thermoelectromotive force is to indicate high temperatures by means of a **pyrometer,** which is a device consisting of an indicating instrument in circuit with a thermoelectric couple or pile. The couple or the pile is so arranged that it can be thrust into the place where the temperature is to be measured and the current thus set up through the instrument causes the needle, or hand, to deflect over a scale on which the temperature can be read. Temperatures in furnaces and ovens can be readily indicated in this way when no other method would serve the purpose. The indicator can be placed at a safe distance from the heat.

ELECTRICAL UNITS

QUANTITY OF ELECTRICITY

20. Measurement of Electricity.—Electricity is measured by its effects, or the work it does; the greater the effect, or the work done, the greater is the quantity of electricity. When it flows through the electrolyte of a voltaic cell, for example, some of the water is decomposed into gases, and the greater the quantity of electricity the greater is the formation of gas. When electricity is caused to flow through an electrolyte containing a metal in solution, some of the metal is deposited on one of the electrodes, the amount being proportional to the quantity of electricity. This is the way in which metallic articles, such as table ware, are plated.

21. The **coulomb** *is the unit quantity of electricity*, and is the quantity that deposits .001118 gram,* or .01725 grain,* of metallic silver from a carefully prepared electrolyte containing silver dissolved in nitric acid. This statement is true regardless of the time required for making this deposit.

CURRENT

22. The **ampere** (abbreviated amp.) *is the practical unit rate of flow of electricity*, or *unit current*, and is the rate when 1 coulomb flows each second; that is, *amperes equals coulombs per second.* The quantity of electricity in coulombs divided by the time in seconds required for this quantity to flow equals the rate of flow in amperes. One *milliampere* is $\frac{1}{1000}$ ampere; 1,000 milliamperes equals 1 ampere.

23. Conversely, if electricity is flowing at the rate of 1 ampere, then 1 coulomb flows per second, and the total number of coulombs equals the product of the number of amperes and the number of seconds. In other words, coulombs = amperes × seconds, or ampere-seconds.

---

*Grams and grains are explained in a later Section.

24. In practical work, electric current is measured by means of an instrument called an **ammeter**, of which Fig. 2 shows one type. The ammeter is connected in circuit by inserting the ends of conductors in the openings in the two binding posts and clamping them with the thumbscrews at the tops of the posts. The current in the circuit causes the pointer to deflect over the scale to a figure indicating the strength of current in amperes or milliamperes.

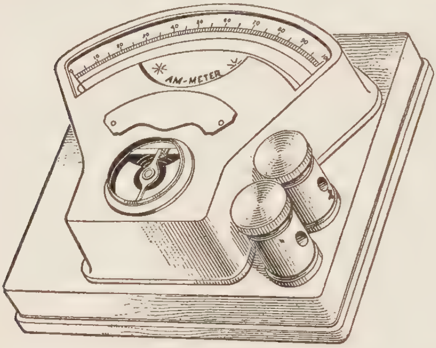


FIG. 2

RESISTANCE

25. The **international ohm** *is the practical unit of electric resistance*, and is determined by the resistance of a column of mercury of specified dimensions or by a standard ohm coil. These standards are of interest only in making accurate scientific measurements; for general practical work, use is made of instruments that indicate either the resistance in ohms or other measurements from which the resistance can be calculated. Such instruments are explained in other Sections. The word *ohm* as generally used in discussing electric circuits refers to the international ohm.

ELECTROMOTIVE FORCE

26. The **volt** *is the practical unit of electromotive force*, and is the electromotive force that will cause electricity to flow at the rate of 1 ampere through a circuit with a resistance of 1 ohm. Because of the name of its unit, electromotive force is commonly called **voltage**, and the word voltage will be much used in referring to electromotive force.

27. In practical work, voltage is measured by means of an instrument called a **voltmeter.** This instrument has much the same appearance as an ammeter. Voltmeters also are made in many styles, or types, and for many different capacities, or ranges of voltage; a type much used is shown in Fig. 3.

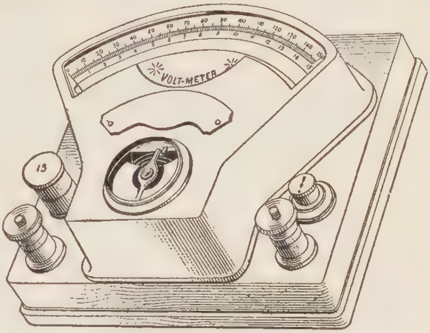


FIG. 3

ELECTRICAL WORK

28. The **joule** *is a unit of electric work*, and is the work done when 1 coulomb of electricity flows under a pressure of 1 volt; the time required makes no difference. The work, in joules, done in any circuit is equal to the product of the number of coulombs of electricity and the number of volts of electromotive force. As coulombs = ampere-seconds, joules = volts × amperes × seconds, or volt-ampere-seconds.

ELECTRIC POWER

29. The **watt** *is a unit of electric power* and is the rate of work when 1 joule is being done each second; that is, watts = joules per second, or joules ÷ seconds. Then the following is true:

$$watts=\frac{joules}{seconds}=\frac{volts\times amperes\times seconds}{seconds}=volts\times amperes$$

Let p = power, in watts;
E = electromotive force, in volts;
I = current, in amperes.

Then, $$p=E\,I$$

This formula is very important and should be remembered.

EXAMPLE 1.—What is the power, in watts, in a circuit in which the current is 12 amperes and the electromotive force 25 volts?

SOLUTION.—Here, $I=12$ and $E=25$; therefore, $p=25\times12=300$ watts. Ans.

EXAMPLE 2.—If the power in a circuit is 550 watts and the voltage 110, what is the current in amperes?

SOLUTION.—Here $550=110\times I$; therefore, $I=550\div 110=5$ amp. Ans.

30. The **kilowatt** (abbreviated K. W.) is also a unit of electric power, and is equal to 1,000 watts; watts ÷ 1,000 = kilowatts. If P = power in kilowatts, then

$$P=\frac{E\,I}{1{,}000}$$

EXAMPLE.—If an electric motor takes 25 amperes from a 220-volt circuit, what is the power, in kilowatts, taken?

SOLUTION.—Here, $E=220$ and $I=25$; therefore,

$$P=\frac{220\times 25}{1{,}000}=5.5 \text{ K. W.} \quad \text{Ans.}$$

31. The **watt-hour** (abbreviated W.-hr.) and **kilowatt-hour** (abbreviated K. W.-hr.) are units used to express either work or energy; each of these units is the product of a unit rate of power, and a unit of time. Electric energy is nearly always bought and sold in kilowatt-hours, which are merely the product of the power, in kilowatts, and the time, in hours.

Let w = work, in watt-hours;
W = work, in kilowatt-hours;
p = power, in watts;
P = power, in kilowatts;
T = time, in hours.

Then,

$$w=p\,T \qquad (1)$$

$$W=P\,T \qquad (2)$$

EXAMPLE.—If work is being done in a circuit at the rate of 1,000 watts, (*a*) how many watt-hours will be done in 5 hours? (*b*) how many kilowatt-hours will be done in the same time?

SOLUTION.—(*a*) Apply formula **1.** Thus, $p=1{,}000$ and $T=5$; therefore, $w=1{,}000\times 5=5{,}000$ W.-hr. Ans.

(*b*) Apply formula **2.** As 1,000 watts = 1 K. W. and in this case also $T=5$, $W=1\times 5=5$ K.-W.-hr. Ans.

32. The relation between the watt-hour and the joule, both units of work, is shown by reducing the watt-hour to watt-seconds, or joules. One hour contains 3,600 seconds; therefore, 1 watt-hour = 3,600 watt-seconds, or joules, or

$$\textit{watt-hours} = \textit{joules} \div 3{,}600$$

HEAT, WORK, AND POWER UNITS COMPARED

33. When work of any kind is done, heat is developed. Some work is required to turn the wheels of a machine, even when the machine is doing no useful work. In this case, practically all the work required to run the idle machine is converted into heat at the bearings. Every mechanic knows that the bearings of machinery must be kept lubricated in order to reduce friction and consequent heating. Too much friction causes overheated bearings; that is, too much work is converted into heat.

34. The *unit of heat* generally used in the United States and Canada is the **British thermal unit** (abbreviated B. T. U.); for practical purposes, it may be defined as the quantity of heat required to raise the temperature of 1 pound of pure water 1 degree (1°) Fahrenheit (F.). The Fahrenheit thermometer is the one generally used for measuring the temperature of the air, and the sign (°) means degree or degrees. The quantity of heat necessary to raise the temperature of any quantity of water may therefore be considered as equal to the product of the weight of water, in pounds, and the change of temperature, in degrees Fahrenheit.

Let H = heat, in British thermal units;
w = weight of water, in pounds;
t = change of temperature, in degrees Fahrenheit.

Then, $$H = w\,t$$

EXAMPLE.—How many British thermal units of heat are required to raise the temperature of $8\frac{1}{3}$ pounds (1 gallon) of water from 32° F. to 212° F.?

SOLUTION.—The change in temperature $t = 212 - 32 = 180°$, and the weight $w = 8\frac{1}{3}$. By the formula,

$$H = 8\tfrac{1}{3} \times 180 = 1{,}500 \text{ B. T. U.} \quad \text{Ans.}$$

NOTE.—The **gram-calorie**, a unit of heat in use in some countries, is $\frac{1}{252}$ B. T. U. One **kilogram-calorie** = 1,000 gram-calories.

35. Mechanical Equivalent of Heat.—By arranging paddles in a vessel of water and a falling weight acting through a system of pulleys to rotate the paddles, the scientist Joule measured both the work done in rotating the paddles and the temperature rise of the water. He found that 778 foot-pounds of work has the same heating effect as 1 British thermal unit. In honor of its discoverer, 778 foot-pounds is called **Joule's mechanical equivalent of heat.** The relation between mechanical work and heat is therefore expressed by the formula

$$\textit{foot-pounds} = 778 \times \text{B. T. U.}$$

Example 1.—How many foot-pounds of work are equivalent to the heat required to raise the temperature of 50 pounds of water 18° F.?

Solution.—According to the formula of Art. **34**, $H = 50 \times 18 = 900$ B. T. U. and according to the preceding equation,

$$\text{foot-pounds} = 778 \times 900 = 700{,}200. \quad \text{Ans.}$$

Example 2.—If 38,900 foot-pounds of work is converted into heat, how much will it raise the temperature of 10 pounds of water?

Solution.—As ft.-lb. $= 778 \times$ B. T. U., $38{,}900 = 778 \times$ B. T. U., and B. T. U. $= 38{,}900 \div 778 = 50$. To raise the temperature of 10 lb. of water 1° F. requires 10 B. T. U., and 50 B. T. U. will raise it $50 \div 10 = 5°$ F. Ans.

36. Heat and Electrical Work.—To produce the same heating effect as 1 British thermal unit requires 1,055 joules of electrical work. This equivalent was also first determined by Joule by arranging a conductor so that all the work done by electricity in it would be absorbed in liquid surrounding the conductor. He then measured the work and the change of temperature of the liquid and divided the number of joules of work by the number of degrees change of temperature. The relation between heat and electrical work is therefore expressed by the formula

$$\textit{joules} = 1{,}055 \times \text{B. T. U.}$$

Example.—(*a*) If an electric heater carrying 12 amperes at 110 volts is submerged in 22.5 pounds of water for 30 minutes (1,800 seconds), how much heat will be applied to the water? (*b*) What will be the final temperature of the water if its initial temperature is 35° F.?

Solution.—(*a*) To apply the formula, the work in joules must be determined. As joules = volts × amperes × seconds, the number in this case is

$110 \times 12 \times 1{,}800 = 2{,}376{,}000$. By the formula, $2{,}376{,}000 = 1{,}055 \times$ B. T. U. Therefore,

B. T. U. $= 2{,}376{,}000 \div 1{,}055 = 2{,}252$, approx. Ans.

(*b*) Since 22.5 B. T. U. are required to raise the temperature of 22.5 pounds water 1°, the total rise will be $2{,}252 \div 22.5 = 100°$ F., approximately, and the final temperature will be $100 + 35 = 135°$ F. Ans.

37. Mechanical and Electrical Work.—Since 778 foot-pounds equals 1 British thermal unit, and 1,055 joules also equals 1 British thermal unit,

$$778 \text{ foot-pounds} = 1{,}055 \text{ joules},$$

$$1 \text{ foot-pound} = \frac{1{,}055}{778} = 1.356 \text{ joules, approximately,}$$

and

$$1 \text{ joule} = \frac{778}{1{,}055} = .737 \text{ foot-pound, approximately}$$

Therefore,

$$joules = 1.356 \times foot\text{-}pounds$$

$$foot\text{-}pounds = .737 \times joules$$

Example 1.—How many joules are equivalent to 500 foot-pounds?

Solution.—Joules $= 1.356 \times$ ft.-lb. $= 1.356 \times 500 = 678.0$. Ans.

Example 2.—A current of 10 amperes from a 110-volt circuit for 30 minutes (1,800 seconds) is equivalent to how many foot-pounds?

Solution.—Joules = volts × amp. × sec. $= 110 \times 10 \times 1{,}800 = 1{,}980{,}000$, and ft.-lb. $= .737 \times$ joules $= .737 \times 1{,}980{,}000 = 1{,}459{,}260$. Ans.

38. Mechanical and Electrical Power.—Since 1 foot-pound equals 1.356 joules, 1 foot-pound per second equals 1.356 joules per second, or watts. One horsepower equals 550 foot-pounds per second, or 550×1.356 equals 745.8, or approximately 746, watts; then,

$$\text{H. P.} = \frac{watts}{746} = \frac{volts \times amperes}{746}.$$

Example 1.—What is the horsepower equivalent of 400 amperes at 2,200 volts?

Solution.— $\text{H. P.} = \frac{2{,}200 \times 400}{746} = 1{,}179.6$, approx. Ans.

EXAMPLE 2.—If a motor takes 94.7 amperes from a 220-volt circuit and converts 89.5 per cent. of the electric power received into mechanical power, what is the output in horsepower?

NOTE.—As commonly expressed, the motor efficiency is 89.5 per cent.

SOLUTION.—The power input in watts is $94.7 \times 220 = 20{,}834$ and the output in watts is $20{,}834 \times .895 = 18{,}646.4$. The output in horsepower is $18{,}646.4 \div 746 = 25$, nearly. Ans.

The work can be indicated thus: $\frac{220 \times 94.7 \times .895}{746} = 25.$

EXAMPLE 3.—If a motor does work at the rate of 17 horsepower, that is, the motor output is 17 horsepower, and this is only .875 of the electrical power received by the motor, how much current does it take from a 220-volt circuit?

SOLUTION.—As only .875 of the electrical-power input is converted into mechanical-power output, the input in horsepower is $17 \div .875$, or $\frac{17}{.875}$. Then, according to the equation, $\text{H. P.} = \frac{\text{volts} \times \text{amp.}}{746}$, or $\frac{17}{.875} = \frac{220 \times \text{amp.}}{746}$; $\text{amp.} = \frac{17 \times 746}{.875 \times 220} = 65.9$. Ans.

EXAMPLES FOR PRACTICE

1. A large room is lighted by 12 fixtures each containing 7 lamps. The current required to light each lamp is .5 ampere from a 110-volt circuit. What is the power, in watts, when all the lamps are lighted? Ans. 4,620

2. At 3 cents ($.03) per kilowatt-hour, what is the cost of lighting all the lamps referred to in example 1 for 1,000 hours? Ans. $138.60

3. At 3 cents per kilowatt-hour, how much could be saved in 1,000 hours by substituting for the fixtures in example 1, 24 tungsten lamps requiring $1\frac{4}{11}$ amperes each from a 110-volt circuit? Ans. $30.60

4. If the work equivalent to lifting 10,000 pounds 200 feet were converted into heat, how much would it raise the temperature of $16\frac{2}{3}$ pounds (2 gallons) of water? Ans. 154.2° F.

5. An electric heater taking 5 amperes at 110 volts is immersed in $8\frac{1}{3}$ pounds of water. How many seconds will be required to raise the temperature of the water from 32° F. to 180° F.? Ans. 2,366−, approx.

NOTE.—Find the number of British thermal units as in Art. **34**, and the number of joules as in Art. **36**, and remember that joules = watt-seconds.

6. If an engine works at the rate of 120 horsepower to drive an electric generator, and the generator converts .9 of this power into electricity at 220 volts, what is the current in the generator? Ans. 366 amp., approx.

7. How many horsepower must a waterwheel develop to drive an electric generator that delivers 1,000 amperes at 500 volts if .92 of the mechanical power delivered to the generator is converted into electric power? Ans. 729

8. At 4 cents ($.04) per kilowatt-hour, what will be the yearly cost of electric energy to run a motor driving the machines of a wood-working plant if the motor develops an average of 6 horsepower for 2,500 hours per year and converts .8 of the electric power received into mechanical-power output? Ans. $559.50

ELECTRIC CIRCUITS

CIRCUIT TERMS

39. As already explained, an electric circuit is *closed* if it affords a continuous path for the flow of electricity and *broken*, or *open*, if the path is only partly complete. Thus, when an electric lamp is switched on, the circuit through the lamp is closed and the lamp lights; when switched off, the circuit through the lamp is open and no light is given off. An electric circuit is said to be *grounded* when any part of it is in electrical connection with the earth or with a conductor leading to the earth. Thus, when an electric-light wire rubs against a water pipe until the insulating cover on the wire is worn through so that the copper wire touches the pipe, the lighting circuit is grounded to the water pipe.

The *external part* of a circuit is the part outside of the source of electromotive force, or external to it; the *internal part* is the part within the source of electromotive force, as illustrated by the voltaic cell.

SERIES GROUPING

40. Electric conductors, sources of electromotive force, or devices using electricity can be connected in several ways. When the connections are such as to form only one path for the flow of electricity, the connection, or grouping, is in **series.**

For example, Fig. 4 represents a closed circuit consisting of simple voltaic cell *B* and four conductors *a*, *b*, *c*, and *d* connected in series; that is, electricity can flow from the positive terminal of the cell to the negative terminal through only one path. An opening in any part of this series circuit would prevent all flow of electricity.

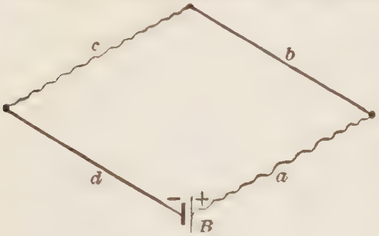


FIG. 4

41. Sources of electromotive force are connected in series when their terminals are joined so that the voltages are added. Fig. 5 represents six voltaic cells connected in series; the positive terminal of each cell is joined with the negative terminal of the next cell in the series. The circuit is represented open and the arrows show the direction of electromotive force, or the direction in which electricity would flow if the circuit were closed. In this case, there would be only one path for the current, and the total electromotive force acting in the path, or circuit, would be the sum of the electromotive forces of all the six cells.

Figs. 4 and 5 serve to show the usual, or conventional, method of representing electric-battery cells; a short heavy line represents the negative terminal of a cell and a longer lighter line the positive terminal.

42. If the terminals of one cell, Fig. 5, were interchanged so that its positive terminal were joined to the positive terminal of another cell and its negative terminal to another negative terminal, this cell would not be in true series with the others, but in opposition to them. Its voltage would then be deducted from the sum of the voltages of the others. For true series connection, sources of electromotive force must not only be in the same circuit but must agree in direction.

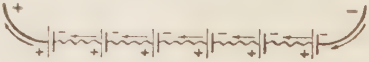


FIG. 5

43. Resistance of Series Group.—The resistance of several conductors connected in series is equal to the sum of the resistances of the conductors.

For example, if, in Fig. 4, the resistances of conductor $a=8$ ohms, $b=12$, $c=22$, and $d=34$, then the four conductors in series have a resistance of $8+12+22+34=76$ ohms, which is the external resistance of the circuit. If the internal resistance between the terminals of the battery B is 5 ohms, the total resistance of the circuit is $76+5=81$ ohms.

PARALLEL GROUPING

44. A circuit that is divided into two or more branches, each branch transmitting part of the current, is a **derived,** or **shunt, circuit,** and the separate branches are said to be connected in **parallel,** or **multiple.** Fig. 6 shows a closed divided circuit consisting of a source of electromotive force B, and conductors a, b, c, and d. Conductors b and c afford two current paths between conductors a and d, and are therefore in parallel. Either path b or c could be broken without interfering with the flow of electricity through the other path, because each is independent of the other. The arrows indicate direction of current.

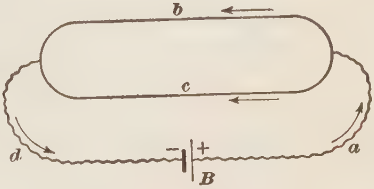


FIG. 6

45. Sources of electromotive force are connected in parallel, or multiple, when all their positive terminals are joined to one conductor and all their negative terminals to another conductor, as in Fig. 7. The electromotive force between the two conductors is the same as that of any one of the equal sources. The arrows indicate the directions of the electromotive forces, or the direction in which electricity would flow if the external circuit were closed. The current in the external circuit would then be equal to the sum of all the currents from the several sources of electromotive force. For example, if the six sources of electromotive force indicated in Fig. 7 were equal,

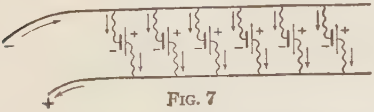


FIG. 7

and if the current through each source were 2 amperes, the current in the external circuit, when closed, would be 12 amperes.

46. Resistance of Similar Conductors in Parallel Groups.—In Fig. 6, the two paths *b* and *c* together are larger, and hence lower in resistance, than either path would be alone. Joining in parallel two conductors of the same material and of the same size and length is equivalent to making one conductor twice as large as either one of the two joined, and doubling the size of the conductor divides its resistance by two.

In a like manner, adding the third conductor of the same size and material would make the combination three times as large as any one of the three, and the resistance one-third as much. When several conductors of the same size, length, and materials are joined in parallel, the combined resistance of all of them is equivalent to the resistance of each conductor divided by the number joined. By formula,

$$R=\frac{r}{n}$$

in which R = combined resistance of a number of equal conductors joined in parallel;
r = resistance of each conductor;
n = number of conductors.

EXAMPLE.—What is the combined resistance of a group of 11 incandescent lamps in parallel, if the resistance of each lamp is 220 ohms?

SOLUTION.—By the formula, $R=\frac{220}{11}=20$ ohms. Ans.

47. Conductance.—The conductance of a substance is its ability to conduct electricity. Conductance is the reciprocal of resistance and is usually expressed in units designated by the word **mho**, which is *ohm* spelled backwards. If the resistance of a conductor is 10 ohms, its conductance is $\frac{1}{10}$ mho; if the resistance is 25 ohms, the conductance is $\frac{1}{25}$ mho; if the resistance is $\frac{1}{2}$ ohm, the conductance is 2 mhos; if the resistance is r ohms, the conductance is $\frac{1}{r}$ mhos; and so on. Conversely, the resistance is the reciprocal of the conductance. Thus, if the conductance is $\frac{1}{r}$ mhos the resistance is r ohms; if the conductance is r mhos the resistance is $\frac{1}{r}$ ohms.

48. Resistance of Unlike Conductors in Parallel Groups.—When conductors are joined in parallel, their individual conductances should be added to get the combined conductance. The reciprocal of this combined conductance is the resistance of the group. For example, if two conductors with resistances of 2 ohms and 3 ohms, respectively, are joined in parallel, their combined conductance is $\frac{1}{2}$ mho$+\frac{1}{3}$ mho $=\frac{5}{6}$ mho, and their combined resistance is the reciprocal of $\frac{5}{6}$, or $\frac{6}{5}$ ohms.

Let R = combined resistance of several conductors in parallel;
r, r_1, r_2, etc. = individual resistances;
G = combined conductance in parallel.

Then,

$$G=\frac{1}{R}=\frac{1}{r}+\frac{1}{r_1}+\frac{1}{r_2}, \text{ etc.} \qquad (1)$$

$$R=\frac{1}{G}=\frac{1}{\frac{1}{r}+\frac{1}{r_1}+\frac{1}{r_2}, \text{ etc.}} \qquad (2)$$

EXAMPLE 1.—Find: (*a*) the conductance, and (*b*) the resistance of four conductors in parallel, their individual resistances being $\frac{1}{3}$ ohm, 2 ohms, 3 ohms, and 6 ohms.

SOLUTION.—Here, $r=\frac{1}{3}$ and $\frac{1}{r}=3$, $r_1=2$ and $\frac{1}{r_1}=\frac{1}{2}$, $r_2=3$ and $\frac{1}{r_2}=\frac{1}{3}$, and $r_3=6$ and $\frac{1}{r_3}=\frac{1}{6}$.

(*a*) By formula **1**,

$$G=3+\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{6}=4 \text{ mhos.} \quad \text{Ans.}$$

(*b*) By formula **2**,

$$R=\frac{1}{G}=\tfrac{1}{4} \text{ ohm.} \quad \text{Ans.}$$

EXAMPLE 2.—Find the combined resistance of four conductors in parallel, their resistances in ohms being 5, 8, 16, and 80.

SOLUTION.—By formula **2**,

$$R=\frac{1}{\frac{1}{5}+\frac{1}{8}+\frac{1}{16}+\frac{1}{80}}=\frac{1}{\frac{32}{80}}=\frac{1}{\frac{2}{5}}=\frac{5}{2}=2\tfrac{1}{2} \text{ ohms.} \quad \text{Ans.}$$

PARALLEL-SERIES AND SERIES-PARALLEL GROUPING

49. When devices are connected in several series and these series are connected in parallel, as in Fig. 8, the devices are said to be connected in **parallel series,** or **multiple series.** If the devices are battery cells, as indicated, each series should contain the same number of similar cells; the positive terminals of all the series are connected to one conductor and the negative terminals to the other. The electromotive force between the two conductors is the same as that of any series, and if the external circuit were closed the current would be the sum of the currents in all the series.

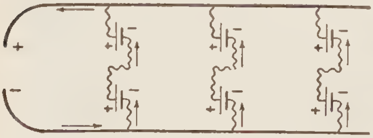


FIG. 8

50. Series-parallel connection means literally a *series of parallels*, as indicated in Fig. 9, in which three groups *a*, *b*, and *c* are connected in series, each group having four devices in parallel.

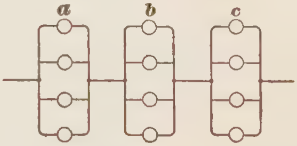


FIG. 9

51. The expressions *parallel series* and *series-parallel* are frequently used without distinction as to meaning, and the distinction is not usually of great importance in practice. Circuits including both methods of connection are sometimes used, as in Fig. 10, in which each part (*a*) and (*b*) is a parallel-series connection and the two parts connected in series form part of a series-parallel circuit.

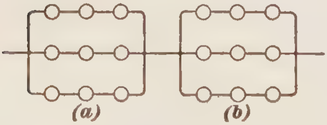

(*a*) (*b*)

FIG. 10

52. Resistance With Combination Grouping.—The combined resistance of devices or conductors grouped in parallel series or series-parallel is obtained by combining the processes already given for series and for parallel circuits. That is, add the resistances of devices in series and the conductances of devices in parallel.

For example, if each device in Fig. 9 has a resistance of 3 ohms, the conductance of each is $\frac{1}{3}$ mho and the conductance of the four devices in each group is $4\times\frac{1}{3}=\frac{4}{3}$ mho. The resistance of each group is therefore $\frac{3}{4}$ ohm, and of the three groups in series, $3\times\frac{3}{4}=\frac{9}{4}$, or $2\frac{1}{4}$, ohms. If the four devices in each group have the same resistance and no two groups have the same resistance, the resistance of each group can be found separately and the resistances of the three groups added to find the total resistance.

THREE-WIRE CIRCUITS

53. A **three-wire circuit,** or **system,** is an arrangement of three conductors in parallel, two called *outside conductors* and the third the *neutral conductor*. The voltage between the two outside conductors is double the voltage between the neutral and either outside conductor. Fig. 11 represents part of such a system with a pressure of 230 volts across the outside wires and one of 115 volts between the neutral and either outside wire. Such systems are much used for electric lighting. The lamps are connected between the neutral and the outside wires, as at l, and when electric motors are run from the same circuit they are usually connected across the outside wires, as at m.

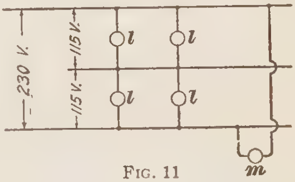


FIG. 11

If the devices are so distributed that the same current is taken from each side of the system, that is, each side of the neutral wire, the system is said to be **balanced.** The neutral wire then carries no current. This is the condition indicated in Fig. 11, in which the same number of lamps are shown connected with each side of the circuit. When the devices are so connected that more current is used on one side of the system than the other, the system is said to be **unbalanced.** The neutral wire then carries the unbalanced current, or the excess used on the more heavily loaded side.

OHM'S LAW

54. The substance of Ohm's law has already been given in the statement that flow of electricity increases with increased electromotive force and decreases with increased resistance. The law as applied to direct-current circuits can be easily remembered, thus: *The strength of a direct current in any circuit is equal to the quotient of the electromotive force acting in the circuit divided by the resistance of the circuit.*

Let I = current;
E = electromotive force;
R = resistance.

Then,

$$I=\frac{E}{R}$$

is the statement of Ohm's law by formula. If the electromotive force is in volts and the resistance in ohms, the current is in amperes; that is,

amperes = *volts* ÷ *ohms*

If any two of these three quantities are known, the third can be found by transposing the equation $I=\frac{E}{R}$ so as to obtain either $E = I\,R$ or $R=\frac{E}{I}$.

EXAMPLE 1.—If an electric lamp has a resistance of 220 ohms, what current will it carry when connected with a 110-volt circuit?

SOLUTION.—Here E = 110 volts and R = 220 ohms; then, by applying the formula,

$$I=\frac{110}{220}=\tfrac{1}{2}\text{ amp.}\quad\text{Ans.}$$

EXAMPLE 2.—What is the resistance of a 550-watt, 110-volt electric flat iron?

SOLUTION.—The first step is to find the current. As before explained, power in watts equals volts × amperes. Then, 550 watts = 110 × amp., and amp. = 550 ÷ 110 = 5. By Ohm's law, $5=\frac{110}{R}$ and, by transposition,

$$R=\frac{110}{5}=22\text{ ohms.}\quad\text{Ans.}$$

EXAMPLE 3.—What voltage is necessary to cause a current of 5 amperes in a device having a resistance of 16 ohms?

SOLUTION.—By Ohm's law, $5=\frac{E}{16}$, and by transposition $80=E$, or $E=80$ volts. Ans.

55. Drop, or Fall, of Potential.—Ohm's law applies whenever direct-current electricity flows. Every conductor has resistance, and electric pressure $E=I\,R$ is required to cause flow of electricity in it. If the current I is constant and the resistance R uniform, the pressure E is proportional to the length of the conductor; that is, the longer the conductor, the greater is the pressure necessary to maintain the current. If 10 volts is necessary to establish 50 amperes of direct current in a conductor 1,000 feet long and of uniform resistance throughout its length, 5 volts will be required to establish the same current in a similar conductor 500 feet long, and 1 volt for a similar conductor 100 feet long.

The electromotive force necessary to maintain current in a circuit is the **drop of potential**, or the *voltage drop*, in that circuit. For example, 50 amperes in the circuit from a generator to a motor may cause a 10-volt drop in the circuit, so that the voltage at the motor terminals is 10 less than that at the generator terminals. If the generator voltage is 230, the voltage at the motor, or the voltage drop in the motor, will be 220. The voltage drop in conductors can always be calculated if the current and the resistance of the conductors are known, for $E=I\,R$.

EXAMPLE 1.—If electric lamps requiring 25 amperes at 110 volts are located 100 feet from a source of electromotive force and the current is led through conductors having a total resistance of $\frac{1}{5}$ ohm, (*a*) what voltage must be maintained at the source? (*b*) what is the voltage drop per foot of conductor?

SOLUTION.—(*a*) The total drop $E=I\,R$, or $25\times\frac{1}{5}=5$ volts; the source of electromotive force must therefore be $110+5=115$ volts. Ans.

(*b*) The voltage drop per foot is $5\div200$ (100 feet each way) $=.025$ volt. Ans.

EXAMPLE 2.—With 12 amperes in a circuit of four parts with resistances as indicated in Fig. 12, (*a*) what is the voltage drop in each part? (*b*) what is the total voltage drop, or the voltage E?

SOLUTION.—(*a*) The voltage drop in *a* is $12\times\frac{1}{2}=6$; in *b*, $12\times\frac{1}{3}=4$, in *c*, $12\times\frac{1}{4}=3$; and in *d*, $12\times\frac{1}{6}=2$. Ans.

(*b*) The total voltage drop, or the voltage *E*, is $6+4+3+2=15$. Ans.

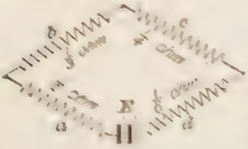


FIG. 12

EXAMPLE 3.—If an electric motor 500 feet from the source of electromotive force must be supplied with 5 amperes of current with a voltage drop of 12 in the line, (*a*) what must be the line resistance? (*b*) what is the resistance of each conductor?

SOLUTION.—(*a*) The line resistance $R=\frac{E}{I}=\frac{12}{5}=2.4$ ohms. Ans.

(*b*) The line has two conductors, and the resistance of each must be one-half of the total line resistance, or $2.4\div2=1.2$ ohms. Ans.

56. Divided Circuits.—If a circuit is divided into several branches in parallel, each branch will carry current in accordance with Ohm's law, and the total current in the circuit will equal the sum of the currents in the branches. A divided circuit with the conditions shown in Fig. 13 will carry current in the 5-ohm branch $\frac{80}{5}=16$ amperes, in the 10-ohm branch $\frac{80}{10}=8$ amperes, and in the 20-ohm branch $\frac{80}{20}=4$ amperes. The total current in the circuit will then be $16+8+4=28$ amperes.

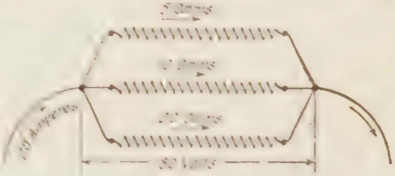


FIG. 13

The combined resistance of the three branches, Fig. 13, is found by adding their conductances and taking the reciprocal of the sum, as previously explained; namely, $\frac{1}{\frac{1}{5}+\frac{1}{10}+\frac{1}{20}}=\frac{1}{\frac{7}{20}}=\frac{20}{7}=2\frac{6}{7}$ ohms. The current set up by 80 volts in a $2\frac{6}{7}$-ohm circuit is $\frac{80}{2\frac{6}{7}}=80\times\frac{7}{20}=28$ amperes, which agrees with the number of amperes found by adding the currents in the three branches. The total current can therefore be found by either method.

EXAMPLES FOR PRACTICE

1. Two resistances of 30 and 20 ohms, respectively, are connected in parallel. Find: (*a*) their joint resistance; (*b*) their joint conductance.

Ans. $\begin{cases} (a) \ 12 \text{ ohms} \\ (b) \ \frac{1}{12} \text{ mho} \end{cases}$

2. Three groups of lamps are connected in series, each group consisting of four lamps of 220 ohms each in parallel; what is the total resistance across the three groups? Ans. 165 ohms

3. What voltage must be impressed upon a 22-ohm circuit in order to establish a current of 5 amperes? Ans. 110 volts

4. It is desired to limit the current through a 70-ohm device to 2 amperes from a 220-volt circuit. What must be the value of the additional resistance connected in series with the device? Ans. 40 ohms

5. Four resistances of 3, 5, 9, and 15 ohms, respectively, are connected in parallel across a 450-volt circuit. Determine: (*a*) the current through each branch; (*b*) the total current.

Ans. $\begin{cases} (a) \ 150, 90, 50, \text{ and } 30 \text{ amp.} \\ (b) \ 320 \text{ amp.} \end{cases}$

ELECTROSTATICS

ELECTRIC CHARGES

57. When a glass rod is electrified by rubbing it with silk, the charge resides on only the parts rubbed; if these parts are touched with anything through which electricity can pass to the ground, the charge instantly disappears. The same statements are true of charges established in any other way, as by rubbing wax with flannel or by the friction of leather belts on iron pulleys.

58. When an electrified glass rod is held near an insulated piece of light material, as a pith ball, Fig. 14, the material will be attracted; if allowed to touch the glass the ball will promptly swing away from it and will recede whenever the glass is brought near. If another pith ball is treated in the same way, the two balls will repel each other.

59. Sealing wax electrified by rubbing it with flannel will attract light pieces that have been repelled by the charge on the glass when rubbed with silk. For example, if the electrified wax is brought near the pith ball, Fig. 14, that has been charged and repelled by the glass, the ball will be attracted to the wax. The charge on the wax is therefore unlike that on the glass.

60. Two kinds of electric charges, or of static electricity, are thus illustrated. The names *positive* and *negative* are used to designate these charges. Neither can be developed without the other. The charge developed on glass when it is rubbed with silk is positive, and an equal negative charge is at the same time developed on the silk. A negative charge is developed on wax when it is rubbed with flannel, and an equal positive charge is simultaneously developed on the flannel.

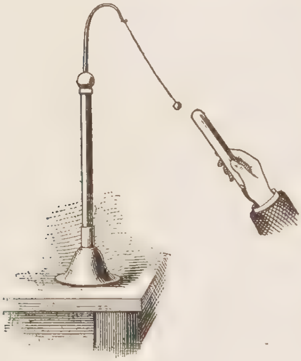


FIG. 14

61. Electrostatic Laws. Electric charges act according to the following laws:

1. When two dissimilar unelectrified substances are brought into contact, one assumes a positive and the other a negative condition.
2. An unelectrified body on coming into contact with an electrified body becomes electrified with a charge similar to that on the electrified body.
3. Similarly charged bodies repel each other; dissimilarly charged bodies attract each other.

62. Electric Series.—The accompanying list of substances, called the *electric series*, is arranged so that each substance is positive to all that follow it in the list and negative to all that precede it; that is, any substance receives a positive charge when rubbed or brought in contact with a substance the name of which occurs later in the list and a negative charge when rubbed with substances named earlier

ELECTRIC SERIES

1. Fur
2. Flannel
3. Ivory
4. Crystals
5. Glass
6. Cotton
7. Silk
8. Leather
9. The body
10. Wood
11. Metals
12. Sealing wax
13. Resins
14. Gutta percha
15. Guncotton

in the list. For example, *glass* when rubbed with *fur* receives a *negative* charge, but when rubbed with *silk* receives a *positive* charge. In some cases, the charges are so small as to be scarcely perceptible.

63. Electrostatic Induction.—If a charged body is brought near an insulated body, a charge will be induced on the latter body, as is proved by the experiment illustrated in Fig. 15. Two conductors *a b* and *c* are supported on insulating stands, several pairs of pith balls being suspended from conductor *a b*. When conductor *c* is charged and brought near conductor *a b*, the pith balls separate, as shown, demonstrating that a charge is induced on conductor *a b*. The two balls of an active pair have the same kind of a charge, and repel each other. When the two conductors are separated again, the balls fall to their original positions, showing that the charge has disappeared.

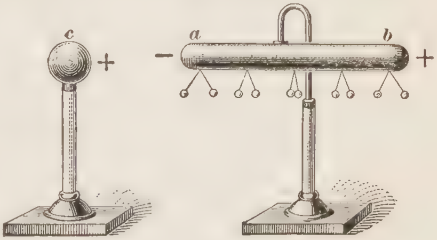


FIG. 15

When the two conductors are close together, the charge on conductor *c* attracts an opposite charge on the end *a* of conductor *a b* and repels electricity of the same kind to the end *b* farthest from *c*. The induced charge on conductor *a b*, being strongest at the ends, the pairs of balls at the ends separate more widely than those nearer the center. When conductor *c* is withdrawn, the two charges on conductor *a b* neutralize each other, and each pair of balls will again hang together. They can thus be made to separate and come together merely by moving a charged body close to conductor *a b* or away from it.

64. If two conductors, one heavily charged, are brought very near each other, the attraction between the opposite charges may become so strong that electricity in the form of a *spark* will jump through the space between the two bodies, exactly similar to a lightning discharge, except on a very small scale. In the case of lightning, opposite charges on adjacent clouds or on a cloud and the earth attract each other so strongly that a very large spark, or *flash*, of electricity jumps across the intervening air space, thus neutralizing some or all of the two charges.

When electricity passes through the insulating medium, for example, the air space separating charged conductors, the insulation is said to be *broken down*, or *punctured*. The insulation does not affect the influence of one charged body on another, but does prevent the discharge of electricity from one to the other until the attraction becomes strong enough to puncture the insulation. For example, a sheet of paper held between conductors *a b* and *c*, Fig. 15, does not influence the induction of a charge on *a b*; but a spark cannot pass between the two conductors until the attraction is strong enough to cause electricity to puncture the paper.

65. Static electricity sometimes accumulates on moving belts, owing to friction with pulleys, and it may become strong enough to cause slight shocks to persons near the belts or when touching the machines with which the belts are operating. In fact, if an electric generator or a motor in a dry room is insulated from the ground and belted, the charge may accumulate strong enough on the frame of the machine to puncture the insulation of the conductors. In order to avoid such accumulations, a grounded conductor can be placed where the belt will pass near it. The charge then escapes to the earth and is neutralized. Best results are obtained if metal points project from the conductor toward the belt; electricity escapes most readily to or from points. This fact explains why lightning rods and other conductors from which static electricity must escape are pointed.

CONDENSERS

66. An **electric condenser** is a device for accumulating, or condensing, electricity by the effect of electrostatic induction between conducting bodies separated by insulation.

An electric condenser is formed when any two conducting substances are separated by an insulating substance; for example, a glass plate with a sheet of tin-foil on each side is an electric condenser. The two metallic substances are termed the **plates**, and the insulating substance the **dielectric** of the condenser.

67. A wire strung on insulators and the earth or any conducting object connected with it are the two plates of a condenser, and the air or any other insulating substance separating the wire from the ground is the dielectric.

Subterranean or submarine cables are condensers; the conductor and the outer steel armor or lead covering are the two plates, and the insulating substance separating them, such as vulcanized rubber, paper, or jute, is the dielectric.

68. When electromotive force is applied to the terminals of a condenser, electricity flows into the condenser until the opposing, or counter, electromotive force of the condenser equals the applied electromotive force. The condenser is then said to be *charged*, and it will discharge through any circuit placed across its terminals. The **capacity** of a condenser has reference to the quantity of electricity that will flow into it with a given charging electromotive force. The voltage across the terminals of a condenser is a measure of the stress to which the dielectric is subjected, or the tendency to puncture the insulation. When a condenser is fully charged, the dielectric stress is equal to the charging voltage. A charged condenser is somewhat like a compressed spring; the spring tends to recoil as soon as it is released, and the charge on the condenser tends to escape the instant a discharge circuit is completed across its terminals.

69. Condensers are essential in some electric circuits. They are made by assembling in a mass, sheets of conducting material separated by a dielectric, alternate conductors being connected with each terminal of the condensers, as indicated in Fig. 16. The sheets of conducting material are indicated at p, the dielectric at i, and the terminals at T. Each terminal and the sheets connected with it form a plate of the condenser.

70. Long transmission lines have considerable capacity as condensers, and this fact must be considered when installing and operating such lines. Serious accidents have happened owing to the discharge from such lines after they are disconnected from the electric generator that charged them. Operators, assuming that the disconnected line is *dead*, proceed to handle it or some device electrically connected with it and may receive fatal shocks due entirely to static electricity. This danger can arise only with high-voltage lines; therefore, after such lines are disconnected from the generator, they should be thoroughly grounded before handling them.

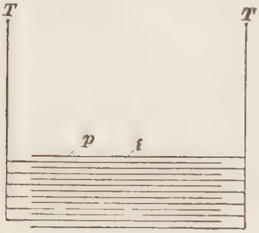


FIG. 16

ELECTRICITY AND MAGNETISM

(PART 2)

PERMANENT MAGNETISM

MAGNETS AND MAGNETIC PROPERTIES

1. A **natural magnet** is a piece of ore (a natural substance containing a mineral) that has the property of attracting pieces of iron, steel, and a few other metals. This ore was first discovered in the province of Magnesia, Asia; the peculiar property was therefore called *magnetism*, and the name *magnet* was applied to a piece of ore possessing the property.

Later the discovery was made that if such magnets were suspended so that they could turn freely, all would come to rest in positions pointing north and south. Small bars of the ore were thus used to guide ships over the seas. They were therefore called *lodestones* (leading stones), a name that is also applied to the ore. These lodestones were thus the forerunners of the modern compass.

2. A bar or a needle of hardened steel rubbed with a lodestone acquires properties similar to those possessed by the lodestone, and is called an **artificial magnet.** Artificial magnets that retain the characteristics of the lodestone for a considerable period of time, are called **permanent magnets.** Fig. 1 shows a common form of permanent magnet, consisting of a bar of steel bent into the shape of a horseshoe and then hardened and magnetized. A piece of soft iron called an *armature*, or *keeper*, placed across the two free ends, helps to

COPYRIGHTED BY INTERNATIONAL TEXTBOOK COMPANY. ALL RIGHTS RESERVED

retain the magnetism. Artificial magnets are also made in the form of a straight bar, as shown in Fig. 2.

3. If a bar magnet is dipped into iron filings, the filings are attracted toward the two ends and adhere there in tufts, as in Fig. 3, while toward the center of the bar no such tendency is noticeable. That part of the magnet where no apparent magnetic attraction exists is called the **neutral region,** and the parts around the two ends are called **poles.** An imaginary line drawn through the center of the magnet from end to end, connecting the two poles is called the *axis of magnetism.*

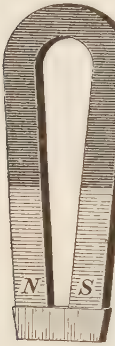


FIG. 1

4. A magnetized steel needle of the form shown in Fig. 4, resting on a fine point so as to turn freely in a horizontal plane, is called a **compass.** When not in the vicinity of iron, steel, or other magnets, such a needle comes to rest with one end pointing toward the north and the other toward the south. The end pointing northwards is therefore called the *north pole*, and the opposite end is called the *south pole*, meaning, respectively, *north-seeking* and *south-seeking*. Every magnet has two poles, and the letters N and S appearing on illustrations of magnets are to indicate north and south polarity.

FIG. 2

5. Magnetic Attractions and Repulsions.—A general law applying to all magnets is that *like magnetic poles repel each other while unlike poles attract each other.* If two bar magnets are suspended so as to swing freely and are brought near together, the repulsion between like poles and the attraction between unlike poles will cause them to turn until unlike poles are as near each other as possible.

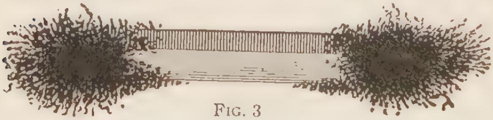


FIG. 3

The earth is a magnet whose magnetic poles coincide nearly, but not quite, with the geographical poles. The north geographical pole may be considered as a south magnetic pole and the south geographical pole as a north magnetic pole. A suspended magnetic needle will point north and south, due to magnetic attractions and repulsions. If a bar magnet is broken into any number of parts, as in Fig. 5, each part will still be a magnet and have two unlike poles.

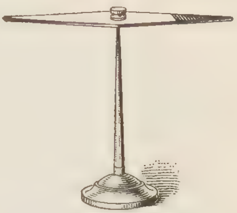


FIG. 4

6. Induced Magnetism.—Magnetism can be induced in a piece of unmagnetized iron by simply bringing a magnet near it. For example, if an unmagnetized bar of iron is laid on a sheet of paper, as in Fig. 6, iron filings sprinkled over it will show no tendency to cling to it, but if a horseshoe magnet is brought into the position indicated and the paper is gently tapped to jar the filings, they will cling to the ends of the bar as if it were a magnet. This is due to magnetism induced in the bar by the magnet.

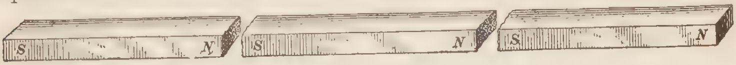


FIG. 5

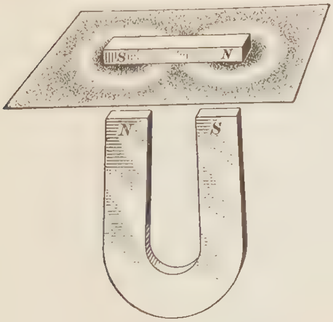


FIG. 6

Whenever a magnet is brought near iron, poles are induced in the iron adjacent to the poles of the magnet, but unlike them; a north pole is induced in the iron adjacent to the south pole of the magnet, and a south pole in the iron adjacent to the north pole of the magnet. Since unlike poles attract each other, the iron and the magnet show mutual attraction; that is, the iron is drawn toward the magnet and the magnet toward the iron.

7. Lines of Force and Magnetic Field.—Iron filings sprinkled on a sheet of cardboard laid over a bar magnet, as shown in Fig. 7, will arrange themselves in lines that seem to issue from each pole in all directions and to curve until they enter the other pole. If the magnet is held endwise to the cardboard, the filings will arrange themselves as shown in Fig. 8. In both cases, gently tapping the cardboard will assist the filings to arrange themselves.

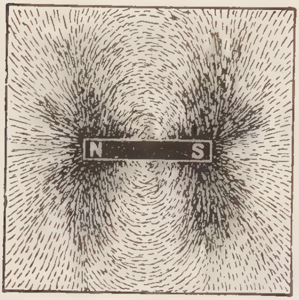


Fig. 7

8. The experiment illustrated in Fig. 9 also shows the action of magnetic force surrounding a magnet. A steel needle is magnetized by rubbing one end on one pole of a bar magnet and the other end on the other pole; the needle is then thrust through a cork so that the ends project, as at *n s*. The bar magnet is laid in the bottom of a glass or earthenware vessel (not metal), the vessel is partly filled with water and the cork is floated on the water so that one end of the needle will project toward a pole of the magnet without touching it.

If the cork does not move, this fact indicates that adjacent poles of the needle and bar are unlike. The cork should then be inverted so as to bring like poles near each other, when the needle and cork will promptly move in a curved path, approximately as indicated by the arrows, until near the other pole of the magnet. The shape of this path depends on the position in which the cork is placed relative to the magnet pole, being different when placed directly over the magnet than when placed so that the needle projects just beyond or at one side of the end of the bar.

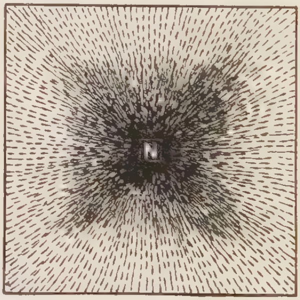


Fig. 8

9. Lines of force are the paths along which magnetic forces appear to act. Lines of force exist within and about all magnets as well as conductors that are carrying an electric current. The word magnetism is frequently used in referring to all the lines of force collectively. These lines of force must not be conceived as invisible threads, but merely as directions in which the forces act. The space in which magnetic forces act around a magnet is called its **field.**

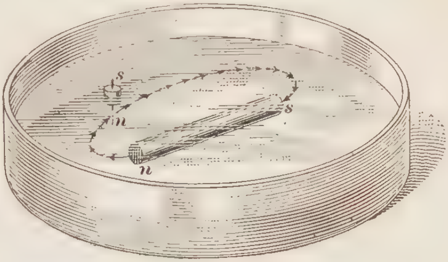


FIG. 9

Any magnetic substance, as iron or steel, in this field is influenced by the magnet and, in turn, influences the magnet. The strength of the field or the influence it can exert decreases as the distance from the magnet increases. The total magnetism, or all the lines of force collectively, surrounding a magnet is

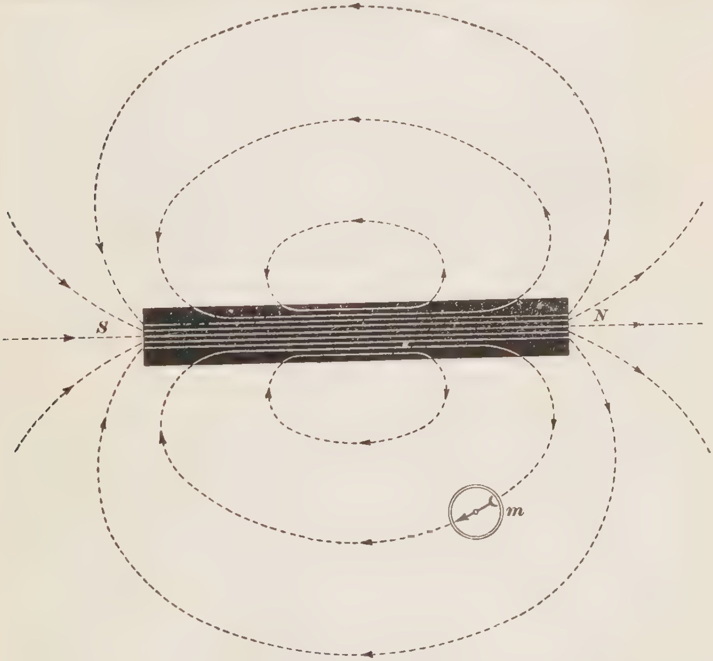


FIG. 10

called the *flux* of the magnet, or the **magnetic flux.** The name *maxwell* has been adopted for one line of force, but it is not in universal use.

10. Direction of Lines of Force.—If a bar magnet is surrounded by air, the magnetic forces act in curved paths, connecting the poles, as indicated in Fig. 10, in which only a few lines of force are represented. The common assumption is that the forces act from a north pole and toward a south pole; that the lines of force pass out from the north pole, through the surrounding air, in at the south pole, and through the magnet to the north pole. This is called the *direction of the lines of force*, and the complete path is called the **magnetic circuit.**

11. The direction of lines of force in a magnetic field can be tested by means of a small, freely suspended magnet, as the needle of a compass. The needle always turns until the direction of the lines of force within its body coincides with the direction of the lines of force in the field, as at *m*, Fig. 10. The north pole of the needle always points in the direction of the lines of force. Lines of force are usually represented by dotted lines and their direction by arrowheads on the dotted lines.

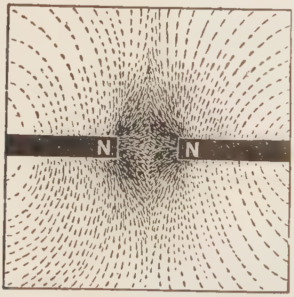


FIG. 11

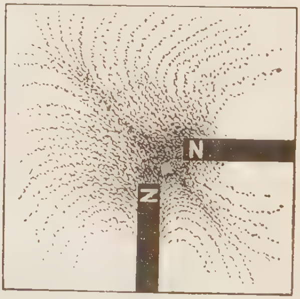


FIG. 12

12. Consequent Poles.—Iron filings on a piece of paper or cardboard over the like poles of two magnets will arrange themselves as in Figs. 11 and 12. The lines of force are thus shown to be distorted from their natural position and to be crowded into new positions, depending on the strengths of the two poles, thus forming what are called *consequent poles*. The lines of force never cross one another, but always change direction to coincide with other lines of force in the same magnetic field.

13. Magnetic and Non-Magnetic Substances.—Substances or materials in which magnetism is easily established are called *magnetic*. Iron and steel are the most important magnetic materials; nearly all other materials offer high opposition to the establishment of magnetism and are called *non-magnetic*.

MAGNETIC CIRCUITS

14. A magnetic circuit may be *closed*, *compound*, or *non-magnetic*. It is **closed** when a complete path for the lines of force is provided through magnetic materials, as in Fig. 1; it is **compound** when the path includes both magnetic and non-magnetic materials, as is the case with a bar magnet; and it is **non-magnetic** when it includes no magnetic materials.

15. The **length** of a magnetic circuit is the length of the average path of the lines of force. The length of a closed magnetic circuit can be easily approximated by measurement. In the case of a bar magnet, the length of the average path is hard to determine, for the lines spread widely in the part of the path through air. If the circuit is nearly all magnetic, as in Fig. 13, the average path will be approximately as indicated by the dotted line.

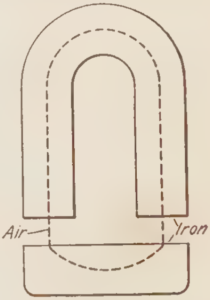


FIG. 13

16. The **cross-section of a magnetic circuit** is a plane at right angles to the lines of force; it is the surface that would be formed by cutting squarely across the path of the lines of force. Such a section may be considered at any point in the circuit; if it is in a magnetic part, it is simply a section of the iron or steel of which the part is composed. For example, if a bar magnet were cut squarely in two, crosswise of its length, either surface thus cut would be a section of the magnetic circuit at that point.

In short air gaps, as in Fig. 13, the lines of force do not spread much when leaving the iron, and a section of the magnetic circuit in the air gap is therefore approximately the same as through the adjacent iron. In longer air paths, as in Fig. 10, the lines of force spread widely and a section of the magnetic circuit is indefinite.

The area of a section, or the *sectional area*, is easily determined with reasonable accuracy in magnetic materials or in short air gaps, but it is difficult to determine in the longer paths through a non-magnetic substance. For example, if a bar magnet is 1 inch thick and 2 inches wide, the cross-sectional area of the magnetic circuit in the bar is 2 square inches.

17. The **magnetic flux density** in any section of a magnetic circuit is the number of lines of force, or maxwells, per unit area of the section. It may be expressed in lines of force per square inch or in maxwells per square centimeter. The unit *gauss*, meaning one line of force per square centimeter, is in some use, and density is sometimes expressed in gausses. The general custom in America, however, is to use lines of force per square inch or per square centimeter. Thus, 50,000 lines of force through a section of 2 square inches, or 12.9 square centimeters, is 25,000 lines per square inch, or approximately 3,876 lines per square centimeter.

18. The **reluctance** of a magnetic circuit is the opposition it offers to the passage of lines of force. Reluctance of a magnetic circuit may be likened to resistance of an electric circuit. Magnetic lines of force take the path of least reluctance, just as electricity seeks the path of least resistance. If the material of a magnetic circuit is homogeneous, or all of the same kind, the lines of force take the shortest possible path.

The reluctance of any magnetic circuit depends directly on its length and inversely on its sectional area; that is, the longer the circuit or the smaller its sectional area, the greater is its reluctance. The reluctance also depends on the materials of which the circuit is composed, being less for magnetic materials than for non-magnetic materials, and less for some magnetic materials than for others.

19. The **unit of reluctance** most generally accepted is the **oersted.** For practical purposes, 1 oersted may be defined as the reluctance between opposite faces of a cube of any non-magnetic material 1 centimeter on each edge.

20. The **reluctivity** of any material is the reluctance between opposite faces of a cubical unit of it. The reluctivity of all non-magnetic materials is practically 1 oersted.

21. In all magnetic materials, the reluctance is greater at high densities than at low densities; but non-magnetic materials have very nearly the same reluctance at all densities. The density in a magnetic substance reaches *saturation point* when it is so high that the reluctance increases very much with a further slight increase of density. Good soft steel and iron will carry more than 100,000 lines of force per square inch before reaching the saturation point, while hard steel or cast iron will carry only about one-half as many.

22. The **permeance** of a substance is the reciprocal of its reluctance, and refers to the readiness with which the substance permits the establishment of magnetic lines of force. A substance that has low reluctance has high permeance. The permeance of a magnetic circuit may be likened to the conductance of an electric circuit.

The **unit of permeance** is the permeance of a cubic unit, usually 1 centimeter cube, of non-magnetic material. No name has been generally accepted for this unit.

23. The **permeability** of a substance is the reciprocal of its reluctivity, and is the permeance of a cubical unit of the substance. For practical purposes, the permeance of a centimeter cube of non-magnetic material is considered as 1, and the permeability of other materials is stated as a number indicating how much more permeable, or less reluctant, it is than non-magnetic materials. Thus, the permeability of some grades of iron is 2,000, because its reluctivity is one-two-thousandth that of air.

24. Magnetomotive force (abbreviated m. m. f.) is the force or influence that overcomes reluctance and establishes the flux of magnetism; it is analogous to electromotive force in an electric circuit.

25. Comparison of Electric and Magnetic Circuits. Electric and magnetic circuits have many analogous features; the names of some of these features appear opposite each other in the following list:

Electric Circuit	Magnetic Circuit
Electromotive force	Magnetomotive force
Electric current	Magnetic flux
Resistance	Reluctance
Conductance	Permeance

ELECTROMAGNETISM

MAGNETIC EFFECT OF ELECTRIC CURRENT

26. Every conductor carrying an electric current is surrounded by a magnetic field. This field has its maximum density in the space next to the body of a conductor, the density decreasing as the distance from the conductor increases. The magnetism thus established by electric current is called **electromagnetism.** It is no different from the magnetism of a permanent magnet, except in the method of establishing it.

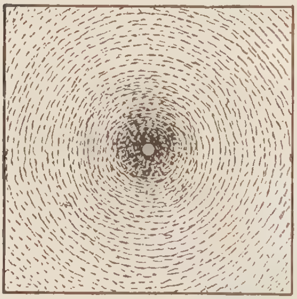


Fig. 14

If the conductor is threaded through a piece of cardboard and iron filings are sprinkled on the cardboard, they will arrange themselves in concentric circles around the conductor, as illustrated in Fig. 14. This effect will be observed throughout the entire length of the conductor; therefore, the field around any conductor carrying an electric current may be imagined as consisting of magnetic whirls, that entirely surround the conductor, as shown in Fig. 15. The condition shown in Fig. 14 can be shown by experiment, but that shown in Fig. 15 is inferred from the other.

27. Direction of Lines of Force Surrounding an Electric Current.—If a conductor carrying electricity in the direction indicated by the long arrow, Fig. 16, is held over a magnetic needle, the needle will deflect as indicated by the curved arrows toward a position at right angles to the direction of the conductor. If the conductor is then held under the needle, the needle will reverse its direction. Fig. 17 shows the indications of two compass needles, one above and one below a conductor carrying electricity, showing that the direction of the lines of force must be as indicated by the curved arrows. These experiments show that *lines of force encircle an electric current clockwise* when looking in the direction of current. This statement is important and should be remembered. The word *clockwise* means in the direction of movement of the hands of a clock or a watch and is made clear in Fig. 17.

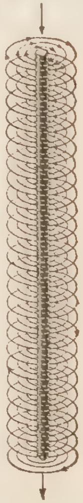


FIG. 15

28. In an actual experiment, the compass needles may not turn so nearly at right angles to the conductor as is indicated in Fig. 17, but they will approach the positions shown, their deflection depending on the strength of the current.

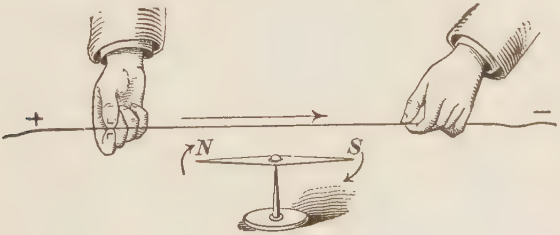


FIG. 16

Reversing the current will reverse the deflections of both needles, showing that the direction of the lines of force has reversed; but they still encircle the conductor clockwise to an observer facing the direction of the current.

29. The compass needle test can also be used to determine the direction of current in a conductor. If convenient, a portion of the conductor should be arranged in a north and south direction, parallel to the normal position of the needle. The compass should then be held near the conductor, and the needle will then indicate which way the magnetic flux encircles the conductor. If clockwise, looking along the conductor, the direction of current is from the observer; if counterclockwise, the direction of current is toward the observer. Fig. 18 (*a*) shows the compass under a conductor and (*b*) shows the compass above the same conductor, causing the needles to point in opposite directions; both positions indicate clockwise direction of flux looking north, showing that the direction of current must be north.

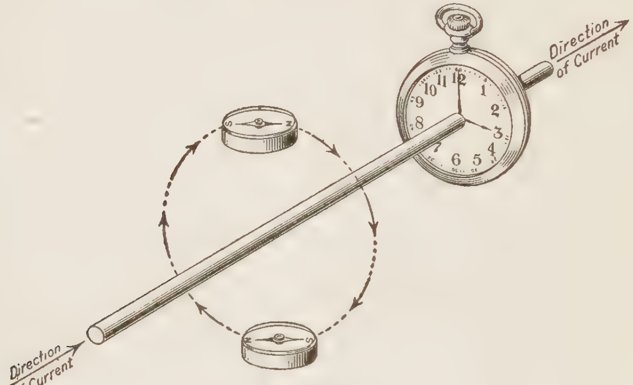
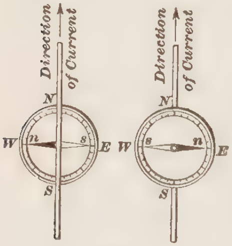


FIG. 17



(*a*) FIG. 18 (*b*)

By looping the conductor over and under the needle, as in Fig. 19, the deflection can be much increased, since the flux surrounding both conductors tends to deflect the needle in the same direction. By looping the wire several times around the needle, that is, by forming a coil with the needle inside, the turning effect is still further increased, so that a very small current will produce a considerable deflection.

30. Mutual Influence of Parallel Currents.—If two or more parallel conductors are carrying current in the same direction, the magnetic flux surrounding them tends to draw them together. The lines of force around each conductor tend to unite with those of the other, forming magnetic flux that encircles both the conductors, as in Fig. 20 (*a*), or all of them if there are several; the tendency of the lines of force to traverse the shortest possible path makes them act like elastic bands drawing the conductors together.

Two parallel conductors carrying current in opposite directions are acted on by a force tending to separate them. The lines of force, being opposite in direction, cannot unite around the conductors, but must all pass between them, as in Fig. 20 (*b*), thus tending to crowd them apart. In some electrical machines, the stresses caused in this way sometimes become enormous when the currents are large. In such machines the conductors must be fastened and braced very securely to hold them in place. For example, short circuits on large generators and transformers may work great injury by tearing the conductors out of place or crushing the insulation

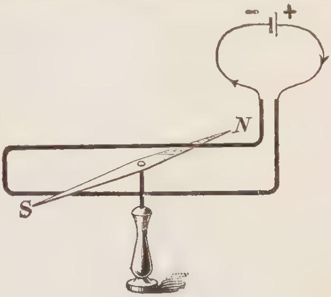
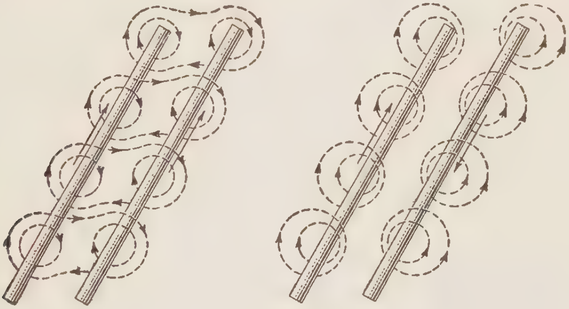


FIG. 19



(*a*) (*b*)

FIG. 20

ELECTROMAGNETIC SOLENOID

GENERAL THEORY

31. If electricity flows in a conductor bent into a loop, as in Fig. 21, all the magnetic flux will pass through the loop in the same direction. With current in the direction indicated, the loop acts similarly to a bar magnet with the north pole downwards through the loop.

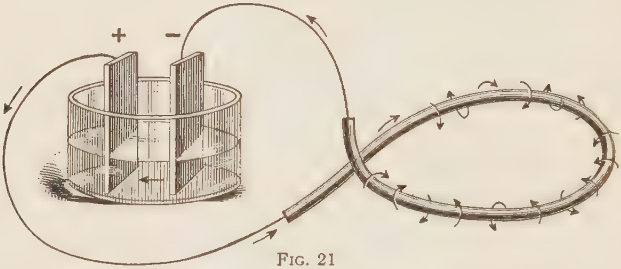


FIG. 21

A conductor coiled into a helix, as in Fig. 22, is called a **solenoid.** With electricity flowing in the direction indicated by the arrows at the conductor ends, lines of force thread through the solenoid, the magnetic circuit being indicated by the arrows forming loops. A solenoid in which electricity is flowing is an **electromagnetic solenoid,** or a *solenoid magnet,* with poles and attractions and repulsions similar to those of a bar magnet; if suspended so as to turn freely, such a solenoid assumes a position with its axis (the line through the centers of its several turns) in a north-and-south direction. The magnetic circuit of a solenoid magnet contains no magnetic material.

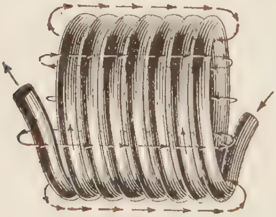


FIG. 22

32. The **polarity** of a solenoid magnet can be determined by the method illustrated in Fig. 17. The lines of force around the conductor combine in paths indicated by the loops, Fig. 22. The compass indicates the direction of the lines of force of these loops. With the direction of the lines of force as indicated in Fig. 22, the north pole is at the left and the south pole at the right end of the solenoid magnet. The polarity can also be tested by holding a compass needle or a bar magnet near the end of the solenoid, remembering that unlike poles attract each other. Still another test is to suspend the solenoid so that it can swing freely to a north-and-south direction.

33. The direction of current in a solenoid magnet can be determined, when the polarity is known, as indicated in Fig. 23; namely, *the direction of current is clockwise when facing the south pole of the magnet.*

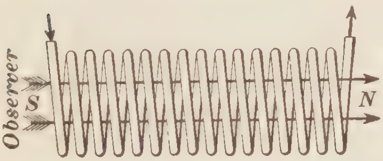


FIG. 23

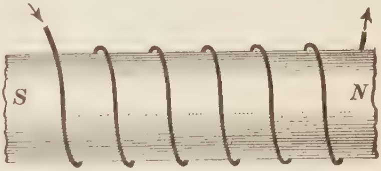


FIG. 24

ELECTROMAGNETS

34. A solenoid magnet with a core of magnetic material is an **electromagnet**. Fig. 24 shows the essential principles; the core is usually made of highly permeable material, such as soft iron or steel. The solenoid is generally called either the *exciting coil* or the *magnetizing coil;* it is made of insulated wire so that the current will be confined to the intended path around the core, instead of escaping to the core itself.

35. Electromagnets are made in the *bar form* indicated in Fig. 24; in the *horseshoe form*, Fig. 25; in the *iron-clad form*, Fig. 26; and in various modifications of these forms. The form shown in Fig. 25 is provided with two cores *M* and a *yoke b*. The coils are wound on insulating spools and slipped over the cores; one coil *c* is shown in section. The iron-clad form, shown partly in section in Fig. 26, is provided with both an inner core *M* and an outer protecting shell *S*.

36. The presence of iron or steel in the magnetic circuit greatly reduces the reluctance of the circuit and increases the number of lines of force established by a given magnetomotive force; and the more nearly the magnetic circuit is closed through magnetic material the less is the reluctance. An electromagnet in the form shown in Fig. 25 is therefore lower in reluctance than one in the straight-bar form, Fig. 24; the iron-clad form, Fig. 26, can be made of less reluctance than either of the others, because its magnetic circuit is nearly all iron. The reluctance of the form shown in Fig. 25 could be decreased by the addition of an iron armature connecting the free ends of the two cores.

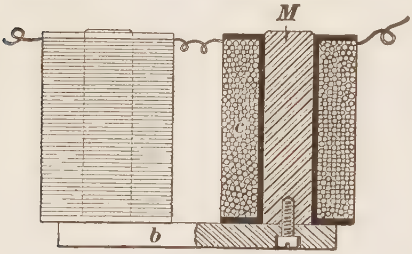


Fig. 25

37. The polarity of an electromagnet and the direction of current in its exciting coil can be determined by the methods previously described. For example, if the bar-type electromagnet, Fig. 24, is held so that the current encircles it clockwise, the end toward the observer is the south pole; or if the polarity is first determined by some other test, as with a compass, and the observer faces the south pole, the direction of the current is clockwise. When facing the poles of a horseshoe electromagnet, Fig. 25, the current must encircle one pole clockwise and the other counter-clockwise in order that one shall be a south pole and the other a north pole. This fact must be observed when connecting the coils of electromagnets.

38. Comparison of Permanent Magnets and Electromagnets.—Both permanent magnets and electromagnets have poles, magnetic fields, and similar attractions and repulsions. Permanent magnets are made of hardened steel, which cannot be magnetized to a very high degree but which holds its magnetism

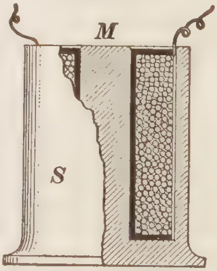


Fig. 26

for years. Electromagnet cores are made of soft iron or of steel, which can be magnetized to a very high degree while the magnetomotive force is applied, but which loses nearly all its magnetism as soon as current ceases in the magnetizing coil.

39. The magnetism retained by a magnet after the magnetomotive force ceases is called **residual magnetism.** By reversing the magnetomotive force, that is, by establishing a magnetomotive force of proper strength in the reverse direction, the residual magnetism can be removed, and the magnet is said to be **demagnetized.** The magnetomotive force necessary to demagnetize a magnet is a measure of its *coercive force*, or power of resisting magnetization or demagnetization. **Permanent magnetism** is residual magnetism in materials having high coercive force. The coercive force of magnet cores depends on the quality of iron or steel and especially on its degree of hardness. Both high permeability and low coercive force are desirable in electromagnet cores for most purposes, and these qualities are found in wrought iron and in soft steel.

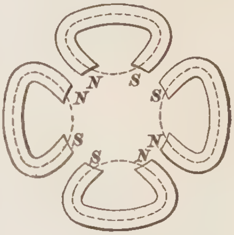


FIG. 27

Magnets can be demagnetized by subjecting them to rapidly reversing magnetomotive force and gradually decreasing this force. Watches in the pockets of those who operate electric machinery sometimes become *magnetized*, that is, some of the steel parts become permanent magnets, thereby injuring the time-keeping ability. This magnetism can be removed by suspending the watch at one end of a string, twisting the string tightly, holding the rapidly whirling watch near a strong magnet, and then gradually withdrawing it from the magnet while it is still whirling. All this is simply to reverse rapidly the magnetism and at the same time weaken it until entirely removed.

40. Generator and Motor Magnets.—The frames, pole pieces, and field coils of direct-current generators and motors form electromagnets. Fig. 27 shows four horseshoe permanent magnets, so placed as to approach very nearly the form of magnet used in street-railway motors. The general shape of the magnetic circuit of each magnet is indicated by the dotted lines. In Fig. 28 the adjacent legs of these horseshoe magnets are merged together, but the dotted loops show that the four magnetic circuits are still complete in themselves. Fig. 29 is the outline of a section through the magnet frame of a street-railway motor. Four magnetizing coils are so connected that the path of the current is around adjacent poles in opposite directions, as can be seen by following the arrowheads on the full lines representing the wires encircling the poles. The poles are thus alternately north and south, the lines of force passing across between the poles, as indicated by the dotted lines with arrowheads, and completing their circuit by way of the magnet cores and the circular yoke inside which the poles are attached.

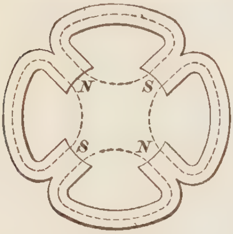


FIG. 28

41. Soft cast steel is extensively used for the magnet frames, or yokes, of generators and motors, because it is much cheaper than wrought-iron forgings of the intricate shapes required. Cast iron of high grade is also sometimes used for the same purpose, though inferior in permeability and of higher coercive force than soft cast steel. Cast-iron pole cores are seldom advisable because of the large cross-section required to carry the necessary flux; cast-steel poles are sometimes bolted to cast-iron frames.

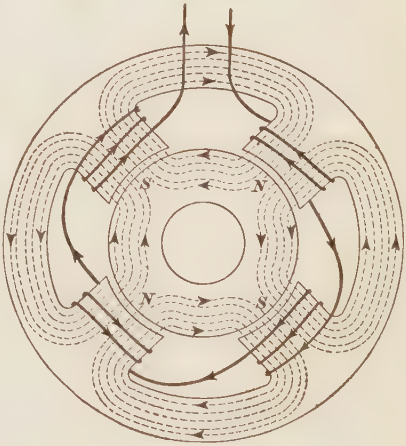


FIG. 29

Frame rings, or yokes, are also made by shearing strips from rolled-steel plate of the proper thickness, and then rolling and electrically welding the strips into rings. Such yokes have the advantages of uniform structure, high permeability, small coercive force, and great strength. The pole pieces for such frames are usually made of thin plates punched from large soft-steel sheets and assembled and riveted together in the required form, and bolted inside the frame rings. Such pole pieces have uniform structure and afford the best possible magnetic conditions.

42. Units of Magnetomotive Force.—The magnetomotive force of electromagnets is generally expressed in **ampere-turns,** which are the product of the number of amperes and the number of turns in the exciting coil. For example, 10 amperes in a coil of 100 turns, or 100 amperes in a coil of 10 turns, or 5 amperes in a coil of 200 turns, all give the same magnetomotive force, namely, 1,000 ampere-turns. A coil of 500 turns having a resistance of 10 ohms, when connected across a 50-volt circuit, would give $\frac{50}{10} \times 500 = 2{,}500$ ampere-turns of magnetomotive force.

43. Magnetomotive force may also be expressed in **gilberts;** 1 gilbert is the magnetomotive force that will establish 1 line of force, or 1 maxwell, through a reluctance of 1 oersted. One ampere-turn equals 1.257 gilberts, or the number of gilberts equals 1.257 times the number of ampere-turns. Gilberts are much less used, especially in America, than ampere-turns.

44. The **magnetic intensity,** or *magnetizing force,* in a magnetic circuit is the magnetomotive force per unit length of circuit expressed in *ampere-turns per inch* or *gilberts per centimeter.*

EXAMPLE.—A coil of 100 turns and 5.5 ohms is connected across a 110-volt circuit and used to excite a magnetic circuit 6.28 inches long. What is the magnetizing force in: (*a*) ampere-turns per inch, and (*b*) gilberts per centimeter?

SOLUTION.—(*a*) The magnetomotive force in ampere-turns is $\frac{110}{5.5} \times 100 = 2{,}000$, and the magnetizing force is $2{,}000 \div 6.28 = 318.5$ ampere-turns per inch. Ans.

(*b*) The magnetomotive force in gilberts is $2{,}000 \times 1.257 = 2{,}514$, and the length of the circuit in centimeters is $6.28 \times 2.54 = 15.95$; therefore, the magnetizing force is $2{,}514 \div 15.95 = 157.6$ gilberts per centimeter.

ELECTRIC CALCULATIONS

CROSS-SECTIONAL AREAS OF ELECTRIC CONDUCTORS

45. A **cross-section of a conductor** is the surface formed by cutting the conductor square across in a plane at right angles, or perpendicular, to its length. If a round, or cylindrical, conductor *a b*, Fig. 30, is cut squarely across, the end is a cross-section. If the conductor is solid, the cross-section is solid, as at *c*, view (*a*); if the conductor is tubular, the cross-section is hollow, as in view (*b*). A cross-section shows the shape of the conductor at the place where the section is taken. For example, if the section is a circle, as at *c*, the conductor *a b* is round wire either solid or tubular; if the section is a rectangle as at *f*, views (*c*) and (*d*), the conductor *d e* is a rectangular bar either in the form of a ribbon, or strip, as in (*c*), or a square, as in (*d*). Conductors in each of the forms shown are used.

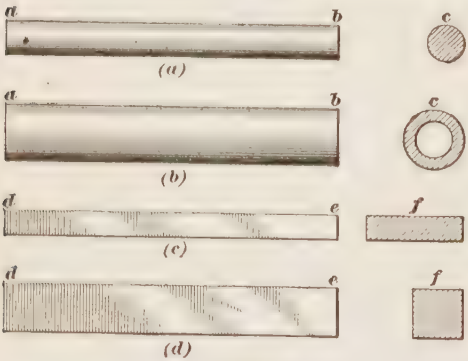


FIG. 30

46. The **mil** is a unit used to express dimensions of conductors and thickness of insulation. Thus, 1 mil = .001 inch; 1 inch = 1,000 mils; $\frac{1}{2}$ inch = 500 mils; 250 mils = $\frac{1}{4}$ inch; etc.

47. The **circular mil** (abbreviated c. m.) is a unit used to express sectional areas of conductors. The distance directly through the center of a round conductor is the diameter of the conductor, as indicated at *d*, Fig. 31, and this diameter is the same in all directions. *The sectional area of any cylindrical conductor, in circular mils, is the square of its diameter in mils.* Thus, if a wire is 100 mils in diameter, its sectional area is $100\times100=10{,}000$ circular mils; a wire 1 inch, or 1,000 mils, in diameter has a sectional area of $1{,}000\times1{,}000=1{,}000{,}000$ circular mils.

48. The sectional areas of rectangular conductors are also expressed in circular mils. Fig. 31 shows, in dotted lines, a square circumscribed around a circle. The side of this square is the same as the diameter of the circle, and the area of the square in square mils is therefore the square of the diameter in mils. The square is evidently larger than the circle, the area of the circle being .7854 times the area of the square. Then, to determine the sectional area in circular mils of a rectangular conductor, find its sectional area in square mils and divide by .7854; that is,

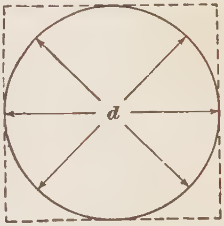


FIG. 31

$$\textit{circular mils}=\frac{\textit{square mils}}{.7854}$$

EXAMPLE 1.—Find the sectional area in circular mils of a ribbon-shaped conductor with a cross-section $\frac{1}{8}$ in. $\times\frac{3}{8}$ in.

SOLUTION.— $\frac{1}{8}$ in. $=.125$ in. $=125$ mils; $\frac{3}{8}$ in. $=.375$ in. $=375$ mils.

$$\text{Circular mils}=\frac{125\times375}{.7854}=59{,}683. \quad \text{Ans.}$$

EXAMPLE 2.—Find the sectional area in circular mils of a conductor with a cross-section 1 inch square.

SOLUTION.— 1 in. $=1{,}000$ mils.

$$\text{Circular mils}=\frac{1{,}000\times1{,}000}{.7854}=1{,}273{,}237. \quad \text{Ans.}$$

49. On the other hand, it is sometimes necessary to calculate one dimension of a rectangular conductor when the other dimension and the sectional area in circular mils are known.

This is done by finding the sectional area in square mils and dividing it by the given dimension in mils. By transposing the equation of Art. 48,

$$square\ mils = .7854 \times circular\ mils$$

Example—If a ribbon-shaped conductor must be 120 mils thick and must have a sectional area of approximately 46,000 circular mils, what must be the width of the section?

Solution.—Square mils $= .7854 \times 46{,}000 = 36{,}128$, and $36{,}128 \div 120 = 301.07$ mils. Ans.

Note.—A section 120 mils by 300 mils, giving a sectional area of 45,837 circular mils, would probably be used in such a case.

EXAMPLES FOR PRACTICE

1. What is the sectional area in circular mils of a conductor with a rectangular cross-section 160 mils by 375 mils? Ans. 76,400, nearly

2. A ribbon-shaped conductor $\frac{1}{8}$ inch thick must have a sectional area of approximately 56,000 circular mils; what must be its width in mils? Ans. 351.8

Note.— 350 mils would probably be used, giving a sectional area of 55,704 circular mils.

3. What is the sectional area in circular mils of a wire $\frac{1}{8}$ inch in diameter? Ans. 15,625

50. Wire Gauges.—A **wire gauge** is a tool for determining the diameters of wires. Wire sizes are ordinarily designated by numbers, known as *gauge numbers*, and in specifying a number the gauge referred to must also be specified. Table I gives wire gauges and gauge numbers in common use. The first five numbers, 6–0, 5–0, etc., are pronounced *six naught*, *five naught*, etc. and are often written with the number of naughts indicated, as *000000* instead of 6–0.

Wire for electrical conductors in the United States is nearly always specified by the *Brown & Sharpe gauge* (B. & S. G.); this gauge is also called the *American wire gauge* (A. W. G.). Each decrease of one in the gauge number designates a wire approximately 26 per cent. larger in sectional area, and each decrease of three gauge numbers designates a wire of approximately two times the sectional area. Thus, a No. 12 (B. & S.) wire is 26 per cent. larger than a No. 13 and No. 10 is twice as large as No. 13.

TABLE I

WIRE DATA

Gauge Number	Brown & Sharpe, or American, Gauge		Roebling, or Washburn & Moens, Gauge		Birmingham, or Stubs', Gauge	Gauge Number
	Diameter Mils	Sectional Area Circular Mils	Diameter Mils	Breaking Stress (Steel)	Diameter Mils	
6–0			462	16,619		6–0
5–0			431	14,522		5–0
4–0	460.0	211,600.00	394	12,130	454	4–0
3–0	409.6	167,800.00	363	10,292	425	3–0
2–0	364.8	133,100.00	331	8,605	380	2–0
0	324.9	105,500.00	307	7,402	340	0
1	289.3	83,690.00	283	6,290	300	1
2	257.6	66,370.00	263	5,433	284	2
3	229.4	52,630.00	244	4,676	259	3
4	204.3	41,740.00	225	3,976	238	4
5	181.9	33,100.00	207	3,365	220	5
6	162.0	26,250.00	192	2,895	203	6
7	144.3	20,820.00	177	2,461	180	7
8	128.5	16,510.00	162	2,061	165	8
9	114.4	13,090.00	148	1,720	148	9
10	101.9	10,380.00	135	1,431	134	10
11	90.7	8,234.00	120	1,131	120	11
12	80.8	6,530.00	105	866	109	12
13	72.0	5,178.00	92	665	95	13
14	64.1	4,107.00	80	503	83	14
15	57.1	3,257.00	72	407	72	15
16	50.8	2,583.00	63	312	65	16
17	45.3	2,048.00	54	229	58	17
18	40.3	1,624.00	47	174	49	18
19	35.9	1,288.00	41	132	42	19
20	32.0	1,022.00	35	96	35	20
21	28.5	810.10	32	80	32	21
22	25.3	642.40	29	62	28	22
23	22.6	509.50	26	49	25	23
24	20.1	404.00	23	42	22	24
25	17.9	320.40	20	31	20	25
26	15.9	254.10	18	25	18	26
27	14.2	201.50	17	23	16	27
28	12.6	159.80	16	20	14	28
29	11.3	126.70	15	18	13	29
30	10.0	100.50	14	15	12	30
31	8.9	79.70	13.5	14	10	31
32	8.0	63.21	13.0	13	9	32
33	7.1	50.13	11.0	9.5	8	33
34	6.3	39.75	10.0	7.9	7	34
35	5.6	31.52	9.5	7.1	5	35
36	5.0	25.00	9.0	6.4	4	36
37	4.5	19.83				
38	4.0	15.72				
39	3.5	12.47				
40	3.1	9.89				

Other wire gauges are the Roebling, or Washburn & Moens (W. & M.), and the Birmingham, or Stub's, (B. W. G.). The Roebling gauge is generally used to specify iron and steel wire. The Birmingham gauge is used to specify wire for other than electrical purposes, except iron and steel wire. As steel wire is used in electrical work more for tensile strength than as conductors, its breaking stress is given in the table.

RESISTANCE OF CONDUCTORS

51. The ohm is sometimes too small to express the value of resistance conveniently, and the **megohm**, which equals one million ohms, is then used. If the ohm is too large, the **microhm**, which equals one millionth part of an ohm, is used. The megohm is therefore a multiple of the ohm, and the microhm is a submultiple of it.

52. The **specific resistance**, or *resistivity*, of a material is the resistance between opposite faces of a cube of the material measuring 1 unit on each edge. Resistivity may be expressed for the centimeter cube or for the inch cube. In either case, the length of the path for which the resistance is expressed is 1 unit and its cross-section 1 unit square, that is, either 1 centimeter long by 1 square centimeter cross-section, or 1 inch long by 1 square inch cross-section. The resistivities given in Tables II and III are for such units.

53. Resistivities are also frequently expressed per **mil-foot**, a unit 1 circular mil in cross-sectional area and 1 foot long. For practical calculations, the resistivity of copper conductors at a temperature of 25° C. (77° F.) is considered 10.8 ohms per mil-foot.

54. Laws of Resistance.—The resistance of any material body changes according to the following laws:

1. Directly with the length.
2. Inversely with the sectional area; that is, directly with the reciprocal of the sectional area.
3. With the temperature, though not in direct proportion.

Thus, doubling the length without changing the sectional area or the temperature doubles the resistance; doubling the sectional area without changing the length or temperature halves the resistance; changing the temperature generally changes the resistance.

TABLE II

RESISTIVITIES OF CONDUCTORS

Material	Resistance, in Microhms, At 0° C. or 32° F.	
	Centimeter Cube	Inch Cube
Brass, commercial	7.200	2.840
Carbon, arc light	5,100 to 7,600	2,000 to 3,000
Copper, annealed	1.594	.627
Copper, hard drawn	1.620	.638
German-silver wire	20.900	8.240
Iron, wrought	13.800	5.450
Lead, compressed	19.500	7.680
Mercury, commercial	94.300	37.200
Platinum, annealed	8.980	3.540
Phosphor-bronze, commercial	8.480	3.340
Silver, annealed	1.490	.587
Silver, hard drawn	1.590	.626
Steel wire	13.500	5.320

If the length, cross-sectional area, and resistivity of a conductor is known, its resistance can be calculated by the formula

$$R=\frac{r\,l}{a},$$

in which R = resistance;
r = resistivity;
l = length;
a = sectional area.

The formula gives the resistance R in the same units, microhms or ohms, as the resistivity r and at the same temperature as that for which r is given. When using values of resistivity from Tables II and III, the length l must be in centimeters when r is for a centimeter cube, and in inches when r is for an inch cube. In calculating the resistance of conductors and using the constant 10.8 ohms per mil-foot, l must be in feet and a in circular mils.

TABLE III

RESISTIVITIES OF INSULATORS*

Material	Resistance, in Ohms, at Ordinary Atmospheric Temperature	
	Centimeter Cube	Inch Cube
Asbestos	160×10^{9}	630×10^{8}
Celluloid	71×10^{9}	28×10^{9}
Ebonite	28×10^{15}	11×10^{15}
Fiber, vulcanized, black	68×10^{12}	27×10^{12}
Fiber, vulcanized, red	10×10^{12}	4×10^{12}
Fiber, vulcanized, white	14×10^{12}	6×10^{12}
Glass, ordinary window	90×10^{12}	355×10^{11}
Glass, flint	20×10^{15}	8×10^{15}
Gutta percha	450×10^{12}	180×10^{12}
Marble	510×10^{6}	200×10^{6}
Mica	84×10^{12}	33×10^{12}
Paraffin oil	8×10^{12}	32×10^{11}
Paraffin, solid	33×10^{15}	13×10^{15}
Paper, ordinary	51×10^{9}	20×10^{9}
Porcelain	540×10^{15}	213×10^{15}
Shellac	9×10^{15}	35×10^{14}
Slate	78×10^{9}	31×10^{9}
Sterling varnish	205×10^{11}	8×10^{12}
Wood, dry, untreated	635×10^{12}	250×10^{12}
Wood, dry, paraffined	380×10^{13}	150×10^{13}
Wood, tarred	167×10^{13}	658×10^{12}

---

* These values are approximate; the resistivity varies greatly with purity and method of preparation.

Example 1.—Find the resistance between opposite faces of a carbon block having the dimensions given in Fig. 32.

Solution.—The resistivity of carbon, as given in Table II, is from 2,000 to 3,000 microhms per inch cube, and either of these values can be used for r in the formula. For present purposes, the average, 2,500, can be used. There are three paths to consider, as follows:

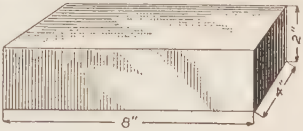


Fig. 32

1. $l_1 = 2$ in., $a_1 = 4 \times 8 = 32$ sq. in., and

$$R_1 = \frac{2{,}500 \times 2}{32} = 156\tfrac{1}{4} \text{ microhms.} \quad \text{Ans.}$$

2. $l_2 = 4$ in., $a_2 = 2 \times 8 = 16$ sq. in., and

$$R_2 = \frac{2{,}500 \times 4}{16} = 625 \text{ microhms} \quad \text{Ans.}$$

3. $l_3 = 8$ in., $a_3 = 2 \times 4 = 8$ sq. in., and

$$R_3 = \frac{2{,}500 \times 8}{8} = 2{,}500 \text{ microhms} \quad \text{Ans.}$$

Example 2.—Calculate the resistance of 2,000 feet of No. 6 (B. & S.) copper wire using a resistivity of 10.8 ohms per mil-foot.

Solution.—The formula $R = \frac{r\,l}{a}$, in which $r = 10.8$, $l = 2{,}000$, and $a = 26{,}250$ circular mils (Table I), gives

$$R = \frac{10.8 \times 2{,}000}{26{,}250} = .823 \text{ ohms.} \quad \text{Ans.}$$

55. The *effect of temperature change* on the resistance of conductors need be considered only when calculating sizes of conductors that will be subject to wide variations of temperature and where the resistance must be known with considerable accuracy. This is the case with many of the conductors in electric generators and motors.

The resistance of copper and of most other metals used for electric conductors increases approximately 1 per cent. for each 2.5° C., or 4.5° F., rise in temperature (more accurately, .42 per cent. for each degree centigrade). Thus, an increase of 25° C. will cause an increase of $25 \div 2.5 = 10$ per cent. in the resistance.

Example 1.—If the resistance of a copper wire is .823 ohm at 25° C., what is its resistance at 60° C.?

SOLUTION.—The temperature rise is $60-25=35°$ C. and the resistance increase is $35 \div 2.5=14$ per cent. The resistance at 60° C. is therefore $.823 \times 1.14=.938$ ohm. Ans.

EXAMPLE 2.—Find the resistance of 500 feet of No. 14 (B. & S.) copper wire at: (*a*) 50° C.; (*b*) 80° C.

SOLUTION.—(*a*) The resistivity of copper at 25° C. is 10.8 ohms per mil-foot; 50° C. is an increase of 25° C. corresponding to $25 \div 2.5=10$ per cent. increased resistance. The cross-sectional area of No. 14 wire (Table I) is 4,107 circular mils. By the formula,

$$R=\frac{r\,l}{a}=\frac{10.8\times1.10\times500}{4{,}107}=1.446 \text{ ohms. Ans.}$$

(*b*) At 80° C., an increase of 55° C., the resistance increase is $55 \div 2.5=22$ per cent. greater than at 25° C. Therefore,

$$R=\frac{10.8\times1.22\times500}{4{,}107}=1.604 \text{ ohms. Ans.}$$

NOTE.—Although the results obtained in this way are not strictly accurate, they are close enough for general purposes, and the method is simple.

56. Specific Conductance, or Conductivity.—The conductance of a specified unit of a substance is called the *specific conductance*, or *conductivity* of the substance. Conductivity is the reciprocal of resistivity. The conductivity of commercial copper is often expressed as a percentage of the conductivity of pure annealed copper, known as *Matthiessen's standard;* thus the statement that copper has a conductivity of 98 per cent. means that its conductivity is .98 that of Matthiessen's standard.

EXAMPLES FOR PRACTICE

1. Calculate the resistance of 1,000 feet of No. 10 (B. & S.) annealed copper wire at: (*a*) 25° C.; (*b*) 80° C.
Ans. { (*a*) 1.04 ohms, approx.
(*b*) 1.27 ohms, nearly

2. Calculate the resistance of 5,000 feet of No. 12 (B. & S.) annealed copper wire at 60° C. Ans. 9.427 ohms, approx.

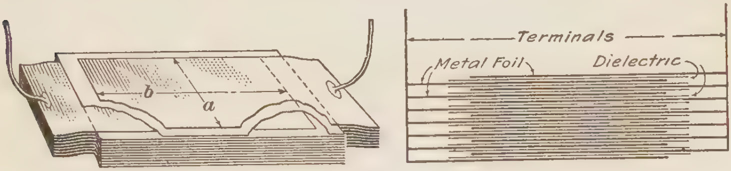
3. Calculate the resistance of 5 miles of No. 0 (B. & S.) annealed copper wire at 25° C. Ans. 2.7 ohms, approx.

DIELECTRIC CIRCUITS

ELECTRIC CONDENSERS

57. Any electric circuit that includes a condenser is a **dielectric circuit.** Practically all electric circuits are therefore also dielectric circuits, for a condenser is formed whenever two conductors are separated by an insulating substance. Condenser effect is sometimes present where undesirable in circuits and at other times is a desirable addition to a circuit. Commercial condensers are made in several forms for use where needed.

58. **Plate condensers** are made by assembling sheets of conducting element and insulation so that the insulation separates the conductors. The conductors are joined electrically in two groups, alternate sheets in each group, and a terminal is connected with each group. Fig. 33 shows one method of constructing such a condenser, (*a*) being a general view with one conducting plate and one insulating sheet partly cut away, and (*b*), a diagrammatic cross-section. Tin-foil is generally used for the conductors, though any other metal in thin sheets can be used. Sheets of mica or oiled paper generally form the insulating material, or dielectric. The active parts of the condenser plates are those separated by dielectric, as indicated by the dimension lines *a b*. The product of these two dimensions and the number of sheets of dielectric is the *effective area*



(a) FIG. 33 (b)

of the dielectric. The complete condenser made as indicated is enclosed between insulating sheets that are firmly pressed together and securely clamped.

59. The **Leyden jar** is a form of condenser that is cheaply constructed and very effective for some purposes. A glass jar free from cracks and flaws is coated inside and outside with metal foil for possibly two-thirds of its height, as indicated at *a*, Fig. 34. A metal rod fitted in the cork extends toward the bottom and carries a piece of chain, some of which rests on the bottom in contact with the inner coating of metal. The end of the rod thus constitutes one terminal, and the outer coating the other; contact is made with the outer coating in any convenient way.

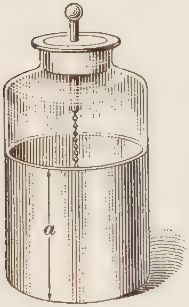


FIG. 34

60. The **electrostatic capacity,** or *permittance*, of a condenser is the ratio between its charge and the electromotive force across its terminals. Its charge is the quantity of electricity that will flow into it with a given electromotive force. The units of capacity are the **farad** and the **microfarad;** the capacity, in farads, is the ratio between charge in coulombs and electromotive force in volts. In making calculations, capacity must always be considered in farads, but the capacities of condensers are practically always stated in microfarads. A microfarad is 1 millionth of a farad, or 1,000,000 microfarads = 1 farad. In practice, condenser capacities rarely exceed a very few microfarads.

Let C = capacity of condenser, in farads;
Q = quantity of electricity, or charge, in coulombs;
E = electromotive force, in volts, across condenser terminals.

Then,
$$C=\frac{Q}{E}$$

EXAMPLE 1.—If an electromotive force of 10 volts establishes a charging current of 1 milliampere for .01 second in a condenser, what is the capacity of the condenser?

SOLUTION.—As coulombs = amperes × seconds and 1 milliampere = .001 ampere, the charge, in coulombs, is .001 × .01 = .00001. By the formula, the capacity

$$C=\frac{.00001}{10}=.000001 \text{ farad}=1 \text{ microfarad. Ans.}$$

EXAMPLE 2.—How many coulombs of electricity will it require to charge a 2.5-microfarad condenser from a 50-volt circuit?

SOLUTION.— 2.5 microfarads = .0000025 farad $=\frac{Q}{50}$;

$Q=50\times.0000025=.000125$ coulomb. Ans.

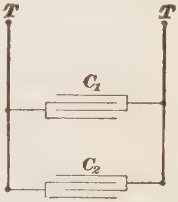


FIG. 35

61. Connections of Condensers.—In diagrams, condensers are usually represented by symbols as at C_1, C_2, Fig. 35. When they are connected in parallel, as shown, their combined capacity is equal to the sum of their individual capacities. For example, if C_1 and C_2 are the individual capacities of the two condensers between the terminals T, the combined capacity between these two terminals is

$$C=C_1+C_2$$

The capacities of condensers in parallel are thus added similarly to the conductances of electric circuits in parallel.

62. When condensers are connected in series, as in Fig. 36, their combined capacity is found by adding the reciprocals of the capacities and taking the reciprocal of this sum. Thus,

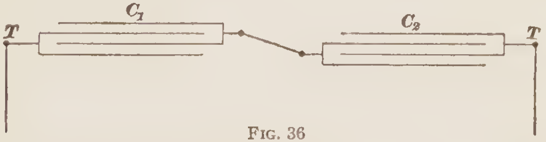


FIG. 36

if two condensers having capacities C_1 and C_2, are in series, the total capacity between the terminals T is

$$C=\frac{1}{\frac{1}{C_1}+\frac{1}{C_2}}$$

EXAMPLE.—Find the combined capacity of three condensers having capacities of 2, 3, and 4 microfarads connected in: (a) parallel; (b) series.

SOLUTION.—

(*a*) $C=2+3+4=9$ microfarads. Ans.

(*b*) $C=\frac{1}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}=\frac{1}{\frac{13}{12}}=\frac{12}{13}=.92$ microfarad. Ans.

63. Condensers connected in parallel are subjected to the same voltage, as appears in Fig. 35. The voltages at the terminals of condensers connected in series are inversely proportional to the capacities of the condensers; the condenser with greatest capacity is subjected to the least voltage. If a high-voltage conductor is separated from earth by a series of insulators with intervening conductors, the weakest insulator may thus be subjected to the highest voltage with danger of break-down.

DIELECTRIC STRENGTH

64. The **dielectric strength of an insulating substance** is the maximum voltage that it can withstand without breaking down or being punctured. A puncture, or disruption, occurs when an electric spark passes through the insulation and thus forms a conducting path through it. A substance may show high resistance to low-voltage current, but may break down easily when high voltage is applied; another substance with comparatively low resistance may show much better ability to withstand puncture by high voltage. As soon as a puncture occurs the material is carbonized, or burned, around the puncture, thus practically destroying its insulating qualities, since carbon is a fairly good conductor. Insulating oils are an exception to this rule, because fresh oil immediately fills the openings caused by punctures, and the lighter carbonized particles rise to the surface.

65. The dielectric strength of a substance is determined by placing a sample between two electrodes and then gradually raising the voltage until the substance breaks down. The voltage that causes this breakdown is the dielectric strength of the material. This voltage divided by the thickness of the material tested gives the dielectric strength per unit thickness, a value by which the merits of different insulating materials can be compared. The dieletric strength per unit is somewhat less in thick samples than in thin samples, probably because of the less uniform structure in the thick samples. Table IV will prove serviceable in selecting some of the more common insulating materials. The dielectric strength of any of these materials of any given thickness can be found approximately by multiplying the thickness in mils by the puncturing voltage given in the table. Some reduction should be made for thicknesses above, say, 100 mils. On account of wide variations of test results, only approximate values can be given. Some of the materials named in the table are compositions of other materials and are known by trade names, such as presspahn and vulcabeston.

TABLE IV

APPROXIMATE PUNCTURING VOLTAGES PER MIL

Material	Volts Per Mil	Material	Volts Per Mil
Asbestos	100	Leatheroid	125 to 300
Calico, treated with rubber	40	Linen cloth	125 to 200
Cambric, oiled	430	Linen, varnished	250
Canvas, oiled	125	Marble	165
Cotton cloth	90	Mica	430 to 700
Ebonite	725	Oiled cloth	450 to 600
Empire cloth	200	Paper, manila	125
Empire cloth, oiled	250	Paper, red rope	240
Fullerboard	400	Para rubber	450
Fiber	60	Paraffin	300
Glass, ordinary	200	Porcelain	230
Glass, lead	140	Porcelain (Locke)	415
Gutta percha	450	Presspahn	100 to 250
Hard rubber	250 to 950	Rosin	280
Jute, impregnated	20	Vulcabeston	60 to 100
Lava	75 to 250	Wax	280

66. The **dielectric strength of air** appears higher with direct voltage than with alternating (rapidly reversing) voltage. It is also higher if the air is compressed than if at ordinary pressure or rarefied. Higher voltage is required to cause a spark through air between parallel disks than between needle points. Fig. 37 shows the dielectric strength of dry air at ordinary pressure with alternating voltage. Curve *A* is for sharp needle points, and curve *B* for polished brass balls $\frac{7}{8}$ inch in diameter. The disruptive, or puncturing, electromotive

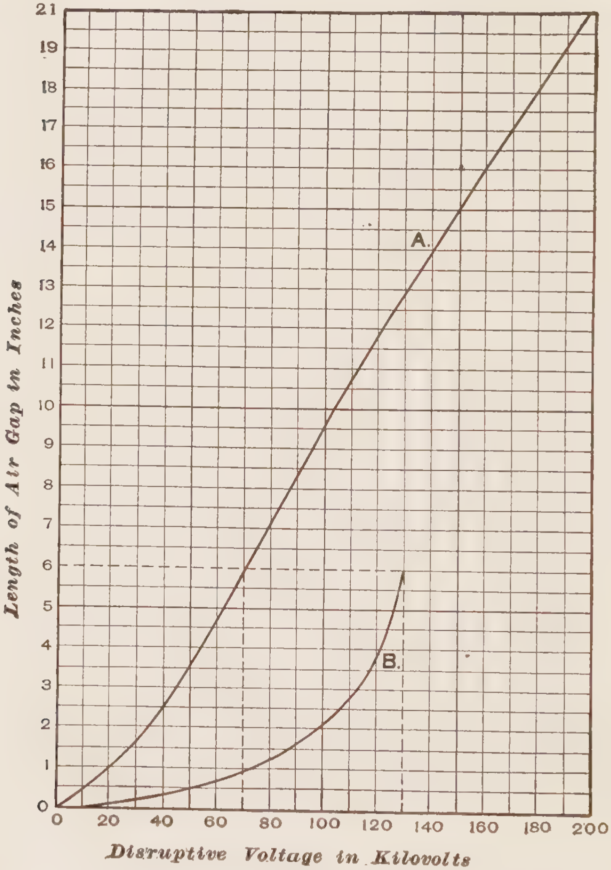


FIG. 37

force is given in kilovolts, the prefix *kilo* meaning 1,000. Dotted lines show the method of reading the curve to find the voltage that will cause a 6-inch spark; 70,000 volts are required between needle points and 130,000 between the brass balls. Voltages for other spark gaps are found in the same way.

EXAMPLES FOR PRACTICE

1. The capacities of four condensers in microfarads are, respectively, .5, 1, 4, and 6. Find their combined capacity when connected in: (*a*) series; (*b*) parallel.

Ans. (*a*) .293 microfarad
(*b*) 11.5 microfarads

2. What is the combined capacity of five condensers of 2 microfarads each when connected in: (*a*) parallel? (*b*) series?

Ans. (*a*) 10 microfarads
(*b*) .4 microfarad

3. What per cent. is added to the insulating quality of empire cloth by oiling it? Ans. 25

THEORY OF DIRECT-CURRENT GENERATORS AND MOTORS

ELECTROMAGNETIC INDUCTION

FUNDAMENTAL PRINCIPLES

1. Electromagnetic induction, broadly considered, relates to the electromotive force generated in a conductor within a magnetic field by relative cutting motion between the conductor and the flux of the field. The conductor may move and the flux remain constant in position, or the flux may move and the conductor be stationary. In either case, if the conductor cuts across the flux or is cut by it, an electromotive force is generated, and if the conductor forms a part of a complete circuit, a current is established throughout the circuit.

The electromotive force for nearly all commercial lighting and power circuits is furnished by a machine called a *generator*, the action of which is based on the principles of electromagnetic induction.

2. Electromagnetic induction may for convenience be classified as *transformer action* and *generator action;* but both actions are closely related and probably identical.

In **transformer action,** both the conductor that bears the exciting magnetomotive force and that in which the electromotive force is generated are stationary, and the electromotive force is induced by causing such motion of the flux of the exciting conductor as to cut across the conductor in which the

COPYRIGHTED BY INTERNATIONAL TEXTBOOK COMPANY. ALL RIGHTS RESERVED

electromotive force is established. A movement of the flux may be set up by increasing or decreasing the exciting current or by changing the reluctance of the path of the exciting flux. The operation of a device called a *transformer* is based on the variation of the exciting current.

In **generator action** also, two sets of conductors are usually employed, one in which the magnetizing current is carried and the other in which electromotive force is generated. Either set of conductors is made movable while the other is stationary.

3. The *direction of motion of a flux* refers to the movement of the flux considered as a group of lines of force; for example, when a current in a wire increases, the flux moves outwards from the wire in the form of constantly expanding circles. The *direction of the flux* refers to the direction of the individual lines of force composing the flux; for example, the direction of the flux may be clockwise or counter-clockwise around the wire.

In order to generate an electromotive force, it is essential that there be a relative cutting motion between the flux and the conductor. No electromotive force will be generated if both the flux and the conductor are stationary, or if the motion of the conductor or the flux is such that the conductor does not cut across the flux or the flux cut across the conductor.

DIRECTION OF THE ELECTROMOTIVE FORCE

4. Generator Action.—The conductor *a b*, Fig. 1 (*a*), is moved toward the right across the magnetic flux of the permanent magnet, as indicated by the full-line, horizontal arrow. The direction of the inducing flux is downwards, as shown by the arrowheads on the dotted vertical lines. An electromotive force is set up in the conductor in such a direction that the current established will always produce a conductor flux that agrees in direction with the inducing flux on the side of the conductor that first comes in contact with the inducing flux and is opposite in direction on the other side of the conductor. This fact holds true whether the conductor or the flux is the moving element. The direction of the conductor flux is indicated by the arrowheads on the curved lines surrounding ab. The direction of the electromotive force, as well as that of the current in this case, is from a to b, as indicated by the arrowheads on the circuit wires. The flux is more dense ahead of the conductor than behind it because of the relation between the direction of the inducing flux and that of the conductor flux.

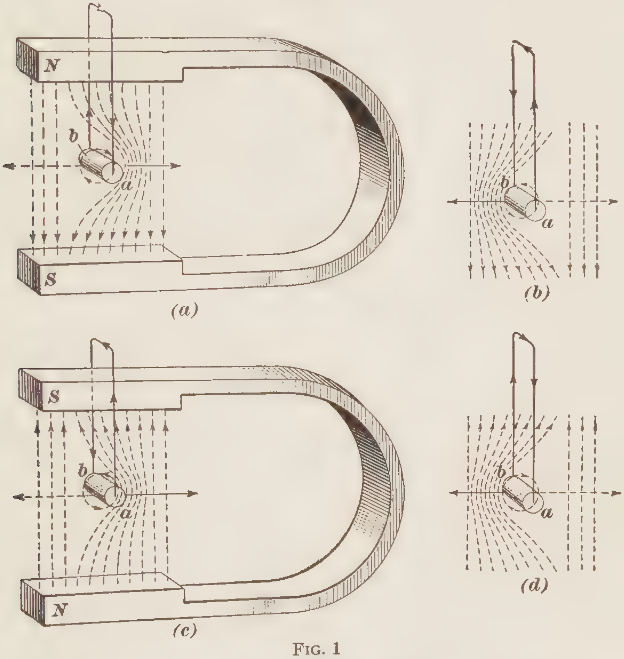


FIG. 1

The direction of the induced electromotive force in any case where the relative direction of the movement of the flux and of the conductor is known may be determined by considering which side of the conductors has the denser flux. The direction of the circular conductor flux on that side agrees with the direction of the inducing flux, and the direction of the induced electromotive force is determined from the relation of the conductor current and its flux, as is explained in *Electricity and Magnetism*, Part 2.

The conductor *a b*, Fig. 1 (*b*), is moved toward the left and the induced electromotive force is from *b* to *a*. In (*c*), a conductor is represented as moving to the right in a field in which the direction of the flux is vertically upwards. The direction of the induced electromotive force is from *b* to *a*. In (*d*), the conductor is moved toward the left in the same field, and the direction of the induced electromotive force is from *a* to *b*.

It should be noted that when either the direction of the motion of the conductor or the direction of the flux is reversed, the direction of the electromotive force is reversed, as in Fig. 1 (*a*) and (*b*) or in (*a*) and (*c*); but, when both the direction of the flux and the direction of the motion of the conductor are reversed, the direction of the induced electromotive force remains unchanged, as in Fig. 1 (*a*) and (*d*) or in (*b*) and (*c*).

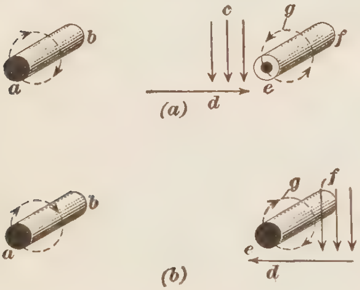


FIG. 2

5. Transformer Action. Fig. 2 (*a*) represents a stationary conductor *a b* in which electricity is just starting to flow from *a* to *b*, or away from the observer, as indicated by the full black circle at the end of the conductor. The flux surrounding this conductor is therefore clockwise, and as a portion *c* of it expands in the direction of the arrow *d* lines of force of *c* move across a neighboring stationary conductor *e f*. The direction of the induced electromotive force in conductor *e f* is from *f* to *e*, as determined from the relation of the moving flux *c* and the induced flux *g* of conductor *e f*. If *e f* is part of a complete circuit, the current established is toward the observer, as indicated by the dot at the end of the conductor.

Consider the case when the current in conductor *a b*, view (*b*), has reached a steady value and is then decreased either by opening the circuit of the conductor or by increasing the resistance of the circuit. The flux *c* is now contracting toward conductor *a b*, as indicated by arrow *d*. The induced electromotive force and the current in conductor *e f* are from *e* to *f*, or away from the observer, as indicated by the full black circle at the end of the conductor.

It should be noted that increasing the current in a conductor induces an electromotive force in the opposite direction in neighboring parallel conductors, and decreasing the current induces an electromotive force in the same direction.

SELF-INDUCTION

6. Suppose that conductors *a b* and *e f*, Fig. 2, are parts of adjacent turns of wire on the same side of a coil, Fig. 3 (*a*). The circuit of the coil is completed by a battery and a small key switch. When electricity starts to flow, the induced electromotive force of conductor *e f*, in the direction indicated by arrow *h*, opposes the impressed electromotive force, the direction of which is indicated by arrow *i*, in forcing current through the circuit. The value of the induced electromotive force depends on the rate at which lines of force cut across the conductor. As the rapidity of this cutting action is maximum at first and decreases as the current becomes steady, the opposing electromotive force decreases in value and becomes zero when the current becomes steady, because there is then no movement of the inducing flux.

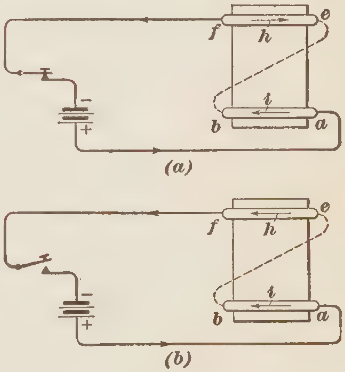


FIG. 3

When the key switch is opened, the current decreases and the direction of the induced electromotive force, as indicated by *h*, Fig. 3 (*b*), agrees with the direction in which electricity was flowing as indicated by *i*, thus tending to prolong the flow, possibly through a spark between the switch contacts.

When the coil in which the electromotive force is induced forms a part of the circuit carrying the inducing current, the electromotive force is self-induced, and is known as the **electromotive force of self-induction.**

7. Self-induction in a circuit always tends to retard any change, either of increase or decrease, in the existing current in the circuit. Circuits containing coils of wire wound on an iron core have comparatively large *inductance;* that is, a change of current causes a considerable electromotive force of self-induction. A single straight conductor without iron near it has small inductance; that is, a change of current induces only a small electromotive force of self-induction, and this is caused by the expanding or contracting flux cutting across a portion of the conductor itself.

A variable current in a circuit in which a considerable electromotive force of self-induction is generated, does not immediately rise or fall to a value indicated by an application of Ohm's law, but takes an appreciable time to arrive at such a value.

MUTUAL INDUCTION

8. When two conductors forming parts of separate circuits are so arranged that variable currents in each cause fluxes to cut across the other, the electromotive forces thus generated in each conductor are said to be due to **mutual induction.**

9. In Fig. 4, conductor *a b* forms a part of a turn that has in circuit a battery and switch. The circuit is known as the *primary circuit.* Conductor *e f* forms a part of a turn in a closed circuit, in which an electromotive force is generated by the action of the primary current. This circuit is known as the *secondary circuit.*

In Fig. 4 (*a*), the current is increasing in the primary circuit. In Figs. 3 (*a*) and 4 (*a*), arrows *h* and *i* have the same direction. The direction of the flux within the iron core, due to the current in conductor *a b*, Fig 4 (*a*), is indicated by arrow *j*; that of the flux within the iron core, due to the current in conductor *e f*, by arrow *k*—opposite in direction to arrow *j*.

When the primary current is increasing, the secondary flux tends to oppose the building up of the primary flux. The self-induction of the primary circuit is decreased because the rate at which lines of force of the primary flux cut across the active conductors of the primary circuit is lessened.

When the current in the primary is decreasing, Fig. 4 (*b*), arrows *h* and *i* are in the directions indicated in Figs. 3 (*b*) and 4 (*b*), and the primary and the secondary flux within the core agree in direction, as indicated by arrows *j* and *k*. The primary flux is decreasing in density, owing to the decreasing primary current, and the secondary flux tends to oppose the rapidity of the decrease in density, thus lessening the electromotive force of self-induction of the primary coil.

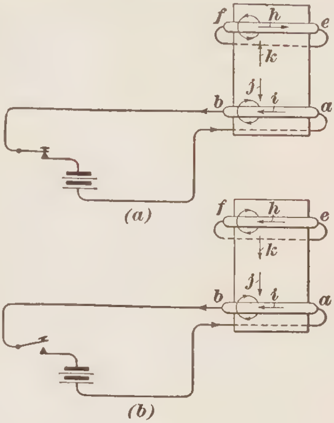


FIG. 4

The effect of mutual induction is therefore to neutralize some of the effect of self-induction, allowing changes of current to occur more promptly than in circuits in which only self-induction is present.

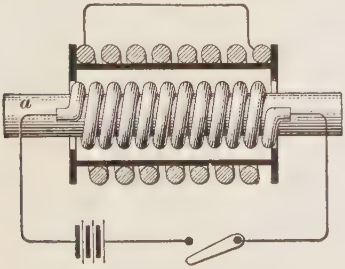


FIG. 5

10. The principles of mutual induction are utilized in induction coils and transformers. Fig. 5 shows a simple form of induction coil: The magnetic core is shown at *a*; the *exciting coil*, also called the *primary coil*, is wound on the core; and the coil in which the induced electromotive force is established, called the *secondary coil*, is wound outside the primary coil. There is no electrical connection between these coils. Any change in the current of the primary coil, such as that due to opening or closing the circuit, causes a movement of the flux due to the varying magnetomotive force of the primary coil, and this flux in expanding or contracting induces an electromotive force in the turns of wire of the secondary coil. The action of the flux due to the varying magnetomotive force of the secondary coil is to decrease the effect of self-induction of the primary circuit

11. If a source of alternating electromotive force is substituted for the battery and switches, Fig. 5, alternating current will be set up in the primary coil and an alternating flux in the core; also, in the secondary coil, there will be generated by mutual induction an alternating electromotive force that will establish an alternating current in the secondary circuit. Such **an arrangement of coils and a core forms a device known as a** *transformer.*

NON-INDUCTIVE CIRCUIT

12. The principle of mutual induction is frequently employed to make a circuit practically non-inductive. When the two wires of an alternating-current circuit are to be installed in iron tubes, called *conduits*, both wires are placed in the same conduit, but they are carefully insulated from each other and from the conduit. At any instant, the current in one wire is opposite to that in the other wire, because while one is acting as the incoming wire the other is acting as the outgoing wire of the same circuit. The effect of mutual induction between the two conductors practically neutralizes the effect of self-induction.

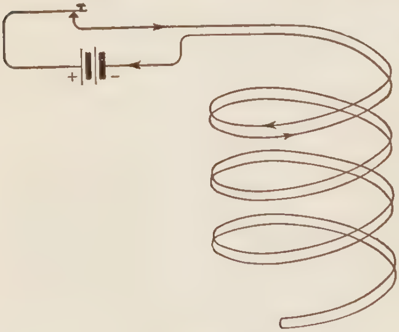


FIG. 6

When winding a resistance coil for a certain type of measuring instrument, the wire is doubled on itself and wound in such a manner that the currents in adjacent conductors at any instant are in opposite directions.

Fig. 6 illustrates one method of winding coils so that their inductance is very small. The effect of mutual induction between the two conductors practically neutralizes the effect of self-induction, thus making the circuit nearly non-inductive, and allowing the current to build up to normal value quickly.

THE ELECTRIC GENERATOR

ELEMENTARY THEORY

13. An **electric generator** is a machine for converting mechanical energy into electrical energy by the application of the principles of electromagnetic induction. The moving element of the generator is usually driven by a steam engine, a gas engine, or a waterwheel, which causes relative motion between a magnetic flux and a group of conductors. An electromotive force is generated in the conductors, and if the circuit external to the generator is complete a current is established throughout the circuit.

A distortion of the magnetic flux occurs near each conductor, as indicated in Fig. 1. Lines of force that have been distorted tend to resume their original position and in so doing endeavor to move the body that causes the distortion in a direction opposite that of the movement required to set up the electromotive force. For instance, the magnetic forces acting on *a b*, Fig. 1 (*a*), tend to move it toward the left, and force must be applied to move the conductor toward the right. It is therefore necessary that force be applied continuously by the engine in order that the relative motion of flux and conductor in the generator be maintained.

14. An electric generator consists, in general, of an electromagnet for providing a magnetic flux; a device on which are mounted the conductors that cut across the flux or are cut by it; and a device for connecting the conductors to the external circuit.

The conductors on generators for direct-current service are mounted on the rotating element, called the *armature*, and connected to the external circuit by a current-rectifying device, called a *commutator*.

Generators for alternating-current service, called *alternators*, may have the conductors mounted on either the rotating element, called the *rotor*, or on the stationary element, called the *stator*.

When electrical connection is necessary with the conductors on the rotating element, it is usually made by means of metal rings, called *collector rings*, or *slip rings*, mounted on the rotor and sliding under stationary brushes.

15. Simple Form of Alternating-Current Generator. In direct-current generators of the usual type, alternating electromotive forces are generated in the armature conductors and changed by the action of the commutator to direct current for the external circuit.

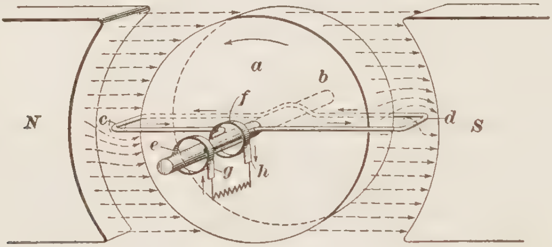


FIG. 7

Fig. 7 shows an elementary type of alternating-current generator. A soft-iron armature core *a* on a shaft *b* carries on its convex surface conductors at *c* and *d* and revolves between magnetic poles *N* and *S* so that the conductors cut the flux of the magnetic poles. Electromotive forces are generated in these *active conductors*, which are connected by other conductors to the slip rings *e* and *f*. Stationary brushes *g* and *h*, which bear on the rings, serve to connect the armature conductors with an external circuit.

With a counter-clockwise direction of armature rotation and the flux in the direction indicated, the electromotive force in *c* is toward the front, and in *d* toward the rear of the armature. The principles of electromagnetic induction, as explained in connection with Fig. 1, should be applied to the conditions of Fig. 7. The two conductors are so connected across the back of the armature that the two electromotive forces act in series to force current through the circuit.

16. The value of the electromotive force generated in a conductor is directly proportional to the rate at which it cuts across lines of force. If either the density of the flux or the speed of rotation of the armature is increased, a higher value of electromotive force is generated. A conductor when midway in the space between the tips of the pole pieces is moving parallel to the lines of force, and, therefore, as no lines are cut, the electromotive force will be zero. When conductor *c*, Fig. 7, is at its highest point, its electromotive force is zero. As it moves toward a position near the middle of the pole piece, it will cut across more and more lines of force for a given distance of circular movement until, at the position of *c* shown, maximum electromotive force is generated. In the next quarter revolution, the conductor cuts across a lessening number of lines of force for a given circular movement; therefore, the value of the electromotive force generated decreases and becomes zero when the conductor is at its lowest position and is moving parallel to the lines. When conductor *c* changes from cutting down to cutting up across the flux, the direction of the induced electromotive force changes, because the relative direction of motion of conductor and flux has been reversed. The electromotive force then increases until conductor *c* is in the position shown occupied by *d*, where maximum electromotive force is generated. For the remaining quarter revolution, the electromotive force decreases and becomes zero when conductor *c* again reaches its highest point.

With the one-turn armature of the simple alternator shown, the current in the whole circuit varies in value and changes in direction practically in unison with the changes in the generated electromotive force. The arrows near the brushes, Fig. 7, indicate the direction of the current in the external circuit only when conductor *c* is moving downwards and conductor *d* upwards, as indicated. The current in the whole circuit is reversed in direction as the conductors *c* and *d* move through their upper and lower positions.

17. Alternating-Current Curve.—The rise and fall of the alternating current established by the simple generator shown in Fig. 7 is indicated by the curve shown in Fig. 8. The distances of the curve above or below the line *a b* represent increasing or decreasing values of electromotive forces and the distance along the horizontal line indicates intervals of time. The part of the curve above the line *a b* indicates a flow of electricity in one direction, called *positive*, through the circuit, and the portion below the line, a flow in the other direction, called *negative*.

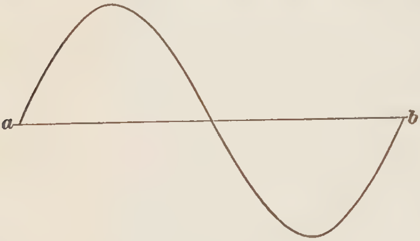


FIG. 8

18. Elementary Direct-Current Generator.—The machine shown in Fig. 7 may be changed to an elementary generator for providing direct current to an external circuit by the substitution of a commutator for the slip rings, as indicated in Fig. 9. The action of a commutator is called *commutation*.

The simple commutator shown consists of two semicircular bars *e* and *f* mounted near, but not touching, each other on the shaft *b* and insulated from it. Conductor *c* is connected to bar *e* and conductor *d* to bar *f*. The brushes *g* and *h* serve to connect the external circuit with the commutator. As the shaft turns counter-clockwise, brush *g*, view (*a*), comes into contact with bar *e* and brush *h* with bar *f*.

The electromotive force generated in the turn, as soon as the conductors start to cut across the lines of force, causes a flow of electricity out through the bar *e*, the *positive brush g*, the external circuit, and into the armature by means of the *negative brush h* and bar *f*. At the end of the first quarter revolution the position of the armature turn, the positions of the commutator bars, and the direction of the current are as indicated in (*b*). At the end of the second quarter, the conductors *c* and *d* are generating no electromotive force and the bars are just reversing their connections to the brushes, as indicated in (*c*). At this time there is no current in the circuit. During the third and fourth quarters, the bar *f*, view (*d*), is in contact with the positive brush *g* and bar *e* is in contact with the negative brush *h*. The direction of current in conductors *c* and *d* is indicated by the arrows. At the end of the fourth quarter, the conductors are again in the positions shown in (*a*), and the commutator bars are about to reverse their connections with the brushes.

As indicated by the arrows near conductors *c* and *d*, views (*b*) and (*d*), the direction of current in each conductor is reversed because the relative direction of motion of the conductors and the direction of the flux are changed; but commutation serves to keep the current uniform in direction in the external circuit.

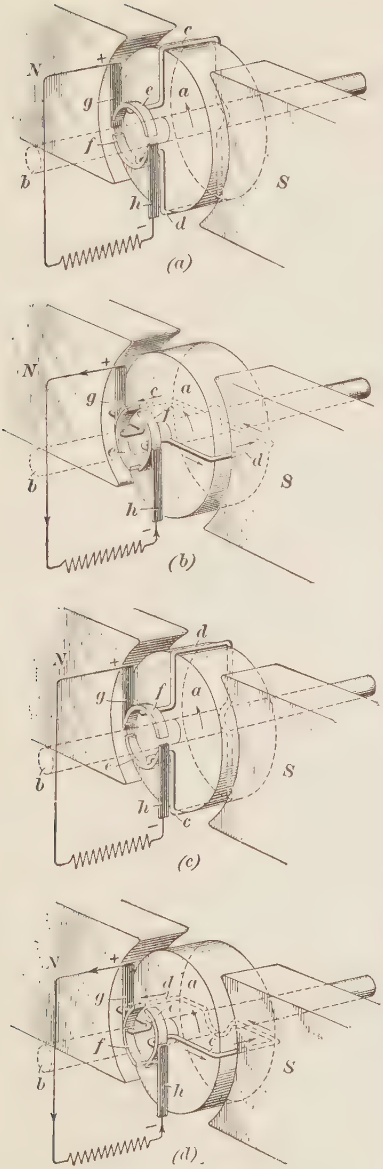


FIG. 9

19. Pulsating-Current Curve.—The alternating electromotive force of the armature, indicated by the curve *a b*, Fig. 8, produces through the action of the commutator an external direct current of the form indicated in Fig. 10. The current starting at zero rises to a maximum value, decreases to zero, and repeats this action as long as the circuit is active, the current being in the positive direction at all times.

A current changing in value in the general manner indicated in Fig. 10 is called a *pulsating current.* The term, as ordinarily employed, refers to a direct current that varies in magnitude through a regular series of changes between maximum and minimum values; the minimum value may or may not be zero.

20. Armature With Several Coils.—The variations of the current, as indicated in Fig. 10, are due to the armature having only a single turn of wire. In an actual armature, there are many coils and commutator bars. The conductors of a few coils are passing through positions in which little or no electromotive force is generated in them, and their connections to the brushes are then reversed; but there are always a number of coils connected in series in the two or more parallel paths between the positive and negative sets of brushes. Therefore, the direct electromotive force impressed on the external circuit does not fall to zero, as indicated in Fig. 10, but assumes a nearly constant value for all positions of the armature conductors during a revolution.

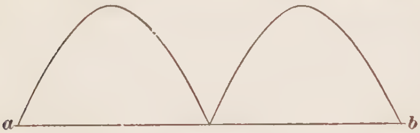


FIG. 10

21. Fig. 11 shows in a conventional manner the arrangement of several coils on an armature intended for a two-pole generator. The armature conductors are indicated by small circles *a*, *b*, *c*, etc., the commutator bars by the outlines numbered *1*, *2*, *3*, etc., and the brushes by $-B$ and $+B$. Blackened circles indicate direction of electromotive forces away from the observer; circles with dots in the center, direction toward the observer; and white circles, conductors temporarily in positions in which practically no electromotive force is generated. Straight dotted lines joining conductors indicate connections on the rear end of the armature, and curved lines, connections with the commutator bars. An external circuit is indicated at R.

Two current paths can be traced from brush $-B$ to brush $+B$ as follows: $-B$–3–g–g'–2–e–e'–1–c–c'–8–$+B$, and $-B$–4–d–d'–5–f–f'–6–h–h'–7–$+B$. Coils 8–a–a'–7 and 3–b–b'–4 are short-circuited by the brushes. These coils are moving through spaces, called *neutral spaces*, where little or no electromotive force is generated in them; therefore, there is little current in the coils. By means of commutation, these coils are transferred from connection with one group of coils to connection with the other group; thus, the electromotive forces that will be generated in them after a little further rotative movement will act in unison with the electromotive forces generated in the group of coils to which they are connected as soon as their commutator bars leave direct contact with the brushes.

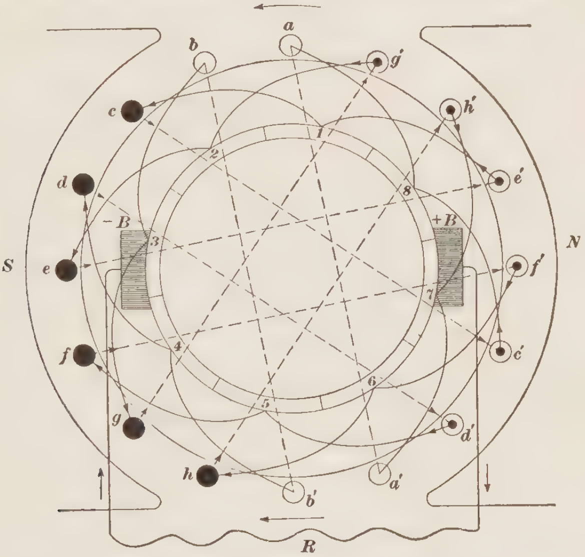


FIG. 11

PARTS OF THE MAGNETIC FIELD

22. Main Field Magnets.—The purpose of the electromagnets that form the field magnets of an electric generator is to establish a magnetic flux. Some types of small direct-current generators have only two pole pieces, and these are known as **bipolar generators.** Large direct-current generators usually have four or more pole pieces, and these are known as **multipolar generators.**

23. In Fig. 12 are indicated by dotted lines the magnetic circuits of a four-pole, field-magnet frame and armature core. The *field-magnet cores*, or pole pieces, are shown at *a*, and the *field frame*, or *yoke*, at *b*. The broadened ends of the pole pieces are called *pole shoes*, and the surfaces of the shoes near the *air gaps c* between the pole pieces and the *armature core d* are called *pole faces*. The *field coils*, or *magnetizing coils*, are shown at *e*.

The field coils, field-magnet cores, pole shoes, and the yoke, taken collectively, are called the *field of the machine*. The field-magnet cores, pole shoes, air gaps, armature core, and field yoke, taken collectively, form the *magnetic circuits of the machine*.

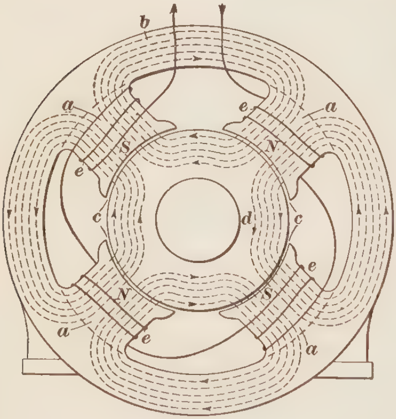


Fig. 12

When the current in the exciting coils is in the direction indicated by the arrowheads, the polarity of the pole faces is as indicated by the letters *N* and *S* and the paths of the magnetic fluxes are as indicated by the dotted lines. The field frame, field-magnet cores, pole shoes, and the armature core are made of very soft iron or of steel. The reluctance of this part of the magnetic circuit is low, and the air gaps are made short to keep the reluctance of the complete magnetic circuit at a low value.

24. The magnetic circuits are so arranged that the flux enters and leaves the armature core at the proper points to establish in each air gap a magnetic field of the required distribution and density of lines. The detailed arrangements of the parts of the magnetic circuit may be varied to conform to the general design of the generator. Fig. 12 shows one arrangement of parts for a multipolar generator, and Fig. 13 an arrangement for a bipolar generator. The reference letters in Figs. 12 and 13 have the same significance.

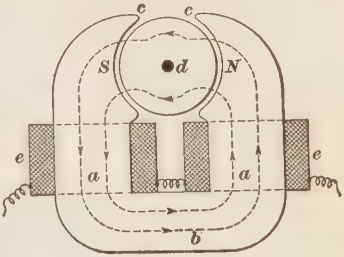


FIG. 13

25. Commutating Poles. In many direct-current generators and motors, the field frame is provided with auxiliary pole pieces located midway between the main pole pieces, and called *commutating poles*, or, rarely, *interpoles*. Their purpose is to neutralize a portion of the disturbing magnetizing effect of the flux due to the armature magnetomotive force, thus improving the commutation of the machine.

CLASSIFICATION BY METHODS OF FIELD EXCITATION

26. Magneto.—There are several methods of connecting the exciting coils of the field magnets to the armature. In some very small generators, called *magnetos*, the exciting magnetic flux is provided by one or more permanent magnets, and in such a case no winding is placed on the field magnets. Magnetos are commonly used for signaling purposes, for blasting, and for the ignition of gas engines.

The armature usually has one coil, and when equipped with a simple two-bar commutator will provide direct current for the external circuit. When two slip rings are used instead of a commutator, an alternating current is established in the external circuit.

27. Separately Excited Generator.—The flux for most commercial generators is produced by electromagnets.

Generators may be classified according to the method employed to *energize*, or *excite*, the field coils. In a separately excited generator, the current for energizing the field coils is provided from a source external to the generator. The connections of both the armature and the field coils are shown in Fig. 14. The exciting coils *a* are placed on the field-magnet cores and connected to a battery or to another direct-current generator. Positive brushes $+b$ and negative brushes $-b$ bear on the commutator *c* and serve to impress the electromotive force generated in the moving armature conductors on the external circuit *d*. There is no electrical connection between the exciting circuit for the field magnets and the armature circuit.

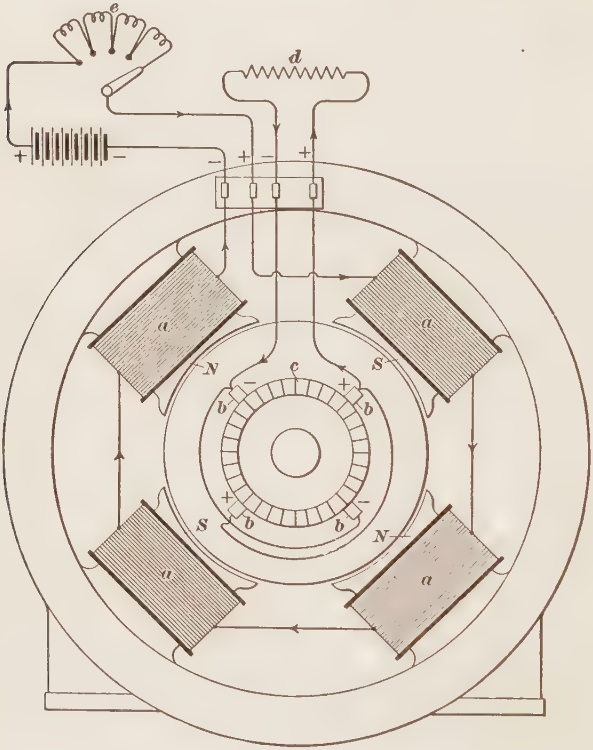


Fig. 14

The value of the electromotive force generated may be varied by changing the speed of the armature or by changing the value of the magnetic flux. The latter change may be effected by means of a variable resistance *e* in the field circuit or by changing the electromotive force causing the exciting current.

28. Shunt Generator.—Another type of machine is called a **self-exciting shunt generator,** or simply a *shunt generator*, from the fact that the exciting current for the field coils is provided by the generator itself. The exciting circuit is connected to the brushes on the armature and is in *shunt*, or parallel, with the external circuit of the generator. A shunt,

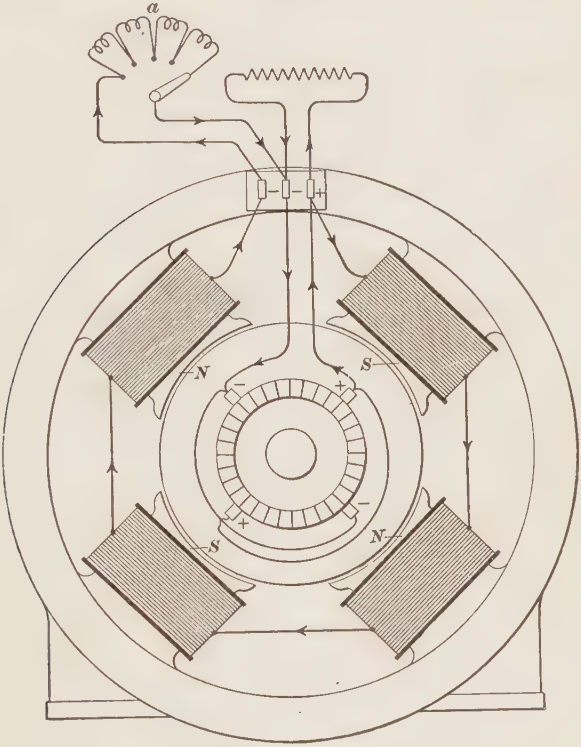


FIG. 15

in its broad sense, refers to a side path; when applied to an electric circuit, it refers to a side path between two points already connected.

The connections of a shunt generator are indicated in Fig. 15. The resistance device a, called a *field rheostat*, is included in the exciting circuit and serves to adjust the exciting current and thus regulate the electromotive force generated by the armature conductors.

The field coils of a shunt generator are formed of many turns of fine copper wire and the individual coils are connected in series, but this group of coils is connected in shunt with the armature. The resistance of the exciting circuit is high and only a very small part of the current from the armature is required to energize the field magnets.

29. Building Up a Magnetic Flux.—In order that a self-exciting machine may start to generate, some residual magnetism is required in the field of the generator. The frame of even a new machine is often slightly magnetized, but if it is not magnetized sufficiently, or if it is of incorrect polarity, the field coils may be separately excited temporarily from another generator or from a few cells of battery. The shunt circuit is then disconnected from the separate source and connected to the brushes of its own armature.

The slight electromotive force generated by the armature will establish a current in the exciting circuit. This will increase the inducing flux, resulting in an increase of electromotive force and further building up of the exciting current and the flux. As the exciting current increases, the cores of the field coils approach saturation; therefore, a given increase in current results in less increase of flux. A point of balance of the electrical and magnetic effects is finally obtained where the generated electromotive force and the exciting current become constant for existing conditions of operation. The operation of setting up the magnetic flux is termed *picking up*, or *building up*, the flux.

30. Series Generator.—Another type of self-exciting machine is the series generator. The exciting coils are connected in series with the armature and the external circuit, as indicated in Fig. 16. No electromotive force, except the slight value due to residual magnetism, is generated in the armature unless the external circuit is closed and a current is established throughout the circuit. The electromotive force generated depends on the value of the current in the circuit, which consists of the

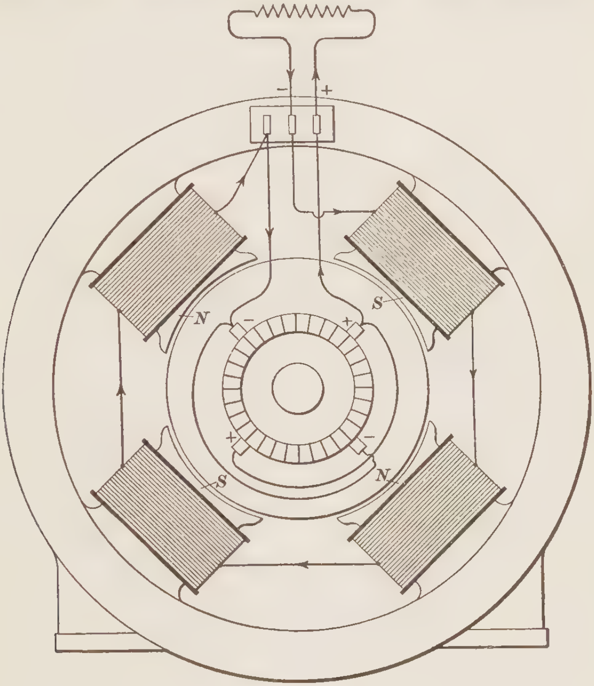


Fig. 16

field coils, the armature, and the external circuit. The field coils are formed of a conductor of comparatively large sectional area, and each coil has comparatively few turns.

31. Compound Generator.—Automatic regulation of the electromotive force of a generator may be effected by a combination of shunt- and series-field coils forming part of a compound-wound generator, connections for which are shown in Fig. 17. The shunt coils are shown at *a*; the main series coils, at *b*; and the coils for the commutating poles, at *c*. The coils for the commutating poles are connected in series with the armature and with the series coils on the main pole pieces. These commutating poles are not always used with compound generators.

In a generator as usually connected, the magnetomotive forces of the shunt coils *a* and the series coils *b* act in unison

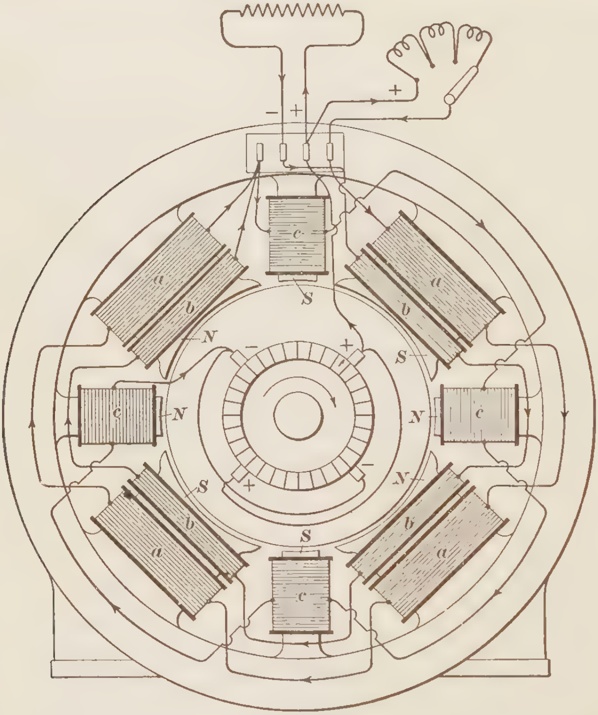


FIG. 17

to set up fluxes in the cores of the main field magnets. The magnetomotive forces of the coils *c* establish fluxes that partly neutralize the effects of the fluxes set up by the armature conductors.

When the generator is running, but is not connected to the external circuit, the magnetomotive forces of the shunt coils *a* produce the flux of the magnetic field of the machine. With the connections of the shunt coils shown, the shunt current passes through the commutating coils *c*, but as both the shunt current and the number of turns of coils *c* are small, the magnetomotive force of coils *c* is of low value.

When the external circuit is closed, current is established in the main series coils *b* and the commutating-pole coils *c*. The fluxes in the cores of the main field magnets are increased and a higher electromotive force is generated. If the load on the generator increases still further, the exciting current through the series coils increases, thus building up the fluxes in the magnet cores and causing a higher electromotive force to be generated. This higher electromotive force is desired because of the increased drop in volts necessary to force the larger load current through the armature, the external circuit, the series-field coils, and the commutating coils.

32. The preceding method of regulating the electromotive force of the generator is called **compounding.** A machine is *flat-compounded* when the magnetomotive force of the series coils is adjusted so that the voltage at the generator terminals remains practically constant for all loads. It is said to be *overcompounded* when the voltage at the terminals rises as the load increases so that some distant point on the external circuit may have nearly constant voltage for all loads.

In an *accumulatively compounded* generator, the shunt coils and series coils are so connected to their respective circuits that they act in unison to establish the inducing flux and thus to increase the generated electromotive force as the load increases.

In a *differentially compounded generator*, the connections of the shunt and series coils are such that the magnetomotive force of the series coils acts in opposition to, or *bucks*, that of the shunt coils. Generators of this kind are used in automobiles and to a limited extent for other purposes. If the speed of a variable-speed generator exceeds a given value, the electromotive force at the terminals does not increase because of the bucking effect of the series coils.

The words *accumulatively* and *differentially* are most commonly used in connection with the field windings of direct-current motors. Compound-wound machines, both generators and motors, are usually connected accumulatively, and this connection is always understood when the other is not stated.

THE ELECTRIC MOTOR

COMPARISON OF GENERATORS AND MOTORS

33. A **motor** may be defined as a machine for converting electrical energy into mechanical energy. The force necessary to maintain rotation of the motor armature and thus cause the motor to perform work is supplied by the interaction of the flux of the field magnets and that of the armature conductors.

Any direct-current generator can be operated as a motor by impressing on its terminals an electromotive force. As in the case of generators, motors may be classified according to the method of energizing their field magnets as *shunt*, *series*, and *compound motors*.

THEORY OF MOTOR ACTION

DEVELOPMENT AND DIRECTION OF TURNING FORCE

34. Motor Action of Conductor Flux.—Fig. 18 shows one arrangement of conductors on an armature intended for a four-pole motor. The conductors, lying in slots in the armature core, and the direction of current in each conductor are represented in a manner similar to that in Fig. 11. Arrows near the pole faces, Fig. 18, indicate the direction of the field flux, and a loop with arrowheads drawn around one pair of armature conductors opposite each pole face indicates the paths of the flux set up by current in these conductors. To avoid confusion of lines, only a few paths for conductor flux are shown. The denser flux on the side of each conductor where the directions of the two fluxes agree tends to move all conductors so as to cause counter-clockwise rotation of the armature, as indicated by the curved arrow near the brushes *A* and *B*.

35. As the armature rotates, successive commutator bars slide under the brushes so as to reverse the direction of current in each conductor while the conductor is passing through the neutral space. The result of commutation, therefore, is that all conductors adjacent to north poles carry current in one direction and all the conductors adjacent to south poles carry current in the opposite direction, as indicated in Fig. 18. The reaction between the flux of the poles and the conductor flux thus maintains the turning effort, or **torque**, tending to rotate the armature. While the torque due to each armature

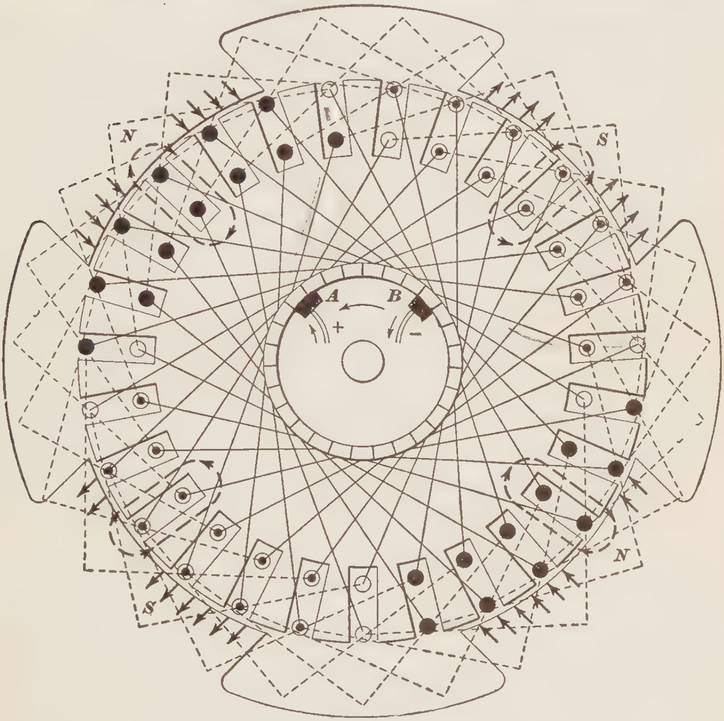


Fig. 18

conductor may be small, the total torque due to all the conductors may be very large.

36. Motor Action of Armature Poles.—The rotation of an armature may also be explained by considering the magnetic action of the poles of the field magnets on the poles formed on the armature core by the magnetomotive force of the armature windings.

In Fig. 19 is shown a four-pole motor. When current enters the armature windings at brush *A* and leaves at brush *B*, the directions of the currents in groups of conductors *C*, *D*, *E*, and *F* are as indicated. The fluxes around the individual conductors combine to form poles on the armature core midway between the tips of two adjacent pole pieces, as indicated at *N′* and *S′*. Unlike poles of the field magnets and of the armature core attract each other; therefore, in Fig. 19, a clockwise rotation of the armature is established.

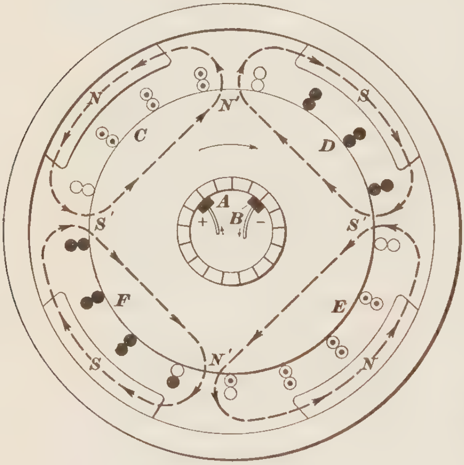


Fig. 19

The connections of conductors and commutator bars in Fig. 19 differ from those in Fig. 18. The direction of current in the conductors near corresponding poles in Fig. 19 is opposite to that in Fig. 18, thus accounting for the opposite direction of rotation.

The individual armature conductors change from one group to the adjacent group as the armature rotates, but as the direction of current in the individual conductors changes during this action, owing to commutation, the direction of the current in a group of conductors, the polarity and location of the poles formed on the armature core, and the direction of rotation of the armature remain unchanged.

The armature poles are not sharply defined, but the lines of force stray somewhat, entering or leaving the core for a considerable space each side of the points indicated by the letters N' and S'.

37. Reversal of Rotation.—The direction of rotation of the armature of a direct-current motor may be reversed either by exchanging the terminals of the exciting circuit of the field magnets, thus reversing the polarity of the pole pieces, or by exchanging the connections of the armature leads to the brushes, thus reversing the polarity of the brushes and changing the direction of the current in corresponding groups of armature conductors. For example, if the brush A, Fig. 19, were made the negative one, the current in group C conductors would be downwards and that in group D conductors, upwards, etc. This results in a reversal of polarity of the poles on the armature core and of the direction of rotation of the armature. If a south pole S' were formed at the top of the armature core, Fig. 19, it would be attracted toward the north pole on the frame, and the rotation would be counter-clockwise. If the terminals of both the field circuit and the armature windings are reversed, the direction of rotation will remain unchanged.

38. The reversal of rotation may be explained by considering the action of a single conductor. In Fig. 1, the direction of movement due to motor action is in each case indicated by the horizontal dotted line arrow near the conductor $a\,b$. This arrow in all cases is opposite in direction to the arrow indicating direction of movement of the conductor when acting as a generator. It should be noted that in (a) and (d), where the flux of the magnet is reversed in direction and the direction of the current in the conductor remains unchanged, the direction of motor action is reversed, as indicated by the dotted horizontal arrows. The direction of motor action is also reversed in (a) and (b), where the direction of the flux from the magnet is unchanged, but the direction of current in the conductor is reversed. In (a) and (c), both the flux of the magnet and the current in the conductor are reversed in direction and the direction of motor action remains unchanged.

39. If a machine that has been operating as a shunt-wound generator is operated as a shunt-wound motor, the polarity of the circuit wires and the connections between the circuit and the machine being unchanged, the armature will rotate as a motor in the same direction as it did as a generator.

Current now passes into the armature at the positive motor brushes, which are the same as the generator positive brushes. This causes a reversal of the direction of the current in the armature windings. The polarity of the pole pieces remains unchanged, because the same end of the field-coil circuit is connected to a circuit wire of the same polarity as before.

In Fig. 1 (*a*), the generator conductor *a b* is moved by some external force toward the right as indicated by the horizontal full-line arrow. In (*b*), the direction of the current in the motor conductor *a b* is reversed, but the flux of the magnet is unchanged in direction, and the direction of the motor action is toward the right, as indicated by the horizontal dotted-line arrow. The movement of the generator conductor and of the motor conductor under these conditions is in the same direction.

COUNTER ELECTROMOTIVE FORCE

40. When an armature conductor is forced by motor action to move across the flux of the field magnets, an electromotive force is generated in it. This electromotive force is usually called **counter electromotive force** (generally written counter E. M. F.), but it is also known as *motor electromotive force*, *back electromotive force*, and *back voltage*.

An armature has but a very low resistance—a fraction of an ohm in many cases—and if the armature is clamped so that it cannot rotate and the full voltage of the line is then impressed on its terminals, the windings would probably be damaged by the resulting large current.

If the armature is free to rotate, the counter electromotive force established in the active conductors acts in direct opposition to the impressed electromotive force from the power circuit and thus limits the current. As the speed increases, the counter electromotive force increases and the armature finally reaches such a speed that the opposing action of the counter electromotive force plus that due to the ohmic resistance of the windings is such that just enough current is taken by the motor to develop the required torque. In the case of a shunt motor, if the load changes, the speed varies slightly and there is automatically established a new value of the counter electromotive force that is suitable for the new value of the current required for the motor load.

The pressure that is actually effective in forcing current through the armature is the difference between the impressed electromotive force and the counter electromotive force. This difference is usually only a few volts, because the ohmic resistance of the armature is so low that only a low effective voltage is required to force the current through the windings.

EXAMPLE.—The armature of a motor has a resistance of .25 ohm between brushes and is designed to operate with an impressed voltage of 500. What would be the current: (*a*) if the armature were held stationary and the full voltage applied? (*b*) if the armature were free to rotate and a counter electromotive force of 490 volts were generated in the windings?

SOLUTION.—(*a*) If the armature is standing still, no counter electromotive force is generated; hence, the current is equal to

$$\frac{\text{impressed electromotive force}}{\text{resistance}} = \frac{500}{.25} = 2{,}000 \text{ amp.} \quad \text{Ans.}$$

(*b*) If the armature were free and generating a counter electromotive force of 490 volts, the voltage that is effective in forcing current through the armature windings is $500-490=10$, and the current is

$$\frac{\text{effective electromotive force}}{\text{resistance}} = \frac{10}{.25} = 40 \text{ amp.} \quad \text{Ans.}$$

PURPOSE OF STARTING RESISTANCE

41. In very small motors, the voltage of the line is impressed directly on the armature terminals, because these armatures have a comparatively high ohmic resistance. In larger motors, the impressed voltage is adjusted to a lower value for starting the motor by the insertion of an adjustable resistance, called a *starting rheostat*, in the armature circuit. As the speed and counter electromotive force of the armature increase, the resistance of the rheostat is gradually cut out of circuit until, finally, the armature is connected directly across the line wires. Rheostats and other auxiliary devices intended for the control of motors are described in a later Section.

COMMUTATING-POLE MOTORS

42. The poles on the armature core formed by the current through the windings, as shown in Fig. 19, cause a magnetic flux in the space midway between the poles, so that the armature conductors of a coil when moving through this space may have an electromotive force generated in them during the time that the coil is short-circuited by the brush. The generation of a very small electromotive force under these conditions may cause a considerable current in the coil and sparking at the commutator under the brushes. The action of commutating poles is to oppose and prevent the disturbing flux in the space where the conductors of a coil are short-circuited. These poles therefore establish between the poles a space in which commutation can occur with practically no sparking.

The relative location of the main poles and the commutating poles is the same in a motor as in a generator, Fig. 17, but a commutating pole of a motor has the same polarity as that of the adjacent main pole behind it, considering the direction of rotation of the armature as forward. The exciting coils for the commutating poles are in series with the armature; therefore, if the polarity of the armature poles is reversed, the polarity of the commutating poles is also reversed. This arrangement permits of good commutation with fixed brushes at all loads and in either direction of rotation of the armature.

DIRECT-CURRENT GENERATORS

CLASSIFICATION

1. An **electric generator** is a machine for converting mechanical energy into electric energy. If the flow of electric energy, or electricity, from the machine is continuously in one direction, the machine is a *direct-* or *continuous-current generator;* if the current of electricity alternates in direction, the machine is an *alternating-current generator*, or an *alternator*. Alternators and other alternating-current machinery are discussed in other Sections.

2. According to output, direct-current generators are classed as *constant-potential*, or *constant-voltage, machines* and *constant-current machines;* constant-voltage machines are much more generally used. Voltages of 110 to 125 are common for electric lighting and for the operation of some comparatively small motors; 220 to 250 volts are used for most industrial motors and for some systems of lighting; and 500 up to 2,400 volts are used for direct-current electric-railway systems. For 2,400 volts, however, two 1,200-volt machines are connected in series.

Constant-current generators are little used in America, except for some old systems of electric lighting in which arc lamps are connected in series. Abroad, however, constant current is used in some large systems for transmitting electric energy long distances. One of these systems of transmission is known as the *Thury system*, from the French engineer who was most active in developing it. It has some decided advantages that will probably bring it into more general use.

COPYRIGHTED BY INTERNATIONAL TEXTBOOK COMPANY. ALL RIGHTS RESERVED

3. According to the usual methods of driving generators, they are classed as *direct-driven*, *belt-driven*, *gear-driven*, etc. Generators direct driven by steam engines are generally called *engine-driven*, or *engine-type*, *generators;* those direct driven by steam turbines are called *turbine-type generators*, or *turbo-generators*. All generators for direct drive must be designed to develop their outputs at the speeds of their drivers, or prime movers; those for belt or gear drive may be designed for any speed within the limits made possible by practicable pulley and gear ratios.

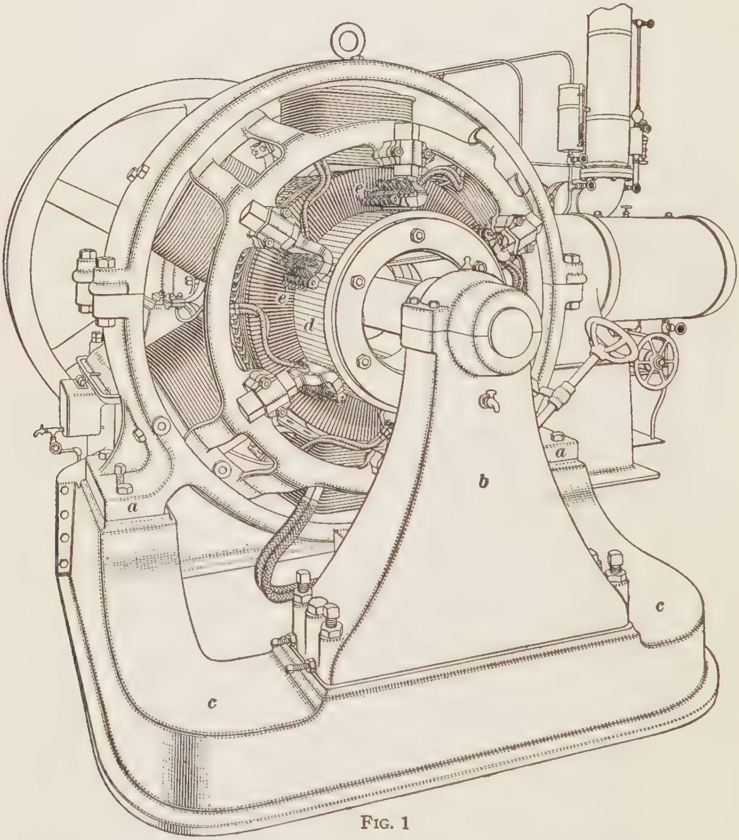


FIG. 1

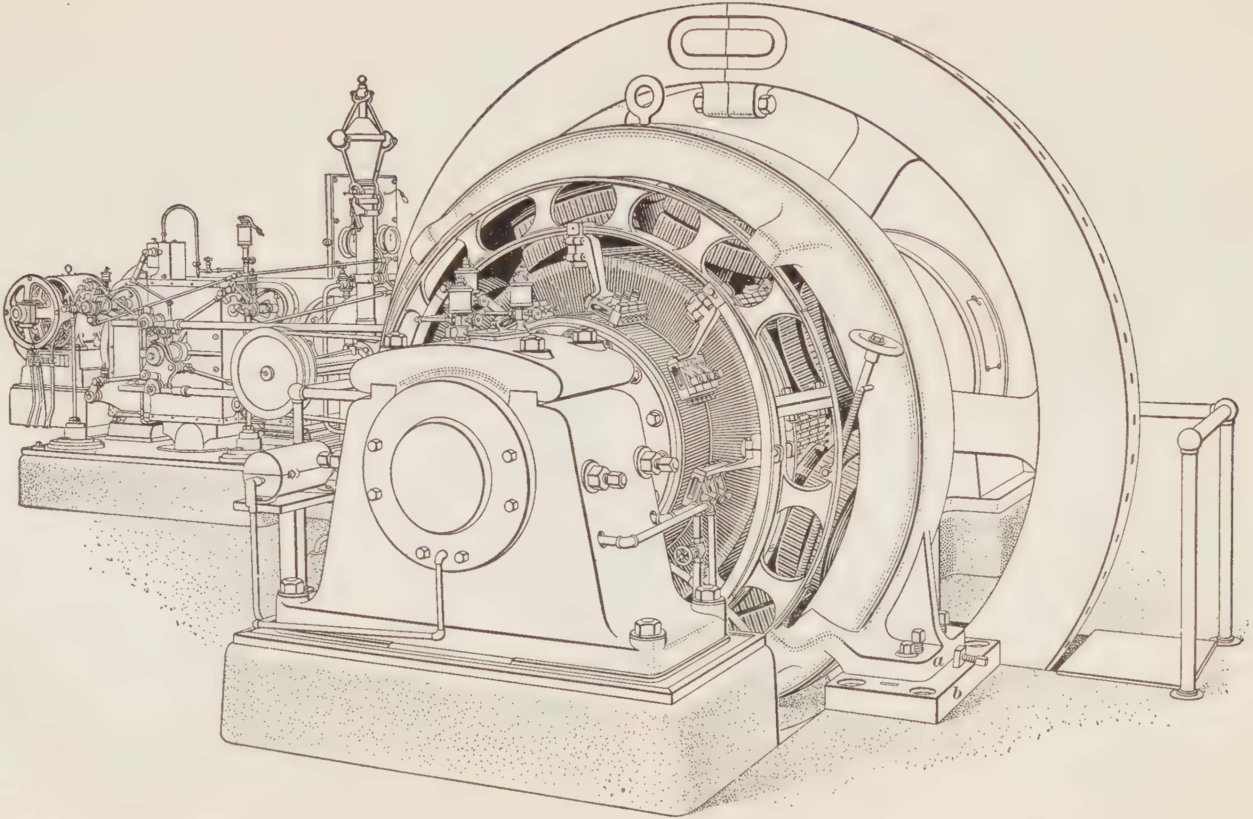


FIG. 2

4. Fig. 1 shows a typical engine-type generator. The feet *a* of the generator frame and the pedestal bearing *b* rest on an extension *c* of the engine base, or bedplate. Fig. 2 shows a larger engine-type generator supported on a separate foundation; the feet *a* rest on *sole plates b*, and the bearing pedestal stands on a concrete foundation. The shafts, bearings, and bearing pedestals of engine-type generators are usually supplied by the engine builders as parts of the engine.

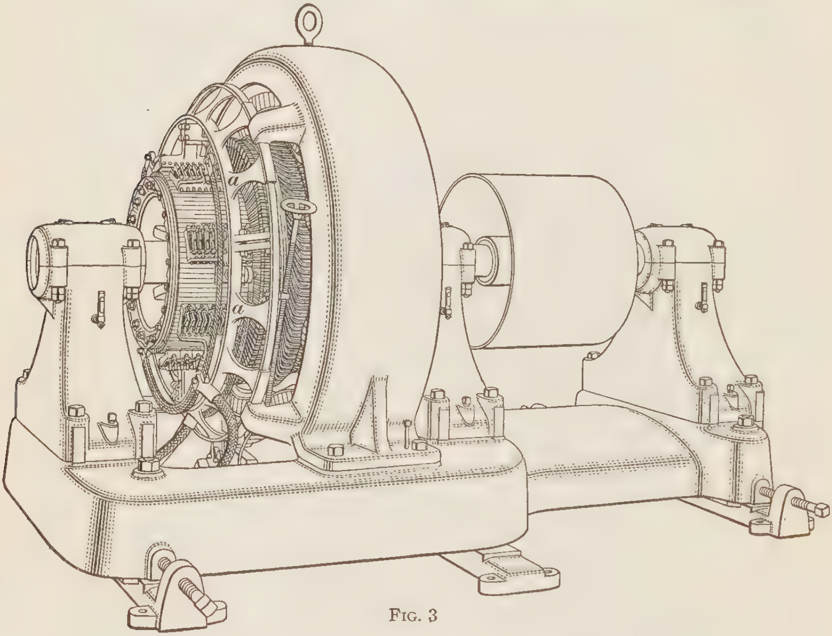


Fig. 3

Fig. 3 shows a three-bearing belt-driven machine with a base, bearing pedestal, slide rails, and tension screws for shifting the generator to adjust the belt pull. Fig. 4 shows a steam turbo-generator resting on a base that also supports the turbine, making the whole a complete, compact unit.

5. Direct-current generators are also classed as commutating-pole and non-commutating-pole machines. On generators for large outputs at high speeds, smaller poles placed midway between the main poles and excited by current in coils connected in series with the armature greatly improve commutation; such poles are called *commutating poles*. In Fig. 4 a commutating pole is shown at *a* and a main pole at *b*. Other special features are added in some cases to aid commutation, as will be explained later. Commutating poles are usually omitted from small generators, especially those operating at slow speeds, because the advantages gained by their addition are not enough to warrant the additional expense involved in their construction.

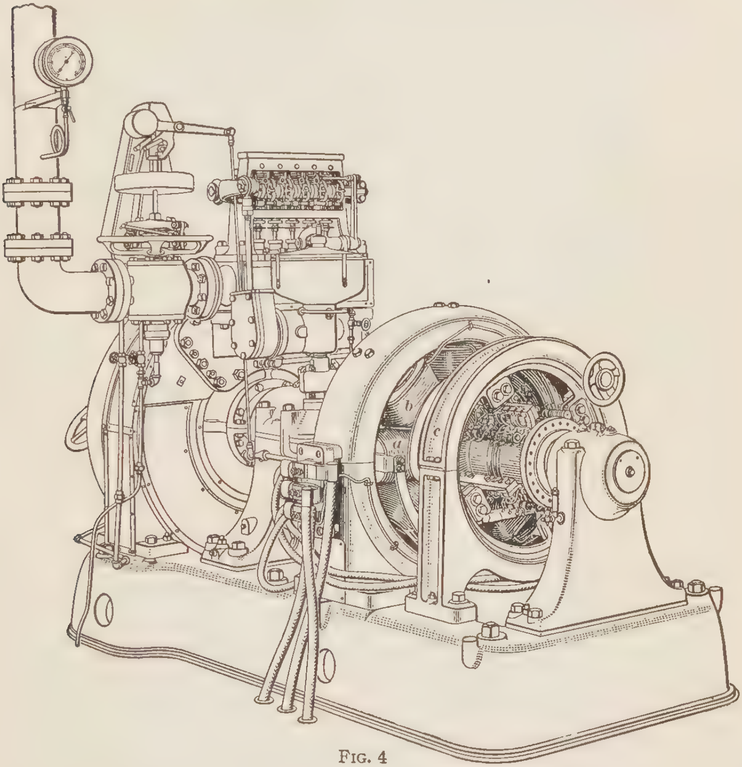


Fig. 4

CONSTRUCTION

GENERAL ARRANGEMENT

6. The essential features of a direct-current generator are a field structure and an armature with a current-collecting device. The armature consists of a laminated iron core, cylindrical in form, on or near the surface of which are conductors forming the armature windings. The current-collecting device is nearly always a commutator, but collector rings are used on one type of direct-current machine described later. The field structure usually consists of a circular yoke with inwardly projecting radial poles; in general, each pole is provided with an exciting coil, or field coil.

ARMATURES

CORES

7. As an armature rotates, the direction of magnetic flux in every part of the armature core changes. Energy is required to change the direction of flux in iron; this energy serves no useful purpose and is therefore a loss, called **hysteresis loss.** The hysteresis loss is lower in some grades of iron and steel than in others, and a special grade of steel, called *electrical steel,* in which the hysteresis loss is low, is used for the cores of direct-current armatures in order to lessen the hysteresis loss in them.

8. Furthermore, the continual change in direction of magnetic flux in the body of the core causes the development of electromotive forces that would set up currents in a solid iron core somewhat as shown in Fig. 5. This illustration represents the lower half of a solid core, and the closed loops with arrowheads represent the direction of local currents in such a core.

These so-called *eddy currents* constitute a loss because they serve no useful purpose. The strength of eddy currents, or the **eddy-current loss,** depends on the resistance of their paths. In order to make this resistance high, and the eddy-current loss low, armature cores are practically always made of thin disks, or laminations, punched from electrical-steel plates. The thinner these laminations are, the lower is the eddy-current loss. For high-speed machines in which the flux changes are rapid, the laminations are usually punched from sheets .014 inch thick. For slower speeds, thicker laminations are permissible, .02-, .03-, and even .04-inch laminations being used in some cases. The punchings are separated from one another by some insulating material, such as paper, japan, paint, varnish, or enamel, in order to prevent the formation of eddy currents between adjacent punchings.

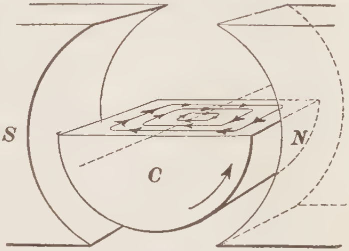


FIG. 5

9. Hysteresis loss and eddy-current loss are difficult to determine separately and are often referred to collectively as **core losses** and sometimes as *iron losses*. The effect of these losses is to heat the core, so that an armature rotating in a magnetic field becomes warm even when no electricity is flowing in the armature conductors.

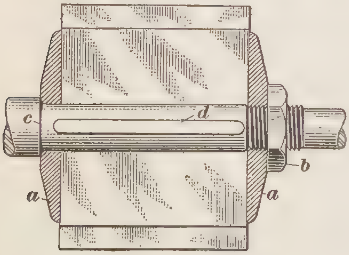
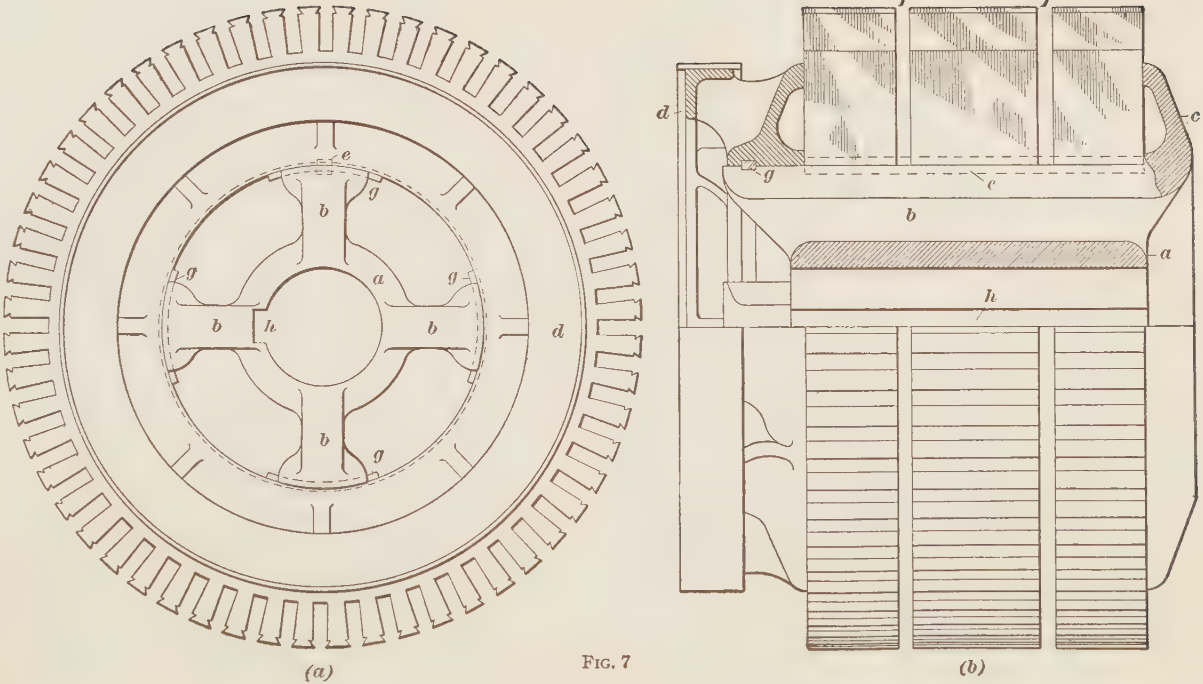


FIG. 6

10. The punchings of the armature core are usually held in line by a shaft or an armature spider passing through a central hole, and they are clamped together between end plates. In Fig. 6 is shown a section of an armature core for a bipolar generator. The punchings are held firmly in place between the end plates *a*, usually of cast iron, which are clamped between a nut *b* and a shoulder *c*, on the shaft. A



(a)

(b)

FIG. 7

key in the keyway *d* projects into a suitably cut notch in each punching, to prevent the core from turning on the shaft.

11. The construction of another form of armature is shown in Fig. 7, in which view (*a*) represents an end view showing the method of supporting the armature core, and view (*b*) a longitudinal, or side, view, the upper part being in section. Each part visible in both views is referred to by the same letter in both. The punchings are mounted on a cast-iron spider consisting of a hub *a*, four arms *b*, and an end flange *c*, all cast in one piece. The rear end flange *d* is a separate casting, and its central hole is bored out to the same diameter as the central hole in the punchings, so that it may slip over the arms of the spider. The punchings are notched to fit the key *e*, which is inserted in the arm *b*. At intervals in the core are spacers, or vent plates, consisting of strips on edge attached to an armature punching. They provide the core with radial air passages, *f* called *vents*, *ducts*, or *flues*, through which air circulates when the machine is running and helps to keep it cool. The rear plate, or flange, *d*, is pressed on and small keys *g* are inserted in the grooves in the spider arms to secure the parts together. A keyway *h* is provided in the hub of the spider for securing it to the shaft. A ring for supporting the rear end windings is cast with the flange *d*, and the arms that support this ring are arranged to fan air through the windings.

12. In Fig. 8 are two sectional views showing the construction of an armature and a commutator for a large engine-type generator. The armature is of too great a diameter for the punchings to be made in a single disk each, so they are made in segments. Each segment has two dovetail lips *a* that fit into accurately machined recesses in the arms of the spider. The punchings are so assembled that the joints *b* in one disk are midway between the joints *c* of the next disk. The punchings are clamped between the end plates *d* and *e* by the bolts *f*. These end plates are centered by providing them with flanges *g* machined to fit the spider.

It is customary to arrange the armature of an engine-driven generator so that it can be shipped complete, either with or

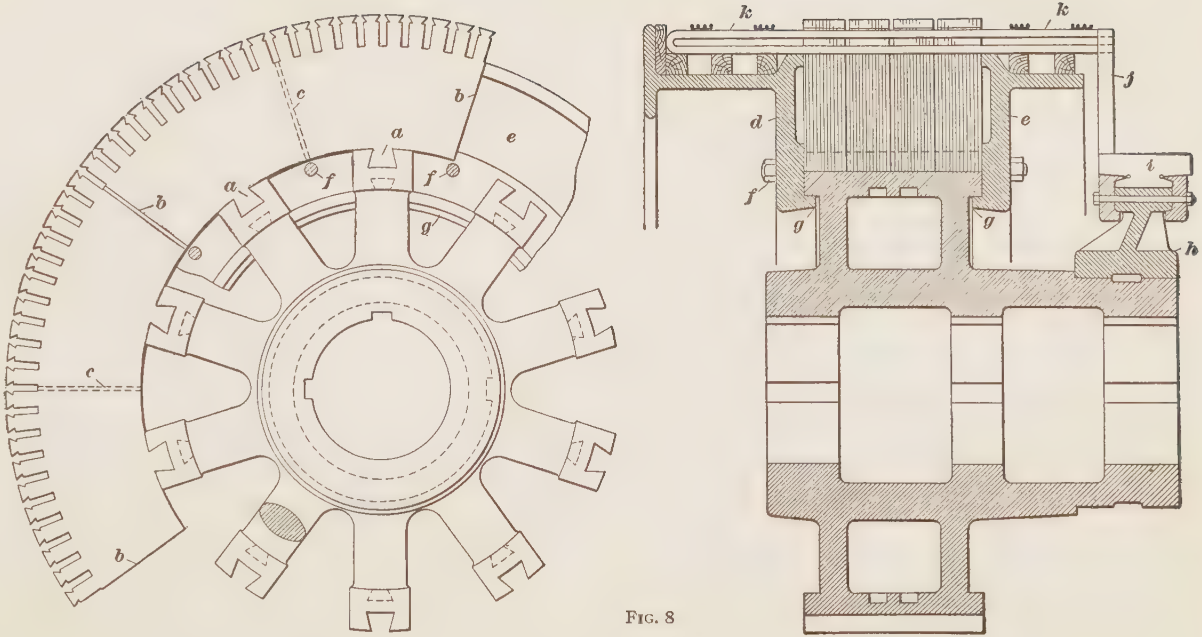


FIG. 8

without the engine shaft. The hub of the armature spider must therefore be extended, as shown in Fig. 8, so as to support the commutator spider *h*. The commutator segments *i* are clamped in place, and the connections *j* lead to the windings *k*.

COMMUTATORS AND BRUSHES

13. The general appearance of finished commutators can be seen in the preceding illustrations, as at *d*, Fig. 1, and the brushes can be seen at *e*. A commutator usually consists of hard-drawn or drop-forged copper segments alternating with mica or micanite segments, all clamped together by *wedge rings*, either at the ends or underneath the segments, or by *shrink rings* over the top of the segments. The wedge rings and shrink rings are insulated from the copper segments by micanite cones and bands. *Micanite* consists of very thin pieces of mica pasted together with shellac or other varnish. The commutator shown in section in Fig. 8 illustrates the method of clamping the segments *i* from the under side. The wedge rings may be made either in a single piece or in segments. Wedge-ring commutators are used for generators of slow and moderate speeds, while shrink-ring commutators are used for high-speed machines.

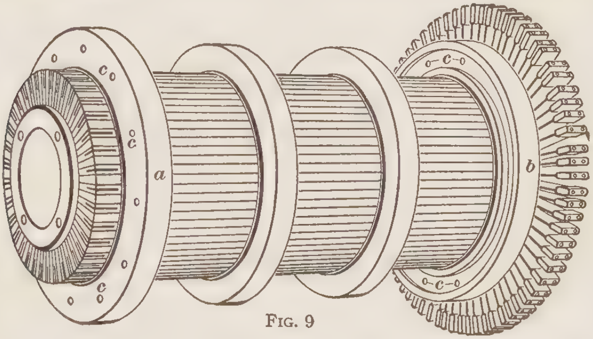


Fig. 9

14. A commutator of the shrink-ring type is shown in Fig. 9. The outer and inner rings *a* and *b* are drilled and tapped at *c* for the insertion of small balance weights. This commutator is intended to be pressed directly on to the shaft, and not to be mounted on the armature spider.

15. The commutator diameter is customarily made from two-thirds to three-fourths the diameter of the armature, and the length must be great enough to give the required current capacity. Low-voltage generators therefore have longer commutators than high-voltage generators of the same power output. The current is usually collected by carbon brushes rubbing on the surface of the commutator, and for the best results the current density in the rubbing surface should not generally be over 40 or 45 amperes per square inch. As the brushes must be of such thickness as to touch not more than two or three commutator bars simultaneously, several brushes must sometimes be arranged side by side to obtain the necessary carrying capacity.

On very low-voltage, large-current generators, brushes of metal or of metal-and-graphite compound are used, and in them the current density may be from 100 to 200 amperes per square inch of conducting surface. This high current density permits the use of commutators that are much shorter than would be required with carbon brushes.

16. High-speed generators, as shown in Fig. 4, must have commutators with comparatively small diameters in order to prevent high centrifugal stresses. The required commutator surface must then be obtained by increasing the length. The commutator shown in Fig. 4 is so long that the brushes are carried by a special yoke *c* over the center of the commutator instead of being supported from the magnet frame in the usual way.

17. The **brushes** of all direct-current generators are stationary and are held against the commutator by springs. The brushes are usually attached to a ring or its equivalent that can be rotated or rocked around the commutator so as to shift them into the position for best commutation. The rocker ring is shown at *a*, Fig. 3, and is inside the special yoke *c*, Fig. 4. In each illustration is shown a hand wheel for shifting the position of the brushes by means of a screw.

ARMATURE WINDINGS

18. The voltage generated in a single armature conductor is usually so small that many such conductors must be connected in series on a generator armature in order to obtain the required voltage. Armature windings are connected with this point in view. They are of two general types, *ring windings* and *drum windings*.

19. Ring Windings.—Ring windings are comparatively little used, but their general features should be understood. Fig. 10 shows the principle. A laminated iron ring forms the armature, the supporting structure of which is not here shown. The winding is a continuous spiral, which is connected at regular intervals with the segments of a commutator. The illustration shows every other turn so connected, but several turns may sometimes intervene between segments.

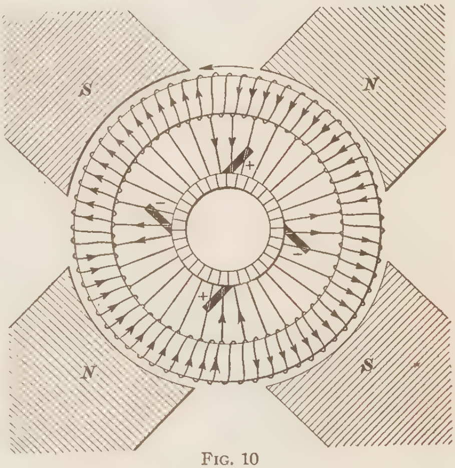


FIG. 10

Only the conductors on the side of the ring nearest the pole faces cut lines of force, except a few stray lines, for practically all the flux follows the ring between adjacent poles instead of crossing the open space in its center. With the direction of rotation counter-clockwise, as indicated by the curved arrow above the ring, Fig. 10, the direction of electromotive force in the armature conductors must be as shown by the arrowheads on those conductors; that is, the flow of electricity is from the negative brushes to the positive brushes through the armature and from the positive brushes to the negative brushes through the external circuit, which is not represented.

20. Drum Windings.—Drum windings consist of coils, both sides of which cross the face of the armature core where they pass close to the pole faces and thus cut lines of force. In rare cases, the coils are laid on the surface of a smooth core and are held in place by projecting pins and by bands around the core over the completed windings. But usually the sides of the coils lie in slots in the surface of the core, as shown in Figs. 11 and 12; wedges in grooves near the tops of the slots or bands of wire over the core hold the coils in place.

21. Drum windings are of two kinds, *series*, or *wave*, and *parallel*, or *lap*. Fig. 11 shows a bundle of four series coils of a single turn each arranged for assembling in the slots. For simplicity, the armature core is shown as being flat instead of in its true cylindrical form, and only the slots occupied by the coil are indicated. Each coil is separately taped to insulate it from the others, and then the bundle is taped to insulate the coils from the core and from the supports for the end windings.

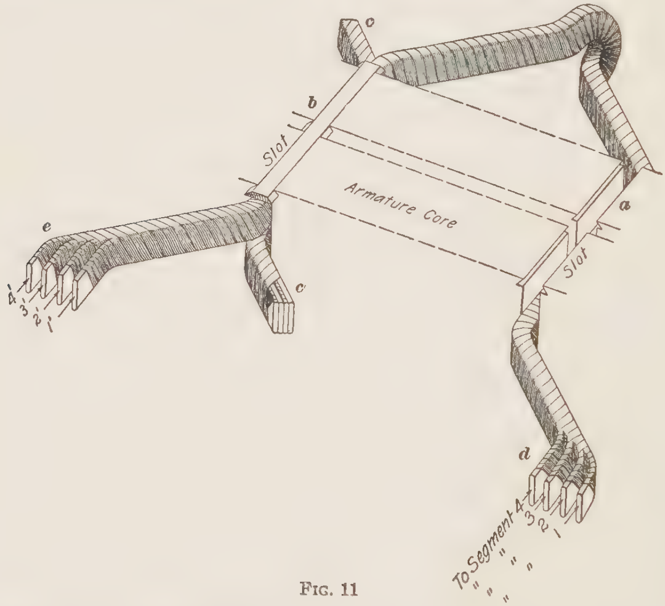


FIG. 11

The bundle of coils in Fig. 11 is so placed on the armature core that one side lies in the bottom of slot *a* and the other side in the top of slot *b*. In the bottom of slot *b* is shown a part of a coil *c* exactly the same as the bundle of coils *a b*. In the completed armature, each slot will have a bundle of conductors in the top and a bundle in the bottom, and is therefore said to have four coils per slot. The slot *b* is shown completely filled with its conductors, and a retaining wedge is shown over the coils. The coils are so made as to span the slots from *a* to *b*, this distance being approximately that of the *angular pole pitch*, or the distance between the center lines of adjacent field poles. The coils at the end *d* are spread out to fit into four commutator segments. At the end *e*, the coils are again spread out, to go into other commutator segments, and *1′*, *2′*, *3′*, and *4′* are, respectively, the other ends of the coils marked *1*, *2*, *3*, and *4* at *d*.

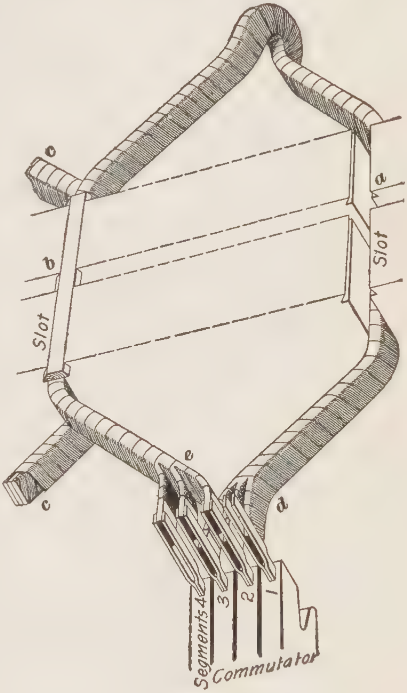


FIG. 12

22. In Fig. 12 is shown a bundle of three parallel coils connected to commutator segments *1*, *2*, *3*, and *4*, The sides of the coils lie in slots *a* and *b*, spaced about a pole pitch apart; a side *c* of another bundle of coils is shown in the bottom of slot *b*, making a winding with three coils per slot. All slots are similarly fitted in the completed armature. The coils at the ends *d* spread out and are connected to the lower part of the commutator necks of segments *1*, *2*, and *3*, while the ends *e* connect to the upper part of the necks of segments *2*, *3*, and *4*, so that coil *1* begins at segment *1* and ends at segment *2*. Coil *2* begins at segment *2* and ends at segment *3*, etc. It is true of all parallel-type windings that if one end of a coil connects to a certain segment, the other end will connect to the next segment, either to the right or to the left of it.

23. In a series winding, the ends of each coil, as at d and e, Fig. 11, are separated by about the pitch of two poles. A series of $\frac{P}{2}$ such coils placed end to end will therefore extend around the commutator, P representing the number of poles. If the ends d and e were separated exactly two pole pitches, the series would end at the exact starting point; but the number of segments in the commutator of a series-wound armature is so selected that a series of $\frac{P}{2}$ coils will extend around to a segment next to the starting point, one side or the other. The spread of the coil from d to e is usually stated by the number of commutator segments between the ends of the coil, as from *1* to *1′*, including the segment to which one end of the coil is attached, as *1* or *1′*. This spread may be called the *commutator connecting pitch*, and may be represented by Y. Then the number of segments in the commutator is one more or less than $\frac{P}{2} Y$; in other words, the number of segments must be $\frac{P\,Y}{2} \pm 1$.

24. The series winding in most common use has only two paths, or circuits, between positive and negative brushes, but a parallel winding has as many paths as there are poles. For a two-pole machine, each winding has two paths, and the two windings are practically identical. If C is the number of coils on an armature and P the number of poles of the field, the number of coils in each path of a series winding is $\frac{C}{2}$ and in each path of a parallel winding $\frac{C}{P}$. When the number of poles is greater than 2, the series winding has fewer paths but more coils in series in each path. If the same size of conductor is used in both windings and the same voltage generated in each coil, the parallel winding has $\frac{P}{2}$ times as much current capacity as the series winding, and the series winding will generate $\frac{P}{2}$ times as much voltage as the parallel winding, but both armatures have the same power capacity.

25. To obtain low voltage from a generator, a parallel winding with one turn per coil, Fig. 12, and a small number of coils is used. In designing machines for higher voltages, the number of coils and segments is usually increased until the limit of the number of segments that can be made into a commutator of a given diameter is reached. Beyond this limit, a series winding with one turn per coil, Fig. 11, can be used. For still higher voltages, the number of segments and

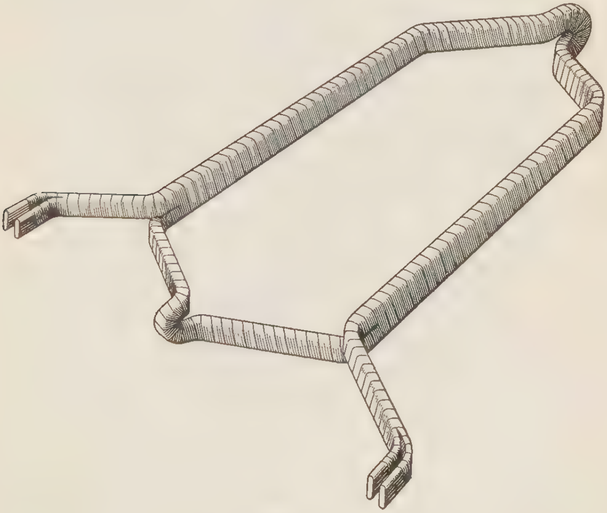


Fig. 13

coils can be increased, and, when the maximum number of segments for the commutator is again reached, then a series winding with two or more turns per coil can be used. Fig. 13 shows a group of two coils with two turns per coil.

As series windings are preferable for high voltages, parallel windings with more than one turn per coil are not often used, and those with more than two turns are very rare. Series windings are seldom used in machines having ten or more poles.

26. Figs. 14 and 15 illustrate completed armatures, that shown in Fig. 14 having a series winding and that in Fig. 15 a parallel winding. The two windings can be distinguished by noting the way in which the ends, or leads, of a conductor turn on leaving a slot. In a series winding, the two ends of each conductor turn in opposite directions, and in a parallel winding they turn in the same direction. In Fig. 14 a conductor in slot *a*, for example, turns up at one end of the slot and down at the other; in the parallel winding, Fig. 15, the conductor in

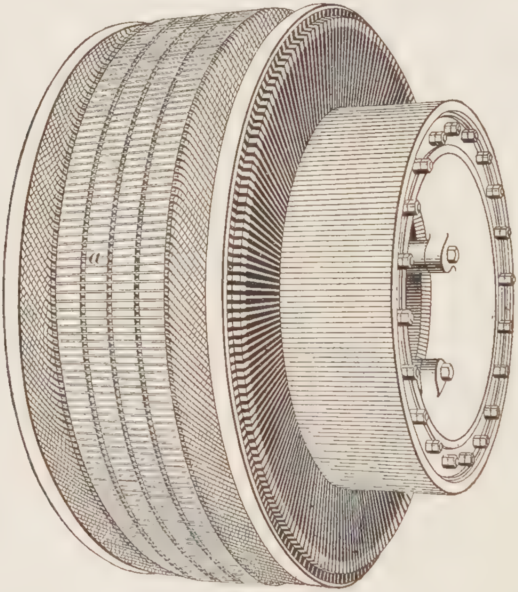


Fig. 14

slot *a* turns up at both ends of the slot. When the leads, or end connections, of the slot conductors form one cylindrical surface with the armature core, as in Fig. 15, the name *barrel winding* is sometimes applied.

27. Connection diagrams for drum windings are shown in Figs. 16 and 18. Fig. 16 shows the connections of a series-wound armature for a four-pole generator. For sake of simplicity, an armature with only seventeen slots, seventeen coils, and seventeen commutator bars is represented, and the brushes A, A' and B, B' appear to be inside the commutator instead of in their true position outside. Brushes of like polarity are cross-connected; these cross-connections on a generator are often called *bus-rings*.

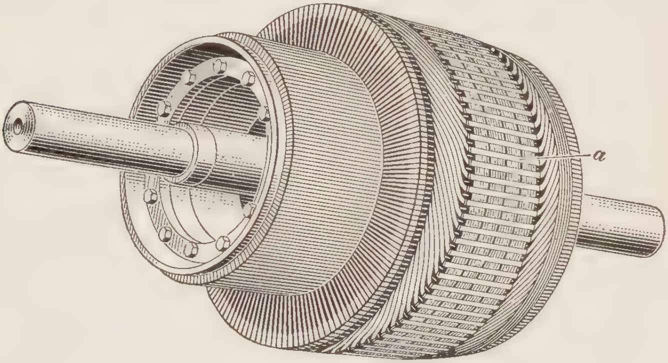


Fig. 15

The commutator segments *a*, *b*, etc. are connected with the slot conductors *1–1*, *28–28*, *3–3*, *30–30*, etc. by the *coil leads*, full lines representing leads in the upper layer and dotted lines those in the lower layer. Two conductors lie in each slot, as shown in Fig. 17, and the coils are completed by the rear end connections, which are represented by curved lines forming the outer part of the diagram, Fig. 16.

The four poles N, S, N, S are represented by dotted outlines, and the armature is assumed to be rotating clockwise, as indicated by the curved arrow across slot *3*. Under these conditions the direction of electromotive force induced in each slot conductor must be from the front end of the armature toward the rear under north poles, and from rear to front under south poles.

At the instant represented, electricity enters the armature winding by way of brush *B* and commutator segment *e* and

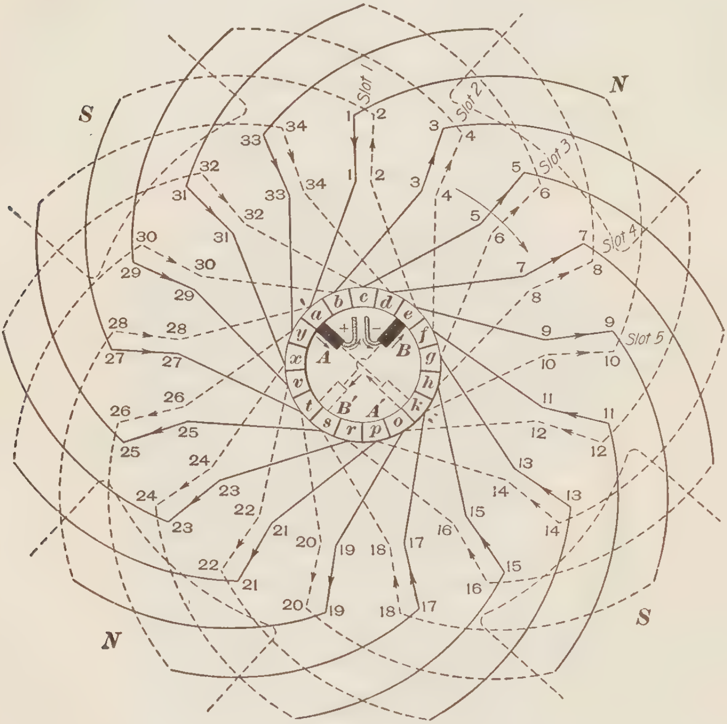


Fig. 16

leaves by way of segment *a* and positive brush *A*. The two paths through the armature are as follows:

1. *e–2–27–t–20–11–f–4–29–v–22–13–g–6–31–x–24–15–h–8–33–y–26–17–k–10–1–a.*

2. *e–9–18–s–25–34–d–7–16–r–23–32–c–5–14–p–21–30–b–3–12 o–19–28–a.*

Only two brushes are needed to make connection with these two paths, but two additional brushes *A′* and *B′* can be used if desirable, to gain additional brush contact surface without lengthening the commutator. If these additional brushes are used, current can enter path *1* at segments *e* and *t* and leave at segments *k* and *a*; path *2* can be entered at *e* and *s* and leave at *a* and *k*.

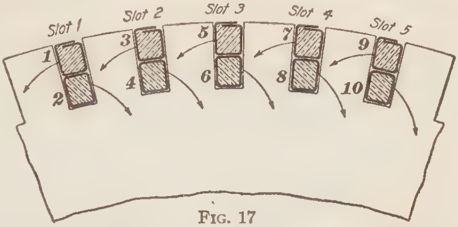


FIG. 17

28. Fig. 18 shows the connections of a parallel-wound armature with seventeen coils, seventeen commutator segments, and seventeen slots. In this case, there must be four brushes in order to make connection with each of the four

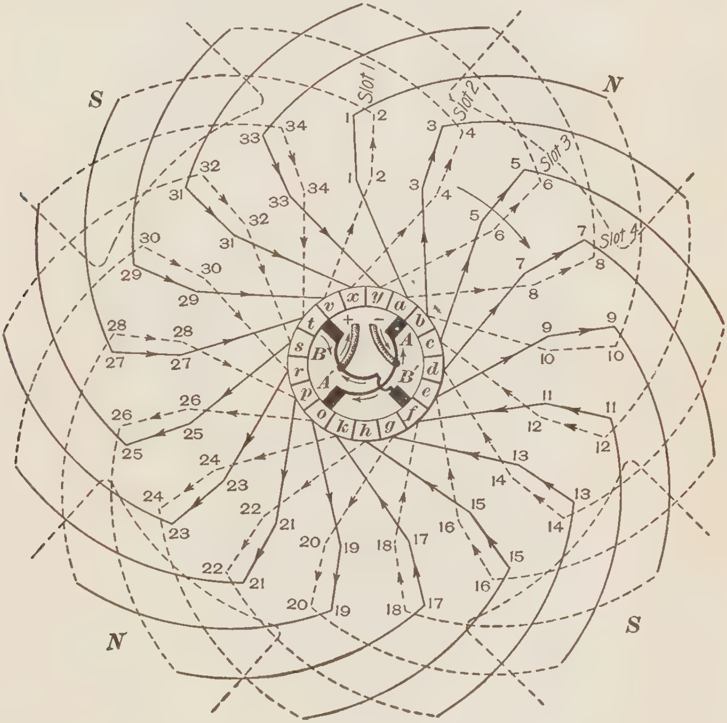


FIG. 18

paths through the armature. At the instant represented, electricity enters the armature by way of commutator segments *a*, *b*, and *o* and leaves by way of segments *f* and *t*. The coil, including conductors *1* and *10*, is short-circuited by brush *A* and therefore carries none of the current of the external circuit; the four paths through the remaining conductors can be traced as follows:

1. *a–8–33–y–6–31–x–4–29–v–2–27–t.*
2. *o–19–28–p–21–30–r–23–32–s–25–34–t.*
3. *b–3–12–c–5–14–d–7–16–e–9–18–f.*
4. *o–26–17–k–24–15–h–22–13–g–20–11–f.*

29. With either type of winding, the direction of current in the circuits remains the same at every instant, but each coil is passed on from one circuit to the next every time the commutator bars with which it is connected pass a brush. For example, in Fig. 16, as soon as segment *d* passes under brush *B*, the direction of current in the coil *d–34–25–s* will be reversed, making it agree with the direction indicated in coil *e–2–27–t*.

30. The coil leads may be of equal length so as to bring the correct position of the brushes opposite the pole centers, as indicated in Figs. 16 and 18, or one lead to each commutator segment may be enough longer than the other to bring the correct brush positions opposite the spaces between poles, the choice depending on which position makes the brushes most accessible. In both cases, the coils short-circuited by the brushes are always in the neutral spaces.

31. The winding shown in Fig. 16 is called **single series.** In addition, another type of series winding, called **double series,** is sometimes used. Double-series windings are not common, however, in modern generators. In such windings, a series of $\frac{P}{2}$ coils ends in a segment next but one to the segment at which it starts. Such a winding is thus in two parts, each connected with alternate commutator segments and each affording two paths for current through the armature. The brushes must be wide enough to make contact with at least three commutator segments, in order that both windings shall be active.

32. With parallel windings and many poles, commutator bars or slot conductors between which the voltage should be equal at every instant, that is, those exactly two pole pitches apart, are usually **cross-connected** in order to equalize the current in the several paths. Fig. 19 shows a generator armature with seven cross-connecting rings connected with the armature winding by leads *S*. Each ring must be connected at equally spaced points two pole pitches apart, and as in this case each ring has seven leads, this armature must be for a fourteen-pole machine. The rings are sometimes placed on the commutator end and connected with commutator segments instead of on the rear end as shown.

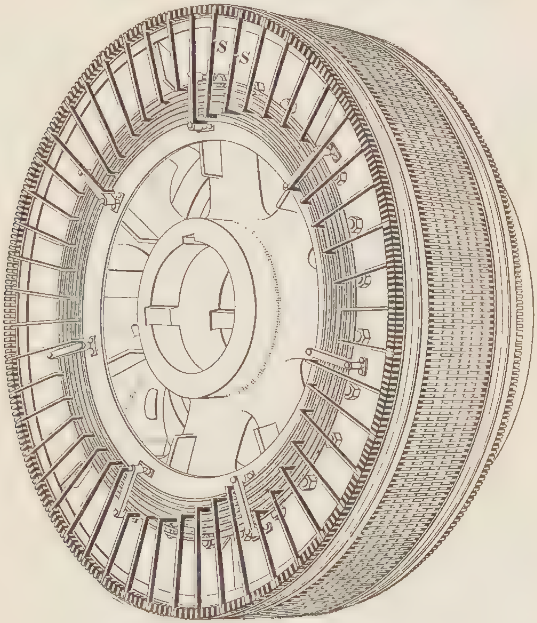


FIG. 19

Without cross-connecting rings, unequal division of current may occur among the several paths; in fact, owing to inequalities in the reluctances of the magnetic circuits, the voltage developed in the conductors of one path may be enough greater than that developed in other parallel paths to cause large local currents in the armature circuits so that part of the winding acts as a generator and part as a motor. Heavy mechanical stresses are thereby set up in the machine; the manifestations are vibration and trembling of the whole structure, with sparking and flashing at the commutator. Properly connected equalizing rings prevent such disturbances.

FIELD MAGNETS

33. The magnet yoke or field frame of a generator is usually made of soft cast steel, of sheet-steel punchings, or of cast iron. The yoke forms a path between poles for the magnetic flux established by the field coils and also supports the poles and their coils. The yoke is bolted either to a cast-iron base or to a foundation, and it must have suitable feet for its support. Yokes made of punchings are provided with cast-iron or steel clamping rings having feet attached.

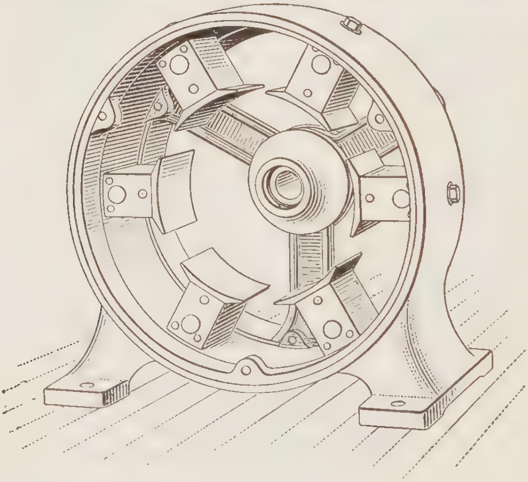


FIG. 20

The pole pieces, except in very small machines, are made of cast steel or of laminations punched from steel sheets. High magnetic permeability of materials used for pole pieces permits the use of small pole pieces and, therefore, small field coils. The poles are usually made separate from the yoke and bolted to it, as shown in Fig. 20. The magnet yoke here shown is a soft-steel casting, and the pole pieces are made of steel punchings riveted together. In assembling the machine, the field coils are slipped over the pole pieces and the latter are then bolted to the yoke. If the pole pieces and the yoke are of the same material, they may be made in one piece; if of different materials, the poles are sometimes placed in the mold and the yoke is cast around them, thus forming a solid unit.

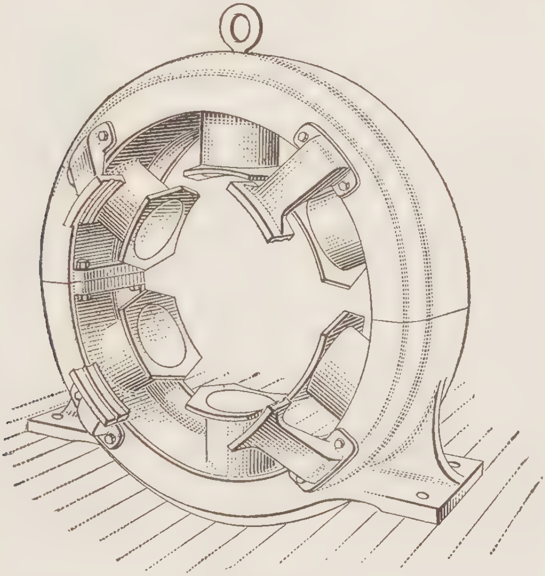


Fig. 21

Fig. 21 shows a cast-iron magnet yoke with solid-steel poles cast in, and Fig. 22 shows similar construction with laminated poles. In each case, cast-iron collars are provided for the ends of the poles next to the armature in order to retain the field coils and to spread the flux over the armature surface.

The adjoining surfaces of all joints in the magnetic circuit are accurately machined and securely bolted together so as to make the reluctance of the magnetic circuit as low as possible. Such joints occur between the poles and the yoke in the construction shown in Fig. 20, and between the halves of the yoke in the construction shown in Figs. 21 and 22.

34. The magnetic flux usually passes across the air gap between the armature core and the pole faces in tufts, owing to teeth of the core. The flux follows the teeth to their ends and spreads in traversing the gap. If the length of the air gap measured from the top of the armature teeth to the surface of the poles is less than a half of the opening between the tops of the teeth, tufts of magnetism will be effective at the pole faces, and if the faces of the poles are of solid iron or solid steel, eddy currents will be developed as these tufts sweep across them. In such machines, the pole pieces may be advantageously made up of laminations in order to reduce eddy currents. These eddy currents are present only in the pole faces, because in the iron the flux promptly spreads to a uniform density. The pole punchings may be riveted together securely, because

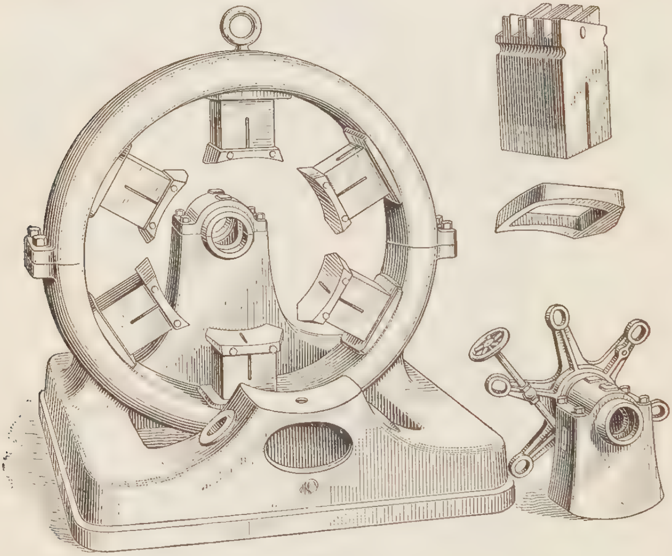


Fig. 22

no change of magnetic flux occurs in the body of the pole to cause eddy currents in the rivets. Rivets cannot be used in the armature core, however, because the flux continually changes throughout the core body. In some constructions, laminated pole shoes are bolted to solid poles.

35. Generators may be provided with shunt, series, or compound field windings, but compound windings are most common because they are more suitable for obtaining the constant voltage generally required. Shunt- and series-wound generators are used only for special purposes. Generators with fields excited by current from some external source are also sometimes used; such generators are said to be *separately excited.*

The field coils may be wound on spools, or bobbins, or on a form, and then be bound together securely with tape, rope, or some other insulating material. Shunt coils are practically always composed of cotton-insulated copper wire; series and commutating pole coils also are sometimes made of insulated round wire and sometimes of rectangular copper or sheet-copper strips. Coils of only a few turns each for carrying large currents are sometimes made of cast copper. Shunt field coils and sometimes other coils of fewer turns and larger conductors are frequently insulated by the impregnation process, whereby hot insulating compound is forced into every opening in the coil, where it solidifies, making the coil a solid mass that is impervious to moisture and capable of rapid heat dissipation.

36. Field-coil spools are made with a sheet-metal body, often sheet iron, but with spool heads, or ends, preferably of some non-magnetic material, such as brass, fiber, wood, etc. If the spool heads were made of iron, some magnetic flux would follow them and leak across between the magnet poles instead of following its useful path through the armature. Field coils are sometimes made with ducts or passages through which air can pass. This practice is resorted to in order to obtain lower operating temperature or to permit the use of lighter field coils with a given temperature.

DESIGN FEATURES DETERMINING OUTPUT

DIMENSIONS

37. The product of the number of conductors on the surface of an armature and the number of amperes in each conductor is the number of **ampere-conductors,** and the quotient obtained by dividing this number by the circumference of the armature in inches is the number of **ampere-conductors per inch.**

On any given armature, the number of ampere-conductors is approximately the same whether there are few conductors with large current in each or many conductors each carrying small currents. Thus, an armature may be wound with 200 face conductors capable of carrying 80 amperes each or 400 conductors capable of carrying 40 amperes each or 800 conductors capable of carrying 20 amperes each. In each case, the number of ampere-conductors will be 16,000. If the first winding develops 125 volts, the second winding in the same field and at the same speed will develop 250 volts, and the third 500 volts. The output, or product of volts and amperes, will be the same in each case, 10,000 watts, or 10 kilowatts.

38. Any variation either in the magnetic flux of the field in which an armature rotates or in the speed of the armature changes the rate at which the ampere-conductors cut lines of force and thus varies the voltage and the output. Also, changing the number of ampere-conductors changes the output proportionately.

Increasing the length of an armature increases the space for magnetic flux proportionately, thus affecting the output directly as the change of length; increasing the diameter increases both the space for flux and the space for ampere-conductors, thus affecting the output as the square of the diameter. Changes in the magnetic density at the armature surface or in the number of ampere-conductors per inch of periphery affect the *activity* of the armature surface.

Let P = output of an armature, in kilowatts;
K = an activity factor, sometimes called an *output constant;*
s = speed of armature, in revolutions per minute;
l = length of armature core, in inches;
d = diameter of armature core, in inches.

Then,
$$P = K\,s\,l\,d^2$$

The formula shows how a change in dimensions affects the output. The factor K is nearly the same for generators of the same output made by different manufacturers. The diameter d is limited by the safe peripheral, or surface, speed, and, therefore, a slow-speed armature, such as is shown in Fig. 2, can have greater diameter than can a high-speed armature like that shown in Fig. 4.

COMMUTATION

ARMATURE REACTION

39. Armature reaction is the name given to the influence that is exercised on the magnetic field by the current in the armature conductors. When electricity is flowing in the windings of a generator armature, all the conductors adjacent to each north pole are carrying current across the armature face in one direction, and all the conductors adjacent to south poles are carrying current in the opposite direction. As the armature rotates, individual conductors are continually passing from the group under one pole into the group under the next pole, but the direction of current in the conductors in each group remains unchanged.

Fig. 23 represents this condition in a four-pole generator. Conductors under north poles are represented as carrying current from the observer, and this current sets up magnetic flux along paths crossing the poles, as indicated by the dotted lines crossing one north pole. Similar paths, not shown, cross the other poles. The path, *a–b–c–d*, includes all the face conductors of the group; but this route is largely through air and other non-magnetic material, and its reluctance will permit only small flux to be established. The path *e–f–g* surrounds a smaller number of ampere-conductors, but is mostly through iron and therefore carries considerably more flux.

40. The flux established by the armature conductors is useless and often troublesome. The flux along the path *a–b–c–d*, Fig. 23, crosses the armature conductors at the point where the coils of which they are a part are short-circuited by the brushes on the commutator. The resistance of the short-circuited coil is very low, and a little flux in this path may cause the development of enough voltage to establish a large current in the coil. This short circuit current enters the brush at one edge and leaves it at the other, agreeing with the direction of the load current in one edge of the brush and opposing it at the other. The current density in one edge of the brush may therefore become very high, while it is low or even reversed in the other edge. The high density thus caused in the edge of the brush may cause sparking.

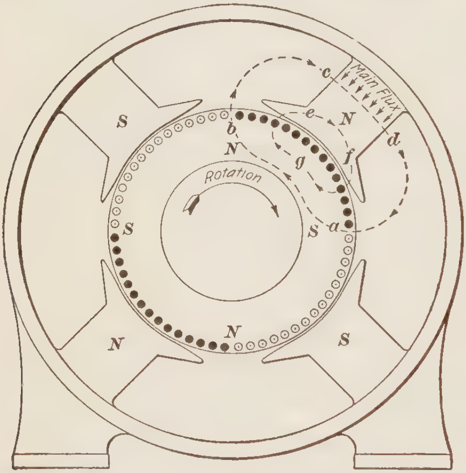


Fig. 23

Furthermore, the flux along the path *e–f–g* agrees in direction with the main flux in one side of the pole face, causing high magnetic density there, and opposes it at the other side, causing low density at that side. This unbalanced magnetic condition may cause unbalanced voltage conditions in the armature coils.

COMMUTATING POLES

41. The undesirable flux in the path *a–b–c–d*, Fig. 23, can be destroyed by means of **commutating poles,** as shown in Fig. 24. These poles make this path almost entirely through iron, and flux may be very readily established along it. The exciting coils, however, on the commutating poles are connected in series with the armature, so that their magnetomotive forces are always proportional and opposed to those of the armature, thus nullifying the effect of armature reaction in this path, so that almost no sparking results at any load. These poles do not, however, prevent the cross-magnetization and field distortion caused by flux in the path *e–f–g*.

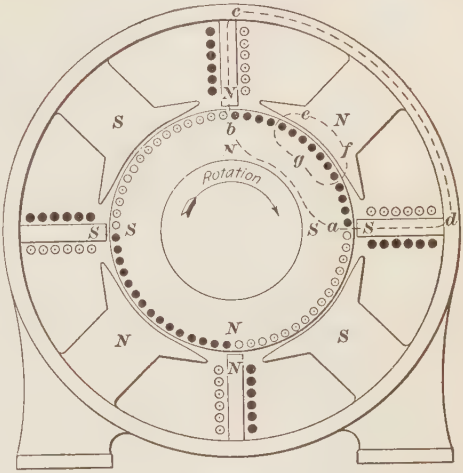


FIG. 24

COMPENSATING POLE-FACE WINDING

42. A **compensating pole-face winding,** as indicated in Fig. 25, arranged in slots in the pole faces and connected in series with the armature can be used not only to overcome the effects of all armature reaction but also to perform the function of a series-field winding in helping to regulate the voltage. The number of these pole-face conductors is chosen so that the number of pole-face ampere-conductors will slightly exceed the number of armature ampere-conductors. The direction of current in the pole-face conductors opposes the direction of current in the adjacent armature conductors; consequently, no undesirable fluxes can be established by the armature ampere-conductors in either of the paths *a–b–c–d* or *e–f–g*.

43. The field structure for a generator with compensating pole-face winding is shown in Fig. 26. For this particular generator, the magnet yoke is made of sheet-steel punchings bolted between heavy cast-iron flanges with supporting feet *a*. Each pole piece consists of two parts, a magnet core *b* and a pole face, or shoe, *c*, each built separately of punchings riveted together. Long bolts passing through the yoke and magnet cores screw into the pole shoes and hold the pole and its shoe securely in place in the yoke. Between every pair of pole shoes is a commutating pole *d* built up of punchings, riveted together and held in place by brass keys driven in between the commutating poles and the adjacent pole shoes, so that these poles are not attached to the yoke. The magnet yoke, the commutating poles, and the pole shoes are assembled with air vents, or flues, at intervals, as can be seen on the pole faces, in much the same way as armature cores are constructed.

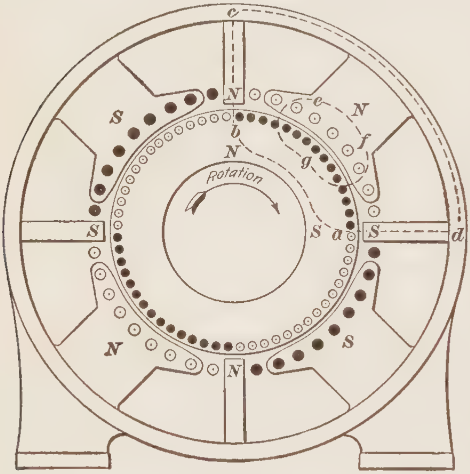


FIG. 25

The commutating poles are shaped to permit some leakage flux between adjacent poles and to pass enough of this flux across the air gap between the poles and the armature to provide a good commutating field for the brushes. Each pole shoe *c* has three slots, and the spaces beside the commutating pole form two more, making five slots per pole for compensating windings.

44. Fig. 27 shows the complete field magnet with shunt field coils at *a* and compensating coils at *b*, each occupying only four of the five available slots. The space on one side of the commutating pole is left vacant, as shown at *c*. Each compensating coil therefore occupies two slots in one pole shoe, one in an adjacent shoe, and a slot space at one side of a compensating pole. This position enables these coils to increase the flux through the armature as the current through them increases, and thus cause the generator to *overcompound;* that is, the generator voltage can be made to rise as the load increases, thus compensating for increased voltage drop in the lines leading to distant lamps or other electric devices.

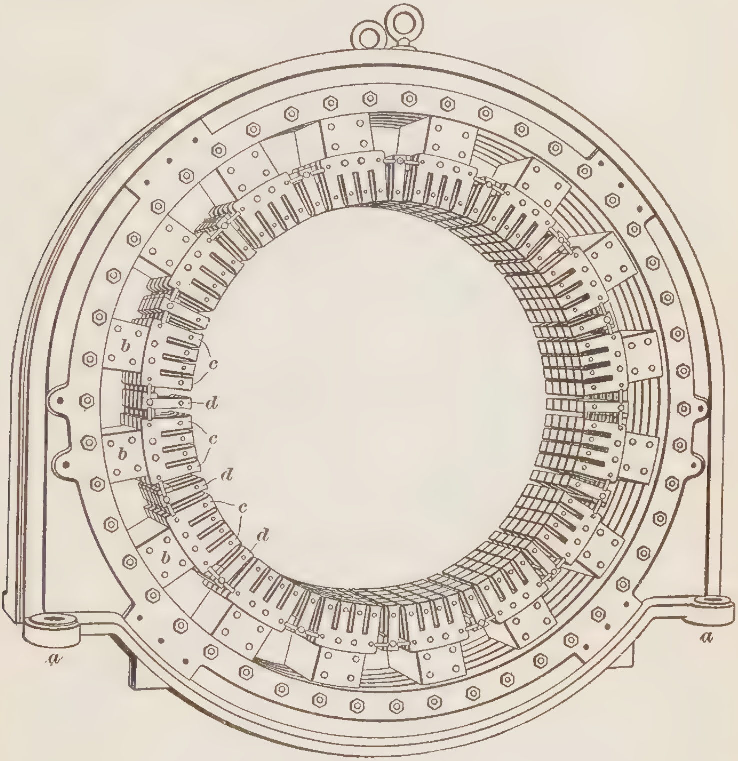


FIG. 26

UNIPOLAR GENERATOR

45. Some attempts have been made to avoid commutation entirely by arranging a magnetic field so that the armature conductors cut lines of force in only one direction. Such a machine is variously called *unipolar*, *homopolar*, and *acyclic*.

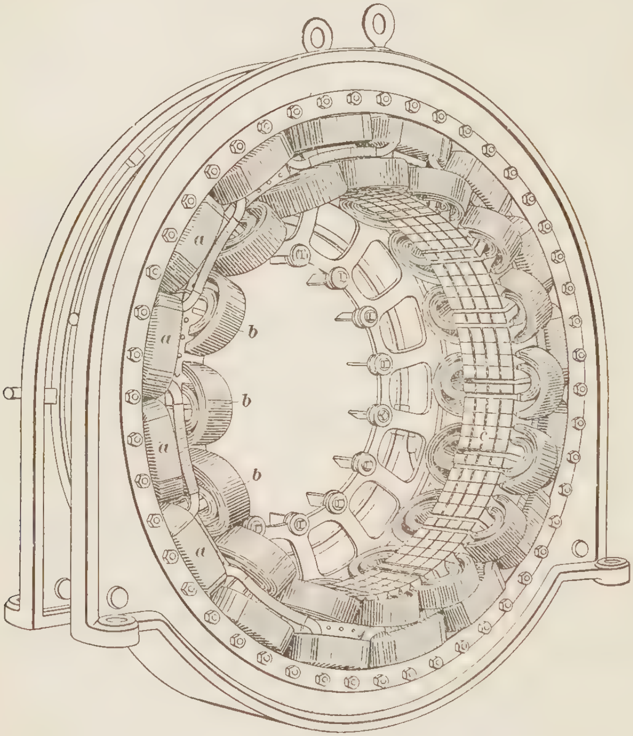
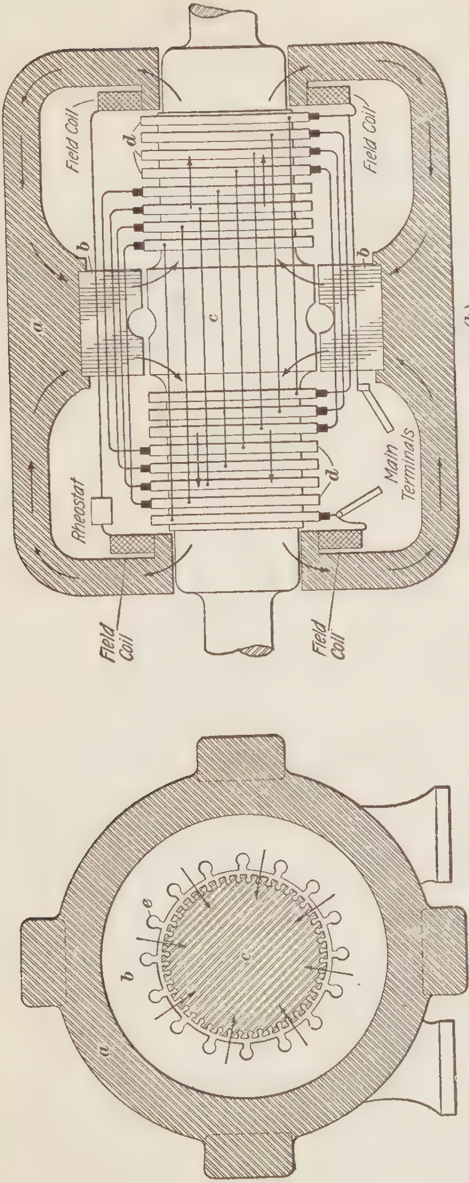


FIG. 27

Fig. 28 shows sections (*a*) and (*b*) of a unipolar generator; many of the parts to which reference will be made can be seen in both views. The frame, or magnet yoke, *a* is a steel casting that carries a laminated pole-face ring *b*. The armature core *c* is a steel forging, and the conductors are carried in slots in its periphery. These conductors terminate in collector rings *d* at each end of the armature. The collector rings slide under



(b)

(a)

FIG. 28

stationary brushes, which are interconnected by return conductors carried in slots *e* in the pole face. A shunt field coil concentric with the armature is carried on an inward projection of the field frame at each end of the armature core, as shown in view (*b*). The leads of the return conductors between the rings are offset, or advanced, enough to carry the armature current around the magnetic path and thus cause compounding as well as prevent armature reaction.

46. The path of the magnetic flux is indicated by the arrows in both views of Fig. 28. Lines of force enter the armature from all directions around the pole face, pass lengthwise of the armature core through the field coils, and return through the magnet frame to the pole face. As the lines of force are cut in only one direction, voltage is developed in the same direction in all armature conductors, and these conductors are so interconnected that their voltages are added to give the required voltage at the machine terminals.

47. The chief advantage of unipolar generators is the avoidance of commutator troubles; but the practical elimination of these troubles by commutating poles and compensating field windings, together with the complications of the unipolar generator, make the commutator type of machine preferable. The only possible field for the unipolar machine is for high-speed, high-current, low-voltage generation, and their practicability even here is doubtful. The sectional views, Fig. 28, were made from drawings for a 260-volt, 7,700-ampere generator to operate at 1,200 revolutions per minute.

ADAPTATION TO SERVICE CONDITIONS

THREE-WIRE SYSTEMS

THREE-WIRE GENERATORS

48. A three-wire system of circuits employs the usual positive and negative conductors, here called **main, or outside, wires,** and an additional conductor called the **neutral wire,** or simply the *neutral*. The voltage between the neutral and either outside wire is one-half of the voltage between the two outside conductors. The circuit between the positive main and the neutral is called the *positive side* of the system, and the circuit between the neutral and the negative main, the *negative side* of the system.

The voltages usually employed are 220-250 between the outside wires and 110-125 between the neutral and either outside wire. Motors are operated between the outside wires, and lamps between the neutral and either outside wire. Electricity can thereby be transmitted at higher voltage than that at which much of it is used, thus effecting a saving of copper.

49. In order to operate a three-wire system successfully, the voltages on the two sides must be kept balanced, or equal. As lamps are switched on and off, the loads on the two sides become unequal, and devices must be employed to compensate for these inequalities in order to keep the voltages equal. The early method was to connect two 125-volt generators in series between the outside wires and to connect the neutral wire to the circuit between the two generators. But an ordinary two-wire generator that develops the voltage required between the outside wires can be equipped with auxiliary parts by means of which a neutral, or middle, voltage can be obtained, making the machine a **three-wire generator**.

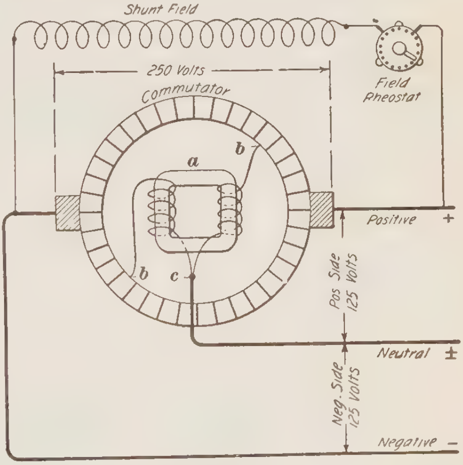


FIG. 29

50. Fig. 29 shows a diagram that will assist in making clear the operation of a three-wire generator. A **compensating transformer** *a*, sometimes called a *three-wire compensator*, or *balance coil*, is shown connected between two diametrically opposite bars *b* of the commutator of a two-pole generator. The middle point of the transformer winding is connected with the neutral wire. The transformer may or may not rotate with the armature. If it rotates, the connection at *c*, joining the neutral wire with the transformer, must be through sliding contacts consisting of a rotating slip ring on which slides a stationary brush. The transformer is then connected with the slip ring and the neutral wire with the brush. If the transformer is stationary, no slip ring is necessary for the

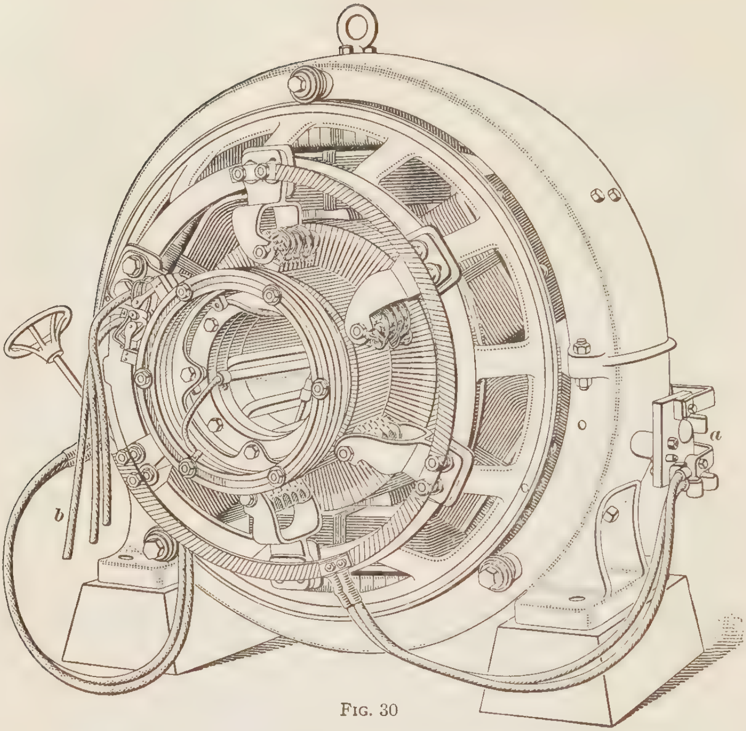


FIG. 30

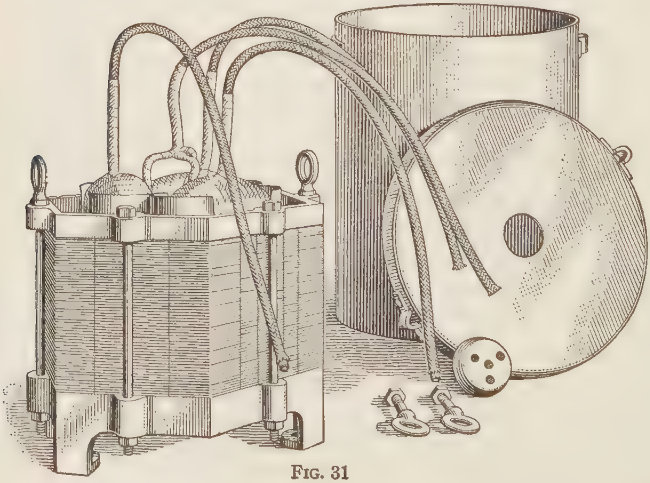


FIG. 31

contact *c*, but two slip rings must be used between the bars *b* and the transformer. The bars are then connected with the rings, which rotate with the commutator, and the transformer leads are connected with the stationary brushes sliding on the rings.

In either case, the transformer remains connected with two bars *b* and with the neutral wire. As the armature rotates, the voltage across the transformer varies from the full voltage of the generator when the bars *b* are under the brushes to zero voltage when these bars are midway between the brushes. The neutral wire, being connected with the middle point of the transformer winding, is always at a potential approximately midway between the potentials of the two brushes or the two outside wires. If the difference of potential between the brushes is 250 volts, the difference of potential between either outside wire and the neutral wire is approximately 125 volts.

51. If the low-voltage devices are so distributed between the two sides of a three-wire system that the number of amperes is the same on both sides, the system is said to be *balanced*, and the neutral wire carries no current to the generator or from it. If the load on the positive side exceeds the load on the negative side the positive wire carries more current than the negative wire and the neutral carries the difference toward the generator. If the negative side has the greater load, the negative wire carries the greater current and the neutral carries the difference from the generator. The neutral current may therefore vary from zero to the full unbalanced current, and its direction may be either toward or away from the generator.

52. Fig. 30 shows a multipolar, compound-wound, engine-type, three-wire generator with three slip rings mounted at the outer end of the commutator. These rings are permanently connected with commutator bars located 120 electrical degrees apart. At *a* is one of two terminal boards attached to the generator frame, the other being near the hand wheel on the opposite side. The three leads *b* are for connection with three of the compensator leads. The compensator is shown removed

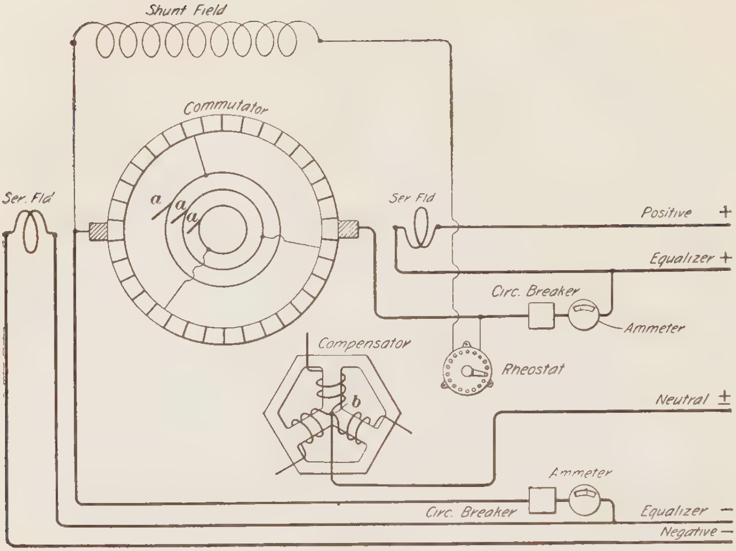

Shunt Field
Commutator
Ser. Fld
a
a
a
Ser Fld
Positive +
Equalizer +
Circ. Breaker
Ammeter
Compensator
b
Rheostat
Neutral ±
Ammeter
Circ. Breaker
Equalizer −
Negative −

Fig. 32

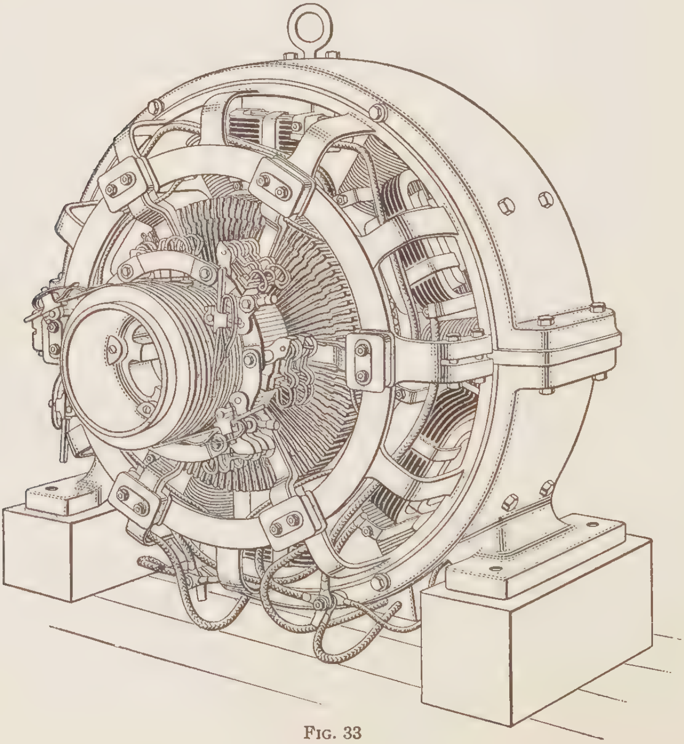


Fig. 33

from its case in Fig. 31, and in Fig. 32 is shown a diagram of connections, in which, for sake of simplicity, the generator is represented as bipolar. Three of the compensator leads are for connection with the three brushes *a* on the collector rings, and the fourth, from the neutral point *b*, is for the neutral wire. The circuit-breakers, the ammeters, and the field rheostat are shown compactly arranged, as on a switchboard, with the necessary leads from the generator. A **circuit-breaker** is a device for automatically opening a circuit when the current becomes too large, and an **ammeter** is a device for indicating the strength of current. One of each of these instruments is connected in each outside wire.

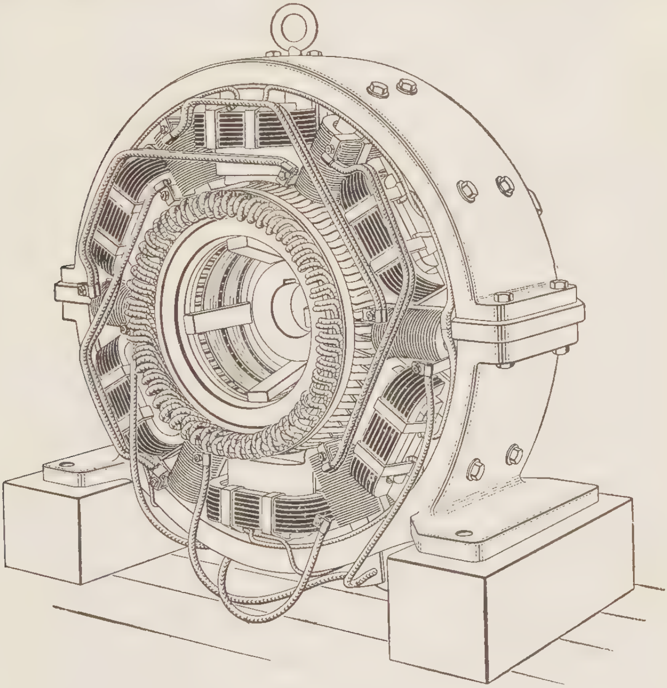


Fig. 34

If this machine is used in parallel with others, equalizer connections are needed to make each machine carry its share of the load. The connection of an equalizer with one terminal of each series field is shown; these conductors extend only between the machines operating together. Equalizer connections will be explained in another Section.

53. Figs. 33 and 34 are front and rear views of a four-ring three-wire compound-wound generator equipped with commutating poles. Both the series coils and the commutating-pole coils are arranged in two groups each, with alternate coils in each group. The current in each main wire must therefore pass through half the series coils and half the commutating-pole coils. The cross connections between coils are plainly shown in the two views. Fig. 35 shows one of the two balance coils used with such a machine.

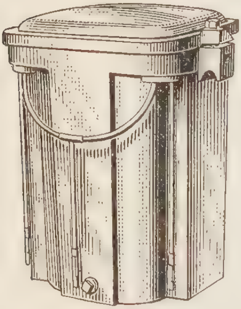


FIG. 35

The diagram of connections is shown in Fig. 36, a two-pole machine being represented because it is more simple and equally representative of the principles. The leads of the two compensators are connected with the brushes *a* and *b*, as indicated by the reference letters, so that each compensator is connected across points on the commutator one pole pitch apart. Both neutral leads from the two compensators are joined to the neutral line wire.

54. Fig. 37 shows a type of armature that includes a compensator, thus necessitating only one slip ring and simplifying the connections. The compensator is built in the form of a ring, which revolves with the armature; connections between the compensator and the armature winding are made direct at *a* and *b*. The collector ring is mounted on the commutator end of the machine, not shown.

55. In another type of three-wire generator, the neutral voltage is obtained by means of an auxiliary winding, usually placed in the bottoms of the armature slots under the main winding. The neutral wire is connected with the neutral point of the auxiliary winding by means of a slip ring, and this winding

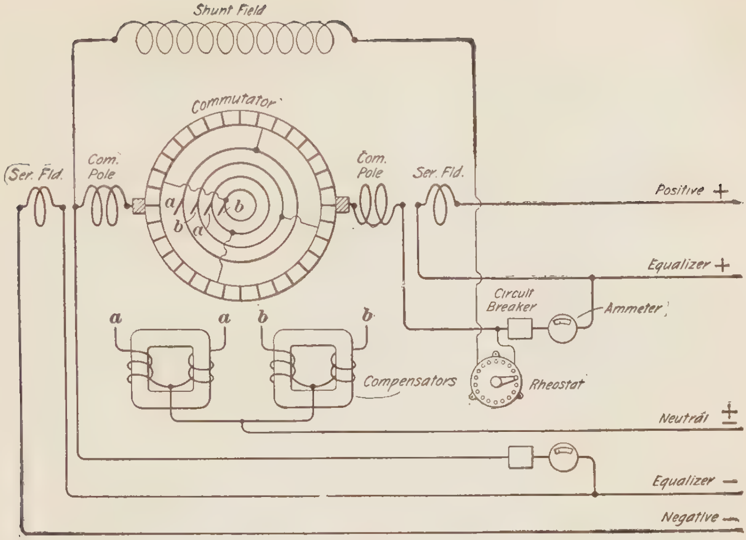

Shunt Field
Commutator
Ser. Fld.
Com. Pole
a
b
a
b
b
a
Com. Pole
Ser. Fld.
Positive +
Equalizer +
Circuit Breaker
Ammeter
a
a
b
b
Compensators
Rheostat
Neutral ±
Equalizer −
Negative −

FIG. 36

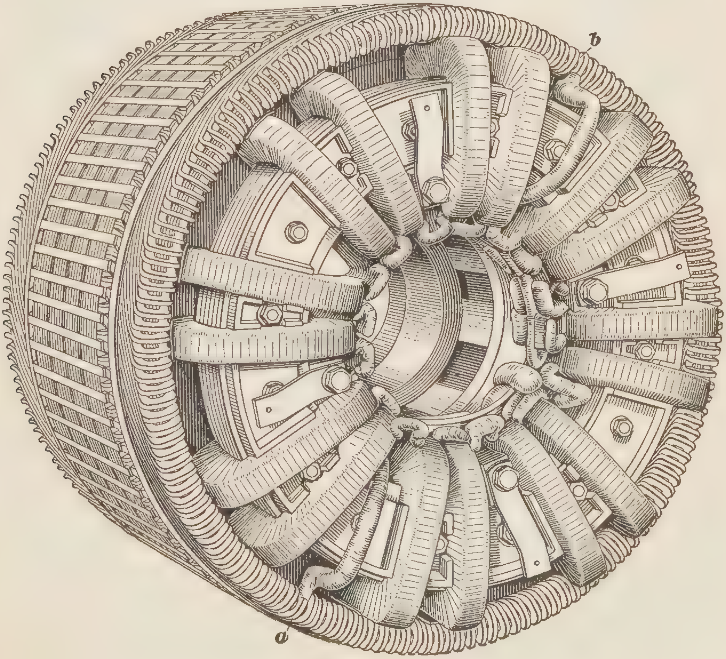

b
a

FIG. 37

is joined to the main winding at the proper points. Such a generator appears much like an ordinary two-wire machine, except that the connections are more complicated.

56. Three-wire generators are usually guaranteed to keep the difference between the voltages on the two sides of the system within 2 per cent. of the line voltage with any current in the neutral wire up to 25 per cent. of full-load current. For example, if the full-load current of a 250-volt three-wire generator is 400 amperes, the difference between the voltages on the two sides of the system will not exceed 5 volts (.02×250) when the neutral wire is carrying 100 amperes (.25×400); that is, the voltage on the heavy-loaded side will not be less than $122\frac{1}{2}$, and on the light-loaded side not over $127\frac{1}{2}$. Closer regulation is sometimes obtained by arranging the circuits so that a switchboard attendant can transfer circuits from one side of the system to the other when necessary to improve the balancing.

Direct-current generators are usually made so that they can be furnished either with or without three-wire parts. The operation—that is, commutation, heating, and voltage regulation—is practically the same in either case, provided the unbalanced load is not greater than 25 per cent. of the full load.

THREE-WIRE BALANCERS

57. A **three-wire balancer** is a combination of two compound-wound, two-wire generators, exactly alike, with shafts coupled or continuous, so that the armatures must run together, the general appearance being as shown in Fig. 38. These generators are comparatively small, and the set is used in connection with a larger two-wire generator to supply a three-wire system, the connections being as shown in Fig. 39. The armatures and series fields of the balancer are connected in series across the terminals of the main generator, and the neutral wire is connected to the circuit between the two armatures. Each armature develops approximately one-half as much voltage as the main generator. For example, if the main generator develops 250 volts, the balancer armatures run as motors at a speed such that each develops a counter electromotive force of nearly 125 volts at no load.

58. The no-load current through the balancer armatures is very small, being only enough to supply their friction and iron losses, and these losses remain practically the same at all loads. The conditions when the neutral wire is carrying current may be explained by reference to Fig. 40. Assume that the positive lead of the three-wire system is carrying 500 amperes and the negative lead 400 amperes, while 100 amperes returns in the neutral wire. The neutral current divides between the two armatures of the balancer, driving the one on the lightly loaded side as a motor and passing through the other as a generator. The direction of current in the armature acting as a motor is always from positive to negative, while in the generator armature the direction is from negative to positive.

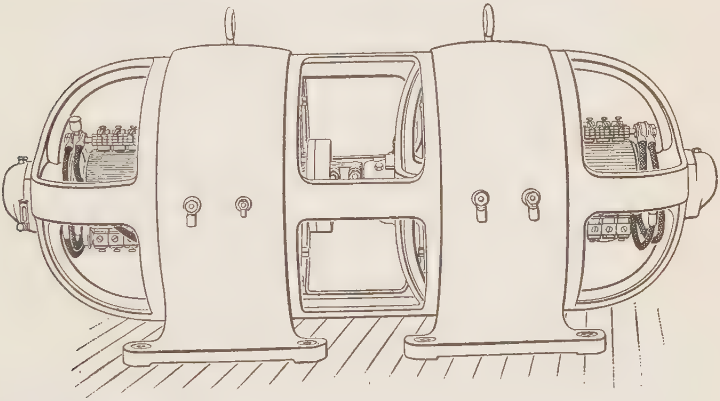


FIG. 38

The motor armature must take enough more than half the current, to supply the losses of both armatures, which for the purpose of illustration, may be assumed to require 5 amperes each. The 100 amperes in the neutral wire must therefore divide so that the motor current is at least 10 amperes greater than the generator current, making 55 amperes in the motor and 45 amperes in the generator. The full-load losses in the armatures and series fields, which were not considered in the assumed losses, may increase this difference between the motor current and the generator current somewhat. The current in the main generator will be approximately 455 amperes.

59. If the conditions in the three-wire system change so that the heavy load, 500 amperes, is on the negative side, as shown in Fig. 41, then 400 amperes must go out in the positive wire and 100 in the neutral wire. The balancer armature on the positive side must now act as a motor, because the direction of current through it is from positive to negative. The other armature acts as a generator because the direction of current through it is from negative to positive. The current in the main generator is still 455 amperes, of which 55 amperes goes through the motor armature and the balance to the three-wire system. Of the 500 amperes returning in the three-wire system, 45 amperes go through the generator armature and the balance returns to the main generator.

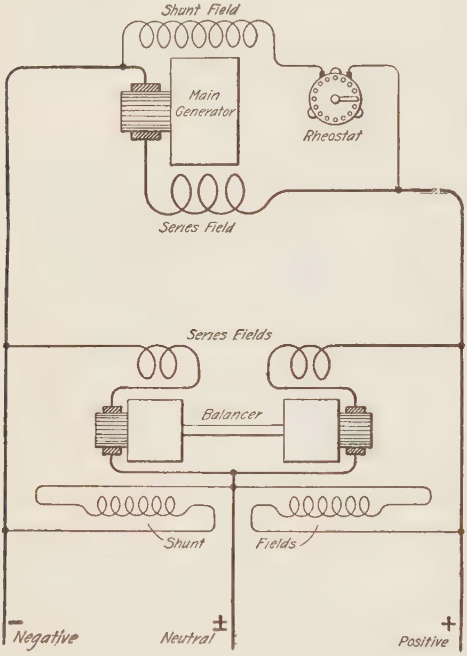


Fig. 39

60. The series fields of the balancer cause a slight rise of voltage on the heavily loaded side, the precise condition desirable for good balance. The shunt and series fields are so connected that their magnetomotive forces are always in the same direction on the generator and in opposite directions on the motor, as indicated in both Figs. 40 and 41. The effects of the series winding are therefore to increase the field strength of the generator and to increase the speed of the set by weakening the motor field, both of which effects increase the voltage of the generator. A comparatively small series winding therefore produces the desired effect.

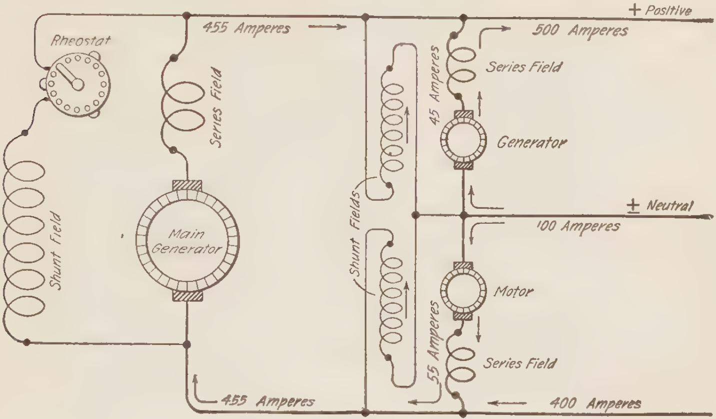


FIG. 40

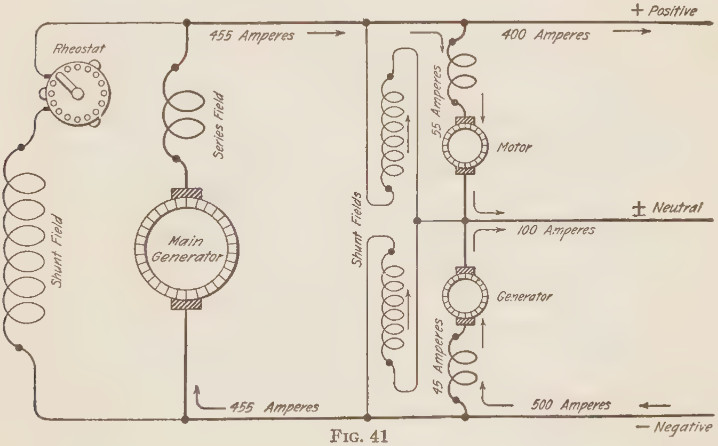


FIG. 41

61. A balancer set can be made to maintain the voltages of a three-wire system within 1 per cent. of equality with 25 per cent. unbalanced load; that is, if a perfect balance is 125 volts on each side, the difference between the voltages of the two sides will not exceed 1.25 volts. In order to obtain such close regulation, however, some additional refinements of connections are essential. One is to arrange the shunt fields so that each is connected across the armature of the other machine, as shown in Fig. 42. If the voltage of either machine rises, it strengthens the shunt field of the other machine, while its own field is excited proportionally to the lower voltage, thus equalizing the two voltages.

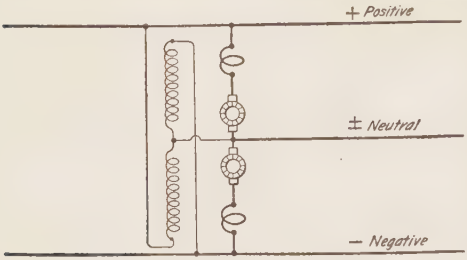


Fig. 42

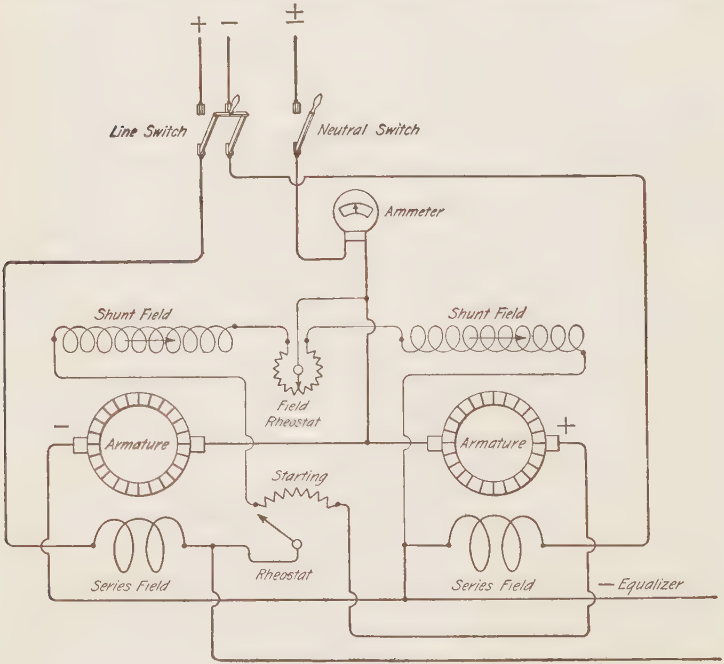


Fig. 43

62. Another refinement is to interchange the series field connections also, as in Fig. 43, so that each armature is in circuit with the series field of the other. The machine acting as a generator then carries in its series field the motor current, which is greater than the generator current, as explained in Art. **58,** thus causing increased compounding effect. On the other hand, the series field of the motor carries the generator current, which, being less than the motor current, causes less speed increase with load.

Fig. 43 also shows a field rheostat so arranged as to weaken one field and strengthen the other simultaneously; this is a good plan and is frequently followed, although a separate rheostat can be used in each shunt-field circuit. A starting rheostat, as shown, is practically always necessary. If two or more such sets are used in parallel, equalizer connections are required between the series fields to insure proper division of current among the sets. An ammeter with its zero reading in the center of the scale is shown in the neutral circuit; the direction of current in the neutral is indicated by the direction in which the needle is deflected, and the strength of current by the extent of the deflection.

MISCELLANEOUS THREE-WIRE BALANCING DEVICES

63. Three-Wire Motor With Booster.—Fig. 44 shows the connections of a balancing device consisting of a three-wire machine that operates as a motor driving as a part of its load a small series-wound generator used to raise, or boost, the voltage. A compensator is connected between the booster and the motor. The neutral circuit is thus through the slip rings, the compensator, and the booster. The direction of the booster voltage therefore depends on the direction of current in the neutral circuit, and the number of volts developed by the booster, seldom more than 5, depends on the strength of this current. A small voltage is thus added to the heavily loaded side of the system, maintaining a balance practically as good as is obtained with the balancer sets previously described. A motor regularly employed for driving other machinery can be equipped for driving a booster in this way.

64. Dynamotor With Booster.—Dynamotors are described later. Briefly, a *dynamotor* is a direct-current machine combining both motor and generator action in one magnetic field, either with two armatures or with one armature having two separate windings and independent commutators. Fig. 45 shows the connections of such a machine with a series-wound booster for balancing a three-wire system. The two armature windings of the dynamotor are connected in series across the outside wires, and the booster is in series with the neutral circuit, which is connected with the dynamotor circuit at a point between the two windings. The booster is direct driven by the dynamotor and serves to increase the voltage of the heavily loaded side of the system enough to compensate for voltage drop, thus maintaining good balance.

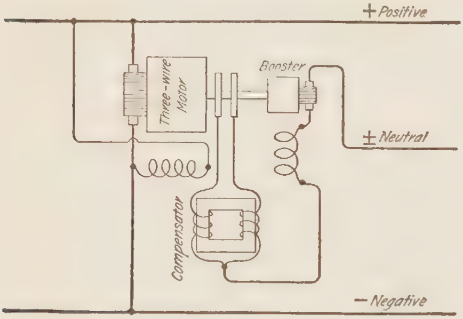


Fig. 44

The two armature windings in the dynamotor are alike, and when the system is perfectly balanced the voltages on the two sides are equal. When the system is unbalanced, the dynamotor winding on the loaded side acts as a generator and the voltage at its terminals falls, owing to the voltage drop in both windings, as is explained later. The voltage of the booster, however, varies with the neutral current in such a way as to keep the two sides of the system at nearly equal voltage.

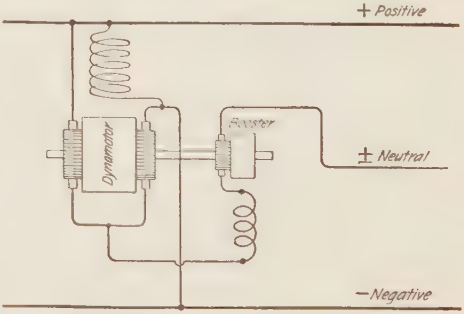


Fig. 45

MOTOR GENERATORS AND DYNAMOTORS

65. A **motor generator** is a combination of a motor mechanically coupled to one or more generators. The form of electrical energy delivered by the generator may be entirely different from that received by the motor. Fig. 46 shows a motor generator consisting of an alternating-current motor and a direct-current generator. The shaft of this set is continuous and is supported by only two bearings, thus necessitating a shaft of large diameter to obtain the requisite stiffness.

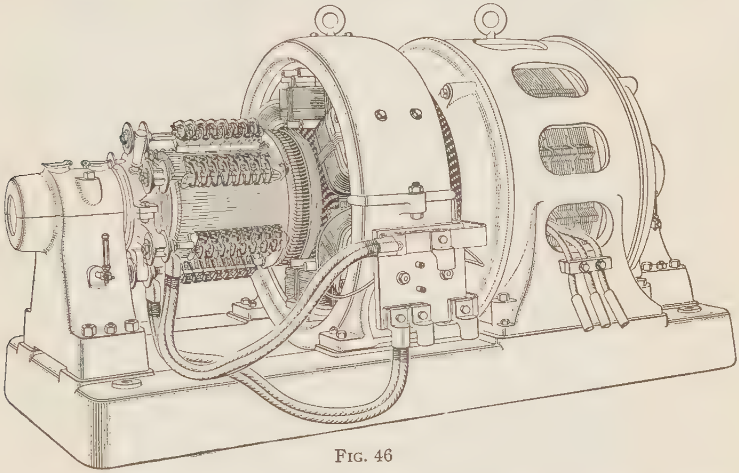


FIG. 46

Fig. 47 shows a 240-volt direct-current motor, on the right, coupled to a 30-volt direct-current generator. The two machines have separate shafts and bearings, and the coupling is flexible. This generator supplies current for a telephone system, and in order to avoid a disagreeable humming noise in the telephone instruments, the armature core is smooth, that is, it has no slots; the commutator also has a large number of segments, so as to obtain smooth, steady current. The presence of slots and teeth or the use of a small number of commutator bars would cause enough variation of current to be objectionable in telephone circuits on account of noise in the receivers.

66. Fig. 48 shows the connections of a motor generator consisting of a 125-volt motor and a 40-volt generator, both for direct current and shunt-wound. This set is arranged for boosting voltage from 125 to 165.

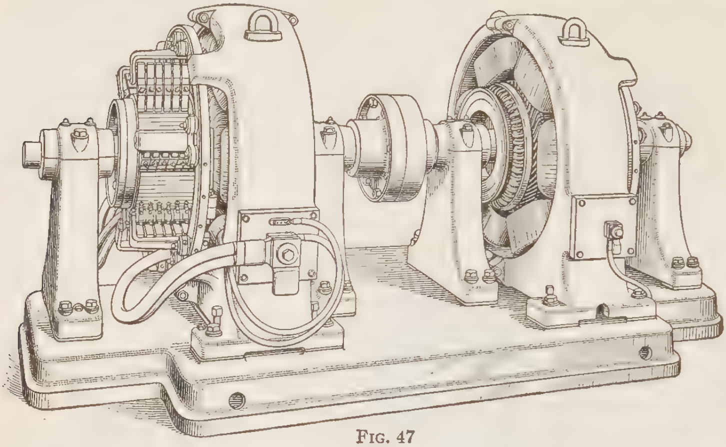


FIG. 47

By reversing the armature connections of the 40-volt machine, its voltage would oppose that of the 125-volt machine and would be deducted from it, leaving 85 volts on the circuit instead of the 165 volts shown in Fig. 48. When current at 85 volts is required and only a 125-volt circuit is available, the reduced voltage can be obtained in this way more economically than to operate a motor generator with an 85-volt generator.

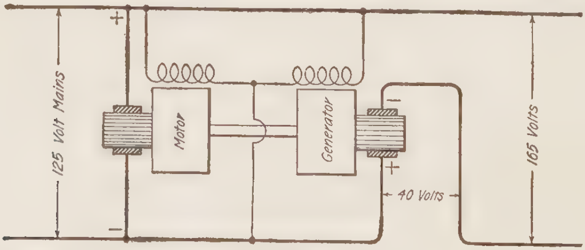


FIG. 48

67. Dynamotors.—Fig. 49 shows one type of dynamotor with the doors opened to give access to the commutator and the brushes. The motor, or primary, winding of a dynamotor is always designed for the voltage of the circuit from which the machine is to receive energy; the generator, or secondary, winding can be designed to develop any voltage required with a given load. This voltage varies widely as the load changes and cannot be adjusted by regulating the field current, because both windings rotate in the same field and are affected alike by changes in the flux.

The voltage generated in the secondary winding bears a fixed ratio to the counter voltage of the primary winding. This counter voltage is less than the voltage of the supply circuit by the voltage drop in the primary winding, and the voltage at the terminals of the secondary winding is less than the generated voltage by the voltage drop in the secondary winding. Every change of current in the secondary winding causes a corresponding change of current in the primary winding, and the voltage at the terminals of the secondary winding is affected by the voltage drop in both windings. Dynamotors are used only in small capacities for operating devices on independent circuits, as large call-bell systems.

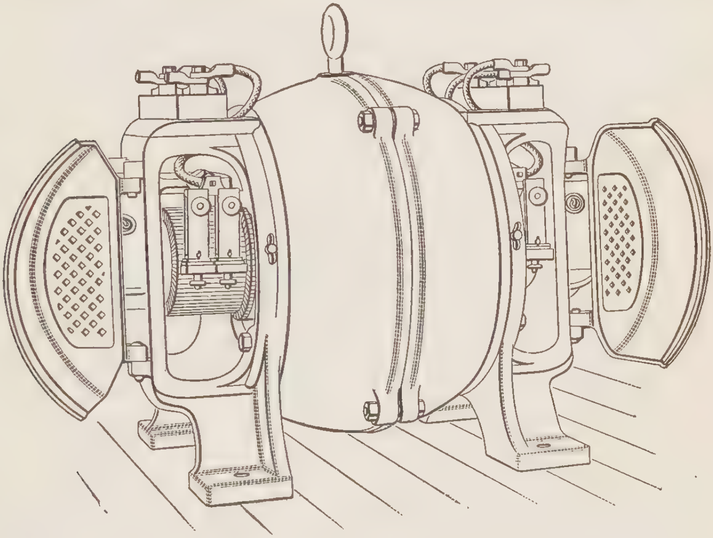


Fig. 49

ELECTROPLATING GENERATORS

68. For electroplating and electrotyping, large currents at about 6 or 12 volts are used, and special generators, often called **electrolytic generators,** are essential for such outputs. The armature conductors must be large, so as to carry the current, and their number must be few, in order to generate the low voltage. The commutator segments must be correspondingly large for the same reason. As a set of brushes can be used for every pole, and since large brush capacity is essential, the number of poles of low-voltage generators is often greater than on generators for the same outputs at higher voltages.

Fig. 50 shows an electroplating generator with two commutators, two separate 6-volt windings, and a terminal board so arranged that the windings can be connected in parallel for 6 volts or in series for 12 volts. The machine can also be connected with two separate 6-volt circuits. Electrolytic generators are provided with low-resistance brushes of metal or a combination of metal and graphite.

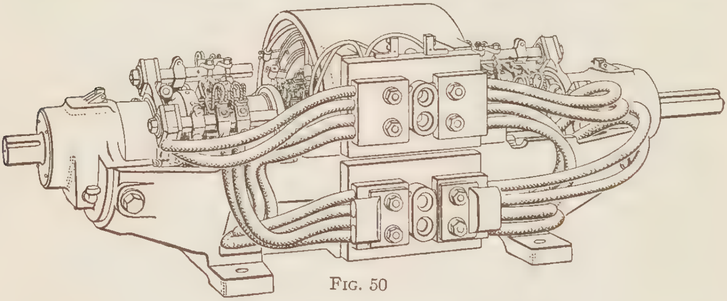


Fig. 50

CONSTANT-CURRENT GENERATORS

69. City streets and the interiors of large buildings were formerly lighted by arc lamps connected in series and requiring direct current at some constant value, usually between the limits of 6 and 10 amperes. A special type of generator was used, and some of these generators are still operating. Each lamp requires about 50 volts, and the machine must be capable of varying its voltage automatically, so as to keep the current constant as lamps are switched in or out of circuit.

70. Fig. 51 shows one type of large constant-current arc-light generator formerly much used. The field has eight coils

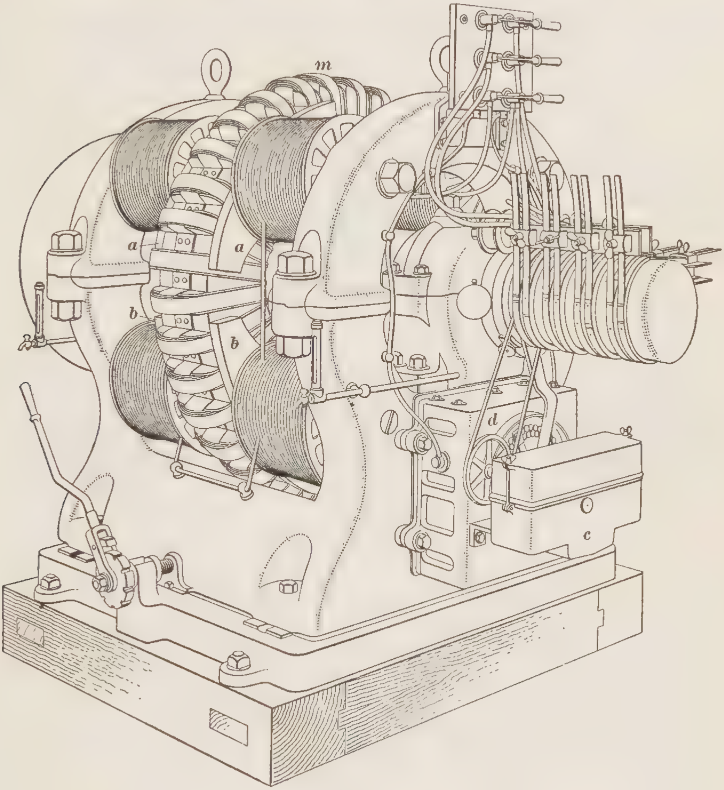


Fig. 51

so placed as to form four magnetic poles. Each pole has two shoes, as at *a* or at *b*, on opposite sides of the armature *m*. The armature is ring-wound and has thirty-two equally spaced coils. as shown in the diagram, Fig. 52. These coils are connected in four equal divisions, each connected with a section of the commutator on one end of the shaft. The connections of only one section are shown; coils *a*, *b*, *c*, and *d* are in series between segments *j* and *l*, and coils *e*, *f*, *g*, and *h* in series between segments *n* and *p*. Segments *j* and *k* are interconnected; so also are *l* and *m*, *n* and *o*, and *p* and *q*. The direction of rotation is assumed to be counter-clockwise, as is indicated by the curved arrows above and below coil *a*.

At the instant represented, brushes *r* and *t* touch segments *n* and *q*, thus connecting coils *e*, *f*, *g*, and *h* in series between the terminals. These coils are now opposite pole faces, represented by dotted outlines, and are generating electromotive force. Coils *a*, *b*, *c*, and *d* are opposite spaces between pole faces, and are therefore inactive at the instant shown. As these coils pass in front of poles, their segments *j* and *m* will slide under

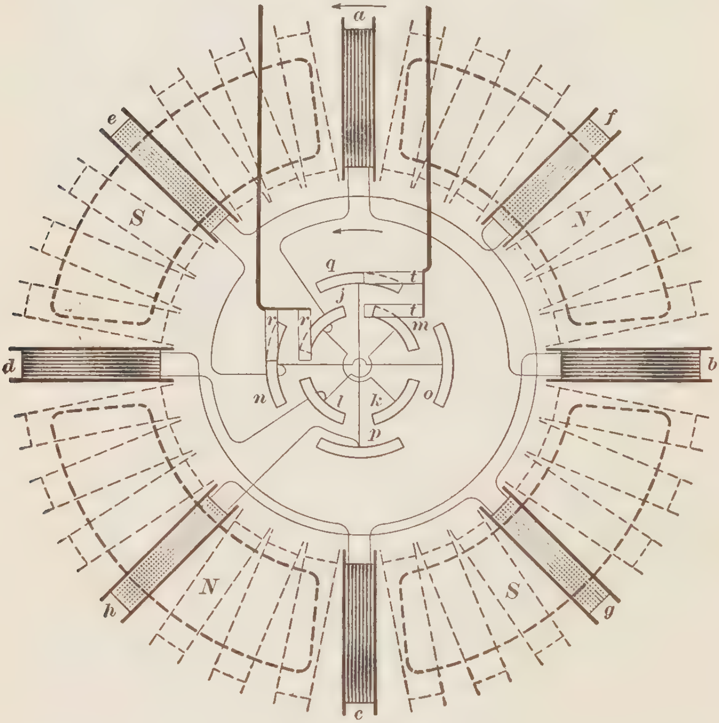


Fig. 52

brushes *r* and *t*. Other coils are similarly connected with other sections of the commutator, and the four sections are so interconnected that the active coils are always in series.

71. These machines are series-wound and are so designed that armature reaction has a comparatively large influence on the field. An increase of armature current therefore distorts and weakens the field, thus tending to keep the current constant. At the same time, an automatic regulator *c*, Fig. 51, adjusts a rheostat *d* and shifts the brushes. The rheostat is in shunt with the series field and is adjusted to divert some of the series-field current and thus weaken the flux on increase of armature current; at the same time, the brushes move to the position for best commutation. The regulator contains an oil pump that operates continuously while the machine is running, and also a magnetically operated valve. When the armature current is at the desired value, the valve remains in a position to allow the stream of oil to discharge into the reservoir without affecting the regulator. If the current changes, the valve moves so that the oil is directed against one side or the other of a vane, so as to turn the regulator and shift the brushes in the direction necessary to restore the current to the desired value.

DIRECT-CURRENT VOLTAGE REGULATION

72. The simplest means of keeping the voltage of direct-current circuits approximately constant with varying load is by the use of compound-wound generators. As the load changes, the field strength is changed, owing to the current in the series-field coils, and the voltage either remains approximately constant or rises slightly with increasing load in order to compensate for increasing voltage drop in the conductors leading to the devices using electricity.

This method of regulation is satisfactory for operating motors or other devices not appreciably affected by moderate voltage changes. It is also satisfactory for some loads consisting wholly of lamps. But when lamps and motors are operated on the same circuits, the lights may be disagreeably affected by the voltage changes unless other means of voltage regulation are employed.

73. The Tirrill voltage regulator automatically adjusts the field strength of a generator by short-circuiting the field rheostat when the voltage is low. Two sets of contacts are employed, each operated by electromagnets, as indicated in Fig. 53. The main control magnet coil is connected across the circuit with external resistance in series. When the voltage is low, this magnet is weakened, and the main contacts are held closed by a spring. The coils of the relay magnet are also connected across the circuit with external resistance in series, but they are shunted by the main contacts. While the main contacts are closed, the coils of the relay magnet are thereby weakened, so that the relay contacts are held closed by a spring. The relay contacts are in a circuit shunting the field rheostat, and while these contacts are closed the rheostat is cut out of the shunt-field circuit, so that the generator voltage builds up rapidly. When the voltage becomes too high, the main contacts open, followed immediately by the opening of the relay contacts, thus placing the rheostat in the field circuit and reducing the voltage.

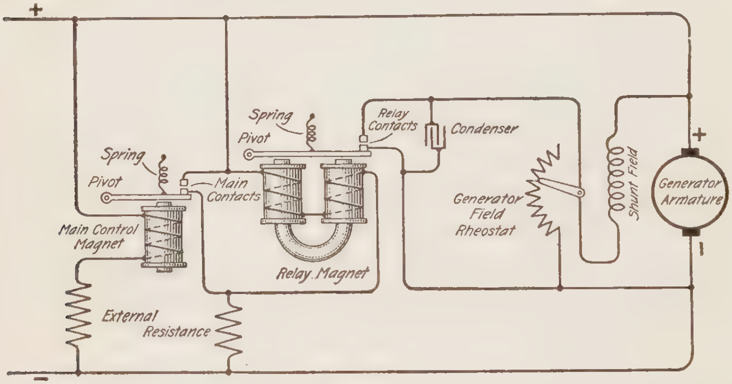


FIG. 53

The resistance in the rheostat is high enough to affect the generator voltage quickly; usually this resistance is enough to reduce the voltage to about two-thirds of its full value if left in circuit continuously. The contacts therefore vibrate rapidly—probably from 300 to 600 times a minute—thus keeping the voltage very nearly constant. A condenser bridges the relay contacts and prevents injurious sparking; the current through the main contacts is so small that such protection is unnecessary. When tested by means of an *oscillograph*, an instrument that records very small voltage changes, the generator voltage is found to vary only a fraction of a volt, and these variations have inappreciable effect on electric lights.

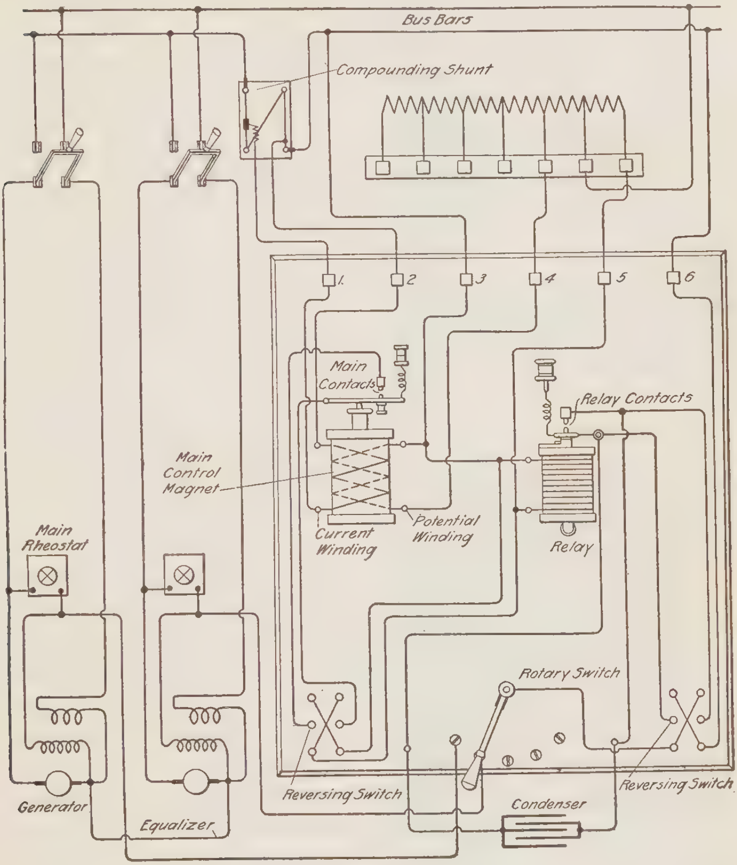
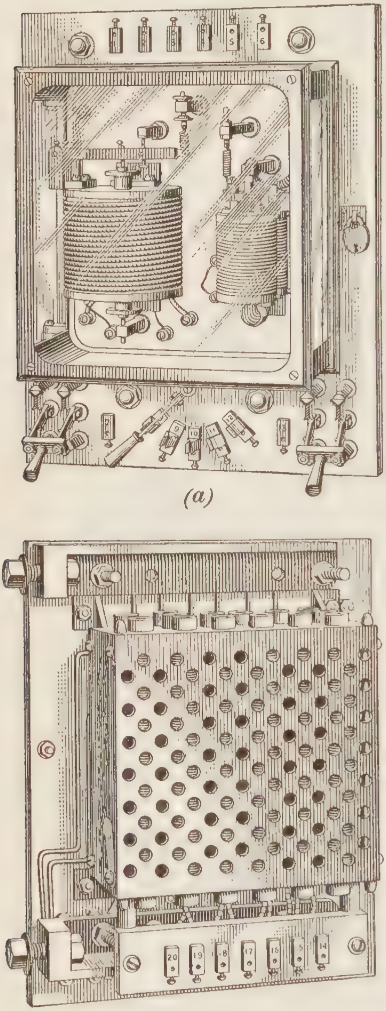


Fig. 54

74. Fig. 54 shows a diagram of complete connections of a regulator for use with either shunt- or compound-wound generators of capacities up to 25 kilowatts, and Fig. 55 shows a front view (*a*) and a rear view (*b*) of the regulator mounted on a marble base. As shown in Fig. 54, the main control magnet has both a potential winding connected across the generator terminals and a current winding connected across an adjustable low resistance, known as a *compounding shunt*, in the main circuit. The current winding therefore carries a fixed portion of the load current, depending on the adjustment of the compounding shunt. The magnetizing effect of this current opposes the effect of the current in the potential winding, thus acting the same as additional tension in the spring, tending to hold the main contacts closed. In order to open the main contacts, the generator voltage must therefore rise higher when the generator is loaded than when running light, and the greater the load the higher must be the voltage in order to open these contacts.



(*a*)

(*b*)

Fig. 55

A resistance with several taps, represented at the top of Fig. 54, is mounted in a case on the back of the panel, Fig. 55 (*b*), and one terminal of the potential winding is connected with the tap that gives the desired voltage. Further adjustment can be made by changing the tension of the spring holding the main contacts closed; a thumb nut is provided for this purpose. At each lower corner of the panel is a double-pole, double-throw switch, by means of which the direction of current through the contact points can be reversed periodically, thus prolonging the life of the contacts.

75. The voltages of several small compound-wound generators operating in parallel can be controlled by a regulator connected with one of the generators. This generator will then be most sensitive to sudden changes of load on the system, but the series fields and equalizers will soon cause the other generators to take their proper shares of the load. The regulator can be connected to any one of several generators by means of a rotary switch shown near the bottom of Figs. 54 and 55 (*a*). Shunt-wound generators do not operate successfully in parallel because of the difficulty of keeping the fields adjusted so as to divide the load properly.

76. Regulators for machines having capacities of more than 25 kilowatts sometimes have several relay magnets operated by one set of main contacts, each relay magnet controlling a section of the field rheostat. The burning action of the field discharge is thus distributed over several pairs of contacts so as to cause less injury. All the relay magnets operate simultaneously, and such a regulator is often used with several compound-wound generators by controlling each shunt field by one or more pairs of relay contacts. A regulator with ten relay magnets can control a total generator capacity of about 400 kilowatts either from a single machine or from several machines in parallel. For controlling the voltage of a larger generator, the generator is usually separately excited and the controller is used in connection with the exciting generator, raising or lowering its voltage in order to adjust the field of the main generator.

DIRECT-CURRENT MOTORS

CLASSIFICATION

SIZES OF MOTORS

1. Widely varying power requirements for industrial processes have led to the development of an endless variety of sizes, types, and classes of direct-current motors with widely varying characteristics. All manufacturers, however, list motors in general classes or types, some of which are comparatively limited in application and others very broad.

Direct-current motors may be classified according to size, field winding, speed, degree of enclosure, mechanical characteristics, service conditions, and the class of application for which they are intended.

2. According to size, motors are usually classified by manufacturers as *small* and *large*. No definite dividing line exists between the two, since what may be called a small motor by one manufacturer, may have an equivalent rating in another classification by another manufacturer. Large motors are seldom listed as such, except occasionally those of very large capacity. The use of small motors has become so common, however, that almost every motor manufacturer has a line of so-called small motors, often called *small power motors* to distinguish them from fan motors.

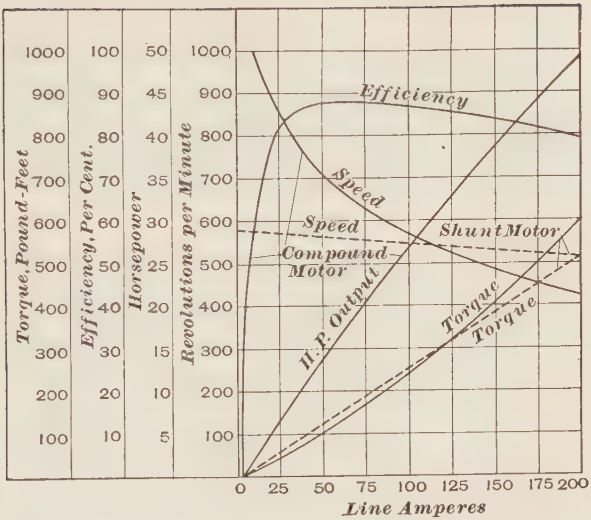
COPYRIGHTED BY INTERNATIONAL TEXTBOOK COMPANY. ALL RIGHTS RESERVED

FIELD WINDING

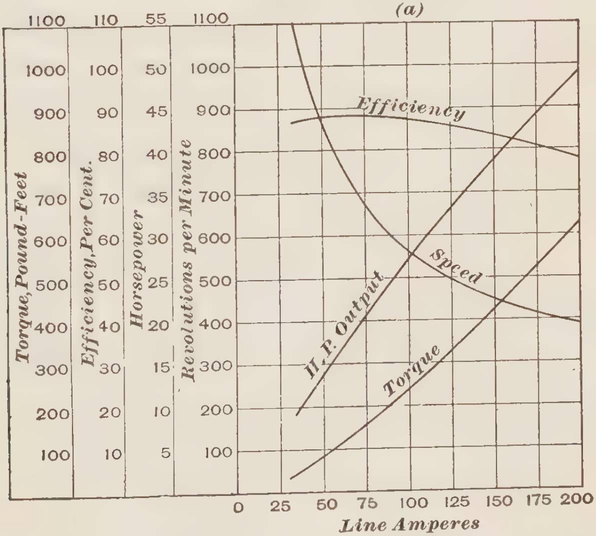
3. According to field winding, all direct-current motors are in three classes, namely, *shunt*, *compound*, and *series*. Shunt-wound motors start and operate with current input proportional to the torque, or turning effort, and run at practically constant speed at all loads. The current input to a series motor varies less than directly proportional to the torque and the speed varies widely with varying load. For example, at twice full-load torque, a shunt motor requires approximately twice full-load current and operates at only a trifle below its full-load speed, while a series motor requires considerably less than twice full-load current, but operates at much below full-load speed. Compound-wound motors have characteristics intermediate between those of shunt and series motors, resembling most closely the one that its field winding most nearly resembles.

4. The **characteristics** of a motor are its variation of speed, torque, and efficiency with varying load. Fig. 1 shows characteristic curves of a 50-horsepower motor; the line amperes, corresponding to the load, are represented by the distance from the left-hand margin; the speed in revolutions per minute, the output in horsepower, the torque in pounds at 1-foot radius (pound-feet), and the efficiency in per cent. are represented by distance from the lower margin. In Fig. 1 (*a*), the efficiency, speed, horsepower, and torque of a compound-wound motor are represented by the full-line curves, and the speed and torque of a shunt-wound motor by the dotted-line curves; the efficiency and horsepower are approximately the same for both windings. The curves in (*b*) show characteristics of a motor of the same capacity with series-field windings.

5. The curves of compound and series motors, Fig. 1, show actual results obtained from standard motors of the same rating, but with the two types of winding; the speed and torque curves of the shunt motor are theoretical, but would be closely approximated in practice. The difference in speed



Shunt and Compound Motor
(a)



Series Motor
(b)

FIG. 1

variation is forcibly illustrated by readings from the curves, as recorded in Table I.

The speed curve of the series motor shows rapid increase of speed with decreasing load. At no load, the speed of such a motor would be too high for safety; this fact accounts for the blanks in the last column of the table. Series motors should never be used where there is a probability of operating at no load or at very light load; for example, they should not be used in belted service, because the belt might run off, leaving no load on the motor and allowing it to race.

TABLE I

INFLUENCE OF FIELD WINDING ON MOTOR SPEED

Amperes	Speed, Revolutions per Minute		
	Shunt	Compound	Series
10	580	1,000	
25	578	850	
50	570	710	860
75	562	630	675
100	555	575	560
125	525	540	495

6. In addition to the foregoing winding classifications are **commutating-pole, or interpole, motors,** in which field poles with series windings are arranged between the main poles to assist in obtaining good commutation. Such motors are usually so designed that the series winding on the commutating poles has practically no effect on the speed. A few compensating series turns are frequently placed on the main poles of shunt motors of this class in order to keep the speed nearly constant at all loads when the motor is operated at high speed. Without such compensating turns and with resistance in the shunt-field circuit for high-speed operation, increased load is in some cases accompanied by increased speed on account of the overbalancing effect of armature reaction and of the commutating poles. These compensating turns serve merely to render the motor operation stable when the shunt field is weakened for speed adjustment.

7. **Consequent-pole motors**, though rarely used, form another class. Alternate field poles have no windings, but are magnetized in consequence of the winding on the other poles.

SPEED

8. According to speed, both alternating-current motors and direct-current motors may be classified in the same way. Alternating current is fully explained in later Sections. The following speed classification applies to direct-current motors only, except where otherwise specified.

9. **Constant-speed motors** are those of which the speed at all loads between no load and full load does not vary more than a fixed per cent.—usually 20 per cent.—from full-load speed. Examples are synchronous and induction motors, both alternating current; shunt-wound motors; and compound-wound motors with comparatively light series-field winding.

10. **Multispeed motors** are those capable of operating at more than one constant speed. Examples are induction (alternating current) motors with field windings capable of more than one grouping and direct-current motors with two armature windings and two commutators.

11. **Adjustable-speed motors** are those capable of speed adjustment by regulation of shunt-field current. Shunt motors in this class operate as constant-speed motors at any field adjustment; compound-wound motors are also in this class as long as the speed does not vary more than 20 per cent. from full-load speed at any load or field adjustment.

12. **Varying-speed motors** are those of which the speed varies widely with the load, decreasing when the load increases and increasing when the load decreases, such as series motors and motors having only enough shunt-field winding to keep the

speed within safe limits at no load. Some alternating-current motors are also in this class.

13. In addition to the foregoing speed classification, motors are sometimes classified as *slow speed*, *medium speed*, and *high speed;* but these classes are not well defined and mean little except when used in connection with the motors of some particular manufacturer.

ENCLOSURE

14. According to the degree of enclosure, motors are *open type*, *semienclosed type*, *entirely enclosed type*, and *enclosed ventilated type.*

15. **Open-type motors** are so made as to obtain the greatest possible cooling effect from air circulation around the parts subject to heating, by leaving those parts open to the passage of air. The skilful designer arranges the rotating element to set air in motion and aims to direct the air-currents around

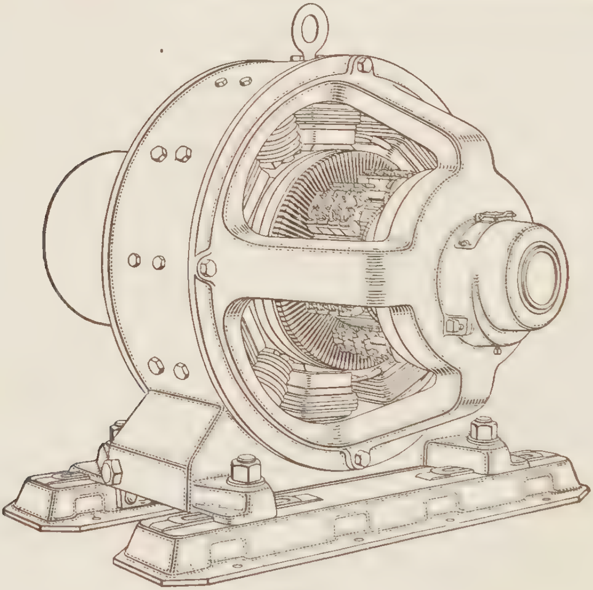


FIG. 2

and over the working parts—field coils, armature, and commutator. Of two equally rated motors, the one emitting a breeze of heated air while operating is better ventilated and generally more desirable than the motor around which the air remains stagnant. Figs. 2 and 3 show open-type direct-current motors.

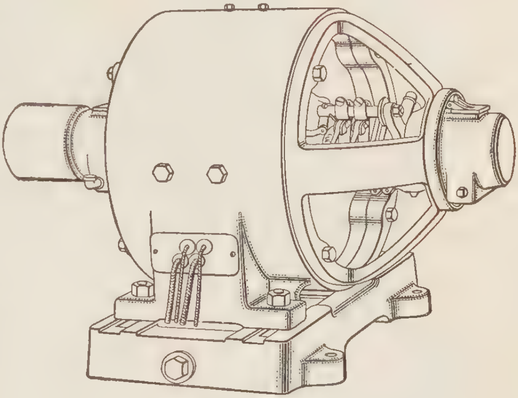


FIG. 3

16. Semienclosed motors differ from open motors only by the provision of covers over all openings to exclude coarse articles. Such covers restrict ventilation, but do not entirely stop it. They are sometimes cast in the form of grids with small openings, as in Fig. 4, sometimes formed of perforated sheet metal, as in Fig. 5, and sometimes made of fine wire gauze, according to the service conditions or the manufacturer's choice. The degree of enclosure depends on the materials to be excluded.

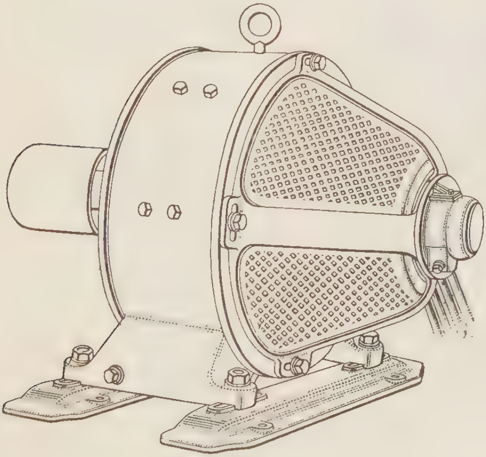


FIG. 4

17. Entirely enclosed motors have no uncovered openings through which foreign particles of any description might enter. All openings are closed by tight-fitting covers, although the motor frames employed may be the same as those for open and semienclosed motors. Gaskets of rubber or some other suitable material may be provided under the covers to keep out dust and possibly to render the motors splash-proof. Such motors are sometimes said to be moisture-proof, but it is doubtful whether they can be made proof against the entrance of moisture if operated in damp places and allowed to stand idle long enough to cool off occasionally.

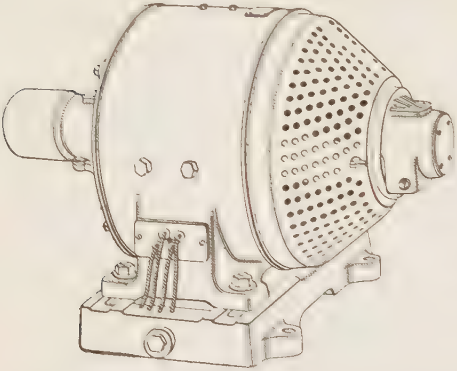


FIG. 5

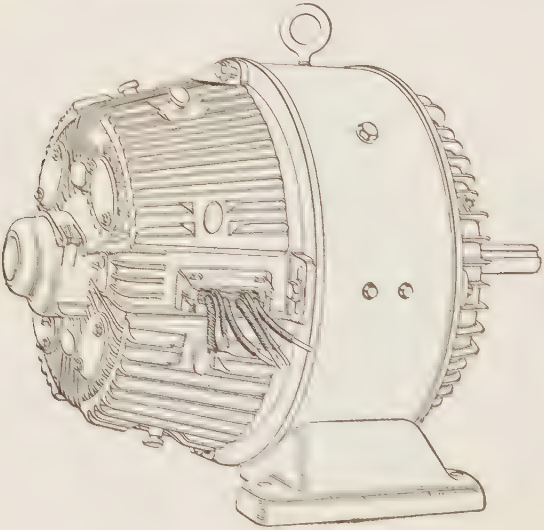


FIG. 6

18. Ordinary enclosed motors have no self-ventilating properties whatever, and the ratings are therefore considerably less than those of open motors of corresponding size. Few open-type shunt or compound motors can be provided with enclosing covers and operated on their rated voltage, even if the motor rating is reduced. In most cases, the shunt-field coils would overheat under such conditions. Sometimes a standard motor can be fully enclosed by reducing its rating somewhat and using some resistance in series with the shunt field or reducing the operating voltage. Ordinarily, special windings are preferable.

Figs. 6 and 7 show types of entirely enclosed motors. The motor shown in Fig. 6 has an unusually small frame for its rating; both bearing brackets and all covers are ribbed in order to increase the radiating surface for heat dissipation. The motor frame shown in Fig. 7 is large enough in proportion to its output to make such construction unnecessary.

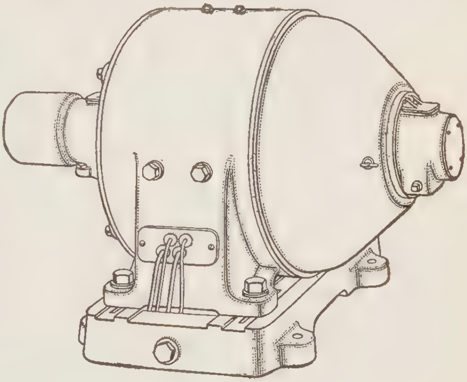


FIG. 7

19. In addition to the foregoing distinct classes, according to enclosure, any combination of these classes may be used. For example, part of a motor may be entirely closed and part semiclosed or open. In some cases, grid covers are provided for the top openings, while the lower openings are not covered.

20. **Enclosed ventilated motors,** Fig. 8, are protected from the entrance of dust, and are provided with ventilation permitting operation in most cases at full open rating. This ventilation results from a blower or a set of fan blades located inside the motor and driven by the armature. Air tubes connect the inside of the motor frame with some outside source. The blower causes air to circulate through the motor and carry away the heat. When several enclosed motors are to operate in the same vicinity, a single independently driven blower with tubes leading to all the motors is sometimes used to good advantage.

MECHANICAL CHARACTERISTICS

21. Among the principal mechanical modifications of motors are *horizontal-shaft motors*, including those with back gears, and *vertical-shaft motors*. These are the general classes; minor modifications are two-bearing motors, three-bearing motors, motors with self-contained bearings (in the end brackets), motors with pedestal bearings, motors with idler pulleys for preventing belt slippage, etc.

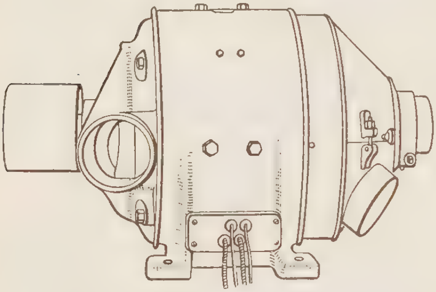


FIG. 8

22. Horizontal motors, Figs. 2 to 12, inclusive, may be arranged for driving by belt, chain, pinion, or coupling. They may or may not require a third bearing outside the pulley, pinion, or sprocket, depending on size, speed, and service conditions.

If the shaft is kept horizontal, such motors can usually be mounted upright on a floor or a foundation, as indicated in preceding illustrations, sidewise on a wall or other vertical support, as in Fig. 9, or inverted on a ceiling, as

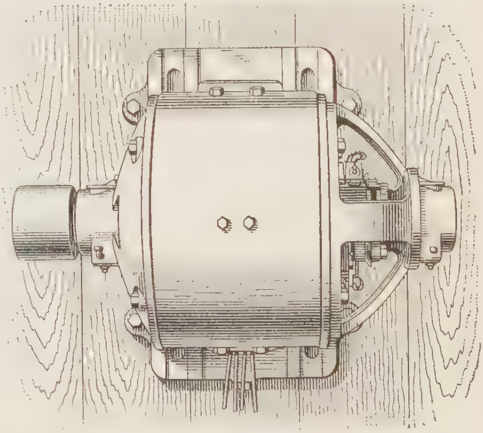


FIG. 9

in Fig. 10, up to sizes too large for side-wall or ceiling mounting. When automatic oil-ring lubrication is employed, the bearings must be so arranged for each mounting that the oil reservoirs are directly under the journals. Motors such as those shown have bearings in brackets attached to the frame so as to be capable of turning 90° or 180°, to suit different mountings.

23. Back-geared motors, Figs. 11 and 12, are applicable in some cases where slow driving speed with comparatively high motor speed is desirable. Such motors have two horizontal shafts, one of which carries the armature and rotates in main bearings a, and the countershaft, or back shaft, b, which rotates in bearings bracketed to the motor frame so as to maintain fixed distance between the two shafts. The teeth of a pinion on the armature shaft mesh with those of a gear-wheel, or gear, c on the countershaft, and as the gear is much larger than the pinion, it rotates at proportionately slower speed.

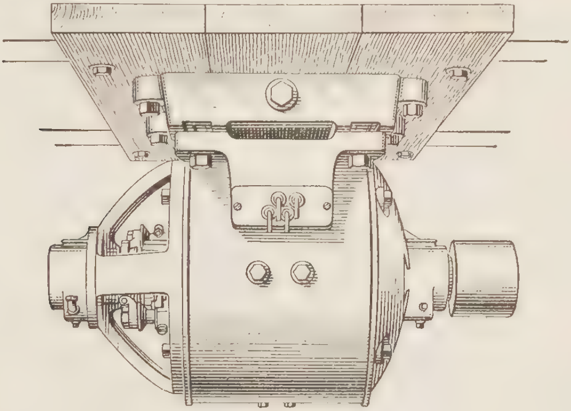


FIG. 10

Let s = speed of pinion, in revolutions per minute;
d = diameter of pinion, in inches;
n = number of teeth of pinion;
S = speed of gear, in revolutions per minute;
D = diameter of gear, in inches;
N = number of teeth of gear.

Then, $$\frac{S}{s}=\frac{d}{D}=\frac{n}{N} \qquad (1)$$

and $$S=\frac{s\,d}{D} \text{ or } \frac{s\,n}{N} \qquad (2)$$

For example, if the diameter of a pinion is 4 inches and the diameter of its gear 24 inches, the gear speed for a pinion speed of 1,700 revolutions per minute is

$$S=\frac{1{,}700\times 4}{24}=283\tfrac{1}{3}\text{ R. P. M.}$$

24. Back-geared motors are usually provided with a countershaft having the free end fitted to receive a pinion or a pulley for connection to the driven machine. The pinion and the wheel may be unguarded, as in Fig. 11, or they may be covered with shields, as in Fig. 12. Unguarded gears are dangerous if they are located where attendants may come near them or where any foreign substances may be caught by them.

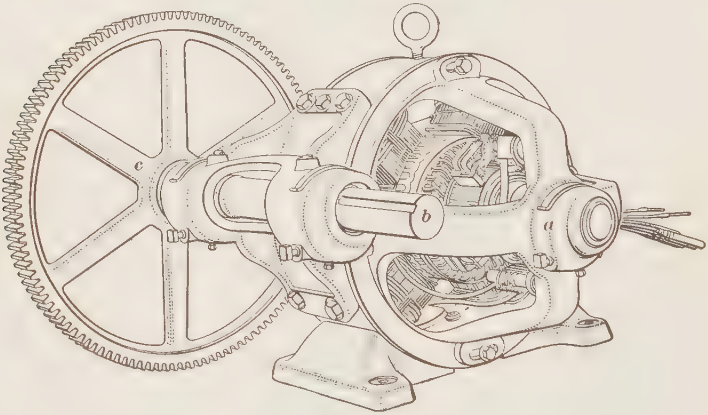


FIG. 11

25. **Vertical-shaft motors** are supplied each with one thrust bearing to carry the armature and at least two guide bearings. The thrust bearing may be placed at either end of the shaft, but it is generally located at the upper end, the armature being suspended. This bearing generally consists of hardened-steel balls rolling between steel rings in an oil bath. A cap, or ring, fastened to the upper end of the motor shaft serves to suspend the armature on the thrust bearing. The thrust bearing is usually designed to support some weight additional to that of the armature; as, for example, the blades of a fan, or blower, or the impeller of a rotary pump.

26. The purpose of the guide bearings is to keep the armature shaft in proper alinement; they are lined with some antifriction metal and are lubricated by means of oil or grease cups that feed the bearing surfaces through suitable channels or by oil reservoirs from which the oil is fed automatically to the bearing surfaces. Automatic lubrication is the later method and has proved to be very successful; bearings of this type in vertical motors require attention no more frequently than do the ordinary ring bearings of horizontal motors.

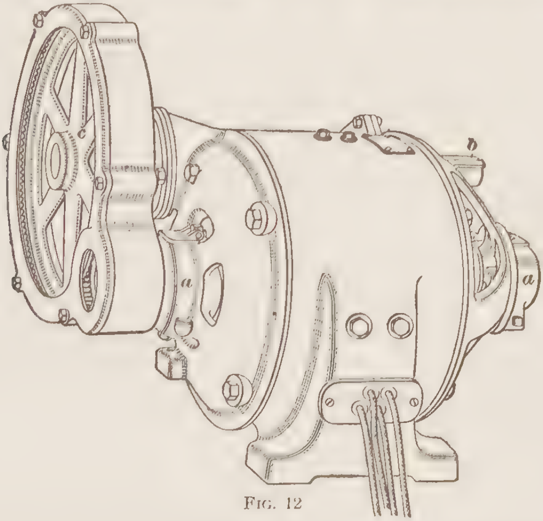


FIG. 12

27. Fig. 13 shows a vertical motor having automatic lubrication. At the bottom of each guide bearing is an oil reservoir, which is filled through a horizontal pipe having an oil gauge *a* at the outer end. When the shaft rotates, oil works

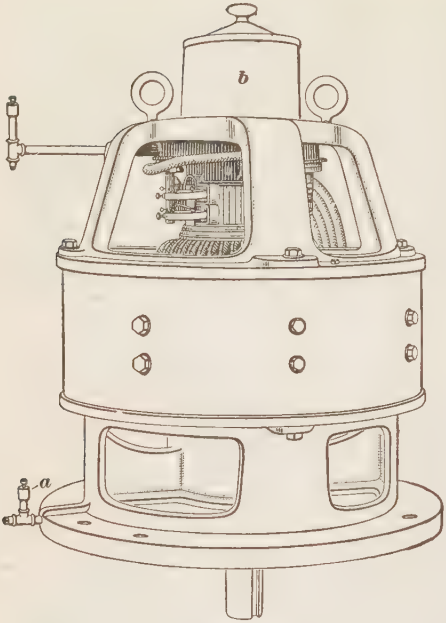


FIG. 13

upwards through spiral grooves in the bearing linings and over the bearing surfaces. Surplus oil flows back into the reservoir. The ball-thrust bearing is inside the cylindrical case *b* at the top of the motor, and the balls are below the surface of the oil in the upper reservoir, thus being effectively lubricated.

28. Vertical motors usually drive from the lower end through direct coupling. They can, however, be arranged for belt drive, but the side pull of the belt may necessitate specially heavy construction of shaft and guide bearings or possibly a third bearing below the pulley.

SERVICE CONDITIONS

29. According to the service for which they are recommended, motors are generally rated for *continuous service*, *intermittent service*, and *periodic service*. Motor manufacturers generally look closely into the conditions of the service to be performed and endeavor to aid in the selection of motors suited to the service.

30. **Continuous-service motors** are ordinarily guaranteed to carry full-rated load continuously for any desired period with temperature rises not exceeding the guaranteed limits. As ordinarily rated, such motors are capable of carrying some overload temporarily without injurious sparking or heating. Continuous service in connection with a motor rating implies constant torque, such as is required for driving fans, blowers, pumps, line shafts, and endless-belt grain elevators. The fact that a motor runs continuously does not always necessitate a continuous-service rating; this rating is necessary only when the load is continuous or nearly so.

31. Intermittent-service motors are generally guaranteed to carry full-rated load continuously for a limited period within specified heating limits. The duration of the guaranteed period depends on service conditions, and the motor windings can be insulated for either high- or low-temperature limits. For example, motors are built for service periods of a few minutes up to continuous service, and for guaranteed temperature rises of approximately 40° C. up to 75° or 100° C. Overload guaranties are not usually given on intermittent-service motors other than momentary overloads. Examples of intermittent service are the operation of some machine tools, passenger and freight elevators, electric vehicles, hoists, cranes, etc. Motors with intermittent-service ratings may be employed where operation is continuous, provided the torque is intermittent, as on many machine tools.

32. Periodic service for motors is also intermittent, but consists of a regular and continuously recurring cycle of operations. A typical cycle for such a motor is that required to operate a shovel for excavating; filling the shovel, hoisting, swinging into position for dumping, and returning for refilling necessitates work at different rates and in both directions of rotation, the same cycle being repeated indefinitely with little variation. Service of this nature is found in several industries.

APPLICATION

33. According to application, motors are classed as *mill motors*, *crane-and-hoist motors*, *elevator motors*, *vehicle motors*, *railway motors*, *mine motors*, etc. Each type named is designed with electrical and mechanical characteristics suited for the application. For example, motors for heavy service, such as in steel and iron mills, in railway work, and on cranes and hoists, are distinguished by very heavy and substantial construction capable of withstanding heavy shocks and great stresses. An example of how motors are modified to suit an application is shown in Fig. 14; this motor is built as a component part of the portable drill.

DIRECT-CURRENT MOTOR CONTROL

TORQUE

34. The **torque,** or *turning effort,* developed by any direct-current motor depends on the current in its armature conductors and on its field strength. The voltage effective in causing current in the armature conductors is the difference between the voltage e applied to the brushes and the counter voltage e' generated in the armature. The current I in the armature conductors is, according to Ohm's law, the quotient of this effective voltage divided by the armature resistance r measured between positive and negative brushes, or

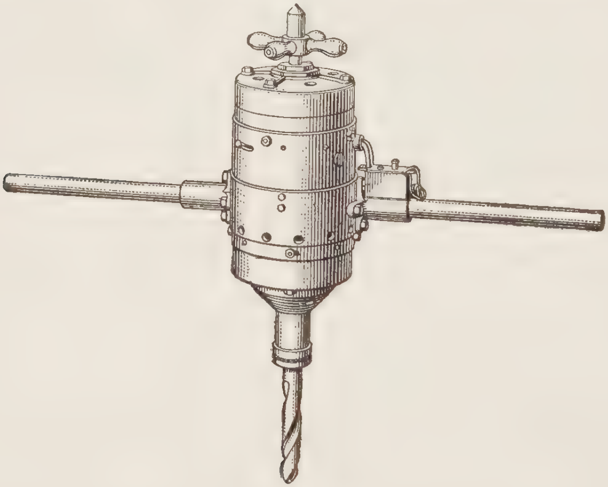


FIG. 14

$$I = \frac{e - e'}{r}$$

35. The counter voltage e' is caused by the motion of the armature conductors in the magnetic field, that is, by the generator action of the armature; the counter voltage is therefore proportional to the speed of the armature and to the strength of the field. The resistance r remains constant, except for small changes caused by temperature variations. At any field strength, the speed automatically adjusts itself until the current is sufficient for the required torque. For example, assuming that e and the field strength remain constant, increasing the load so as to require more torque causes the speed to decrease until $e-e'$ is large enough to cause the necessary increased current. On the other hand, assuming that e and the torque remains constant, decreasing the field strength causes the motor speed to increase so as to keep $e-e'$ at the proper value.

A trolley car in ascending a grade always slows down because additional torque is required of the motors. This additional torque necessitates more current, and, consequently, less counter voltage, and as such motors are series-wound the increased current strengthens the field and causes still further speed reduction, as is explained in the early part of this Section.

MOTOR STARTING

36. When at rest, the counter voltage of a motor is zero and the formula of Art. 34 becomes $I=\frac{e}{r}$. Since in all except very small motors the armature resistance r is very small, the voltage e at the brushes must be low in starting in order that the current I may be kept within safe limits. Switching full-line voltage E on to a motor to start it would in most cases result in current in the armature conductors so large as to overheat them and ruin their insulation.

The applied voltage must therefore be reduced when starting a motor. For general purposes, the only practicable way of obtaining this reduction is by passing the armature current through a resistance R external to the armature. Since the voltage drop in this resistance is the product IR, the voltage applied to the armature is $e=E-IR$.

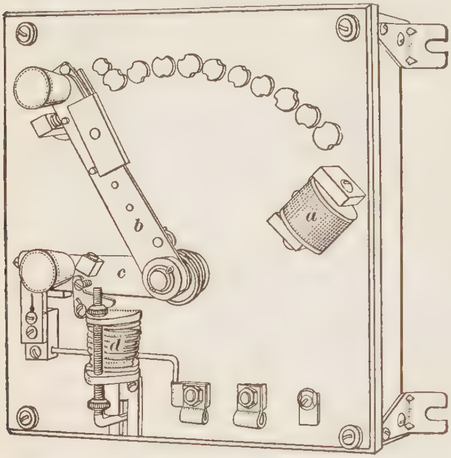


Fig. 15

37. As the counter voltage e' builds up, the impressed voltage can be increased by reducing the resistance R. In practice, this reduction is effected in steps until R is all out of circuit. The number of steps depends on the size of the motor, more steps being required for a large motor than for a small one. More than ten or twelve steps is rarely necessary.

38. A **motor starter** is essentially a resistor with a switching device to remove the resistance in steps from the armature circuit as the speed increases. The starter may be self-contained in the form shown in Figs. 15 and 16, sometimes called *starting rheostat*, or *starting box;* it may also consist of the same essential parts with the resistor and switching device mounted separate from each other.

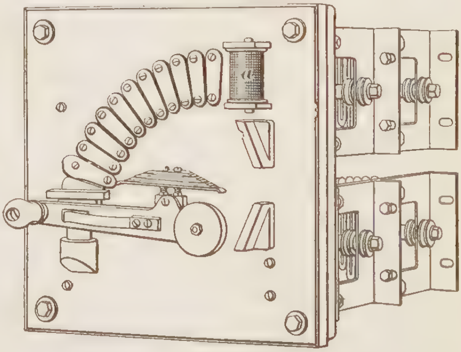


Fig. 16

Fig. 17 shows a *universal starting panel*, so called because the motor starter is mounted on a panel beneath a circuit-breaker, which opens automatically if the motor current becomes too great. The circuit-breaker can also be *tripped*, or opened, by hand, thus serving both for overload protection and as a line switch. Panels of this type are also made with a line switch and line fuses instead of the circuit-breaker.

39. The most important part of a motor starter is the switching device, which may be of the **face-plate type** with contacts arranged as shown in Figs. 15, 16, and 17, or with contacts in the form of series of hand-operated switches, known as the **multiple-switch type,** Fig. 18. The switching device may also be of the **automatic switch, or contactor, type,** Fig. 19. In this case, the switches close automatically, each cutting out a section of resistance when the motor speed has accelerated to the proper point.

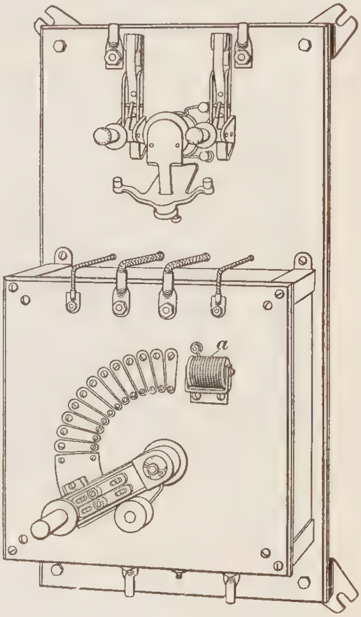


FIG. 17

40. Protective Devices. Nearly all direct-current motor starters are so arranged that in case the line voltage fails the motor is automatically disconnected from the line, and the switching mechanism returns to a position such that the motor must be again started in the usual way after the voltage returns. The return of voltage after temporary failure cannot then injure the motor by the sudden application of full voltage. This automatic **no-voltage** or, more correctly, **low-voltage, release mechanism** consists of a magnetic latch that holds the switching device closed until the voltage falls too low, when the latch releases and a return spring throws the handle or the switches to the off-position. These magnetic release coils can be seen at *a* on the starters shown in Figs. 15, 16, 17, and 18; the starter shown in

Fig. 19 is operated by magnets depending for excitation on the voltage, and failure of voltage allows the switches to open.

41. Many starters also include protective features to preserve the contacts from injury. Among these are **arc shields** of refractory material, as at *a*, Fig. 19, to prevent the formation of destructive arcs when the circuit is opened; also **magnetic blow-out coils** that create a strong magnetic flux across the path of the arc, blowing it sidewise until disrupted. In this case the arc is a conductor, and any conductor carrying current in a magnetic field is forced sidewise by the reaction of the current and the flux.

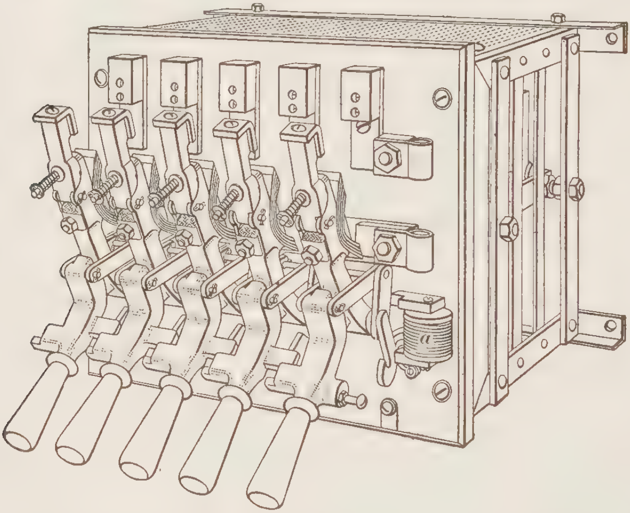


FIG. 18

42. **Overload protection** is also incorporated with starting devices by arranging a magnetic latch to release the switching device if the current in the armature becomes too great for safety. A release of this sort that operates by demagnetizing the low-voltage release magnet is not effective against overloads while starting a motor, since it affords protection only when the switching device is in the running position. The starter shown in Fig. 15 has an overload release that is effective whenever the starting lever *b* is over any of the resistor contacts.

The connections are shown in Fig. 20, in which the reference letters are the same as those in Fig. 15. The low-voltage release coil *a* is energized when the starting lever *b* rests over any of the resistor contacts *1*, *2*, *3*, etc., provided the overload release lever *c* remains in the position shown in both illustrations. If the current becomes too great for safety, the overload coil *d* releases lever *c*, which is thrown open toward lever *b* by a spring, thus opening the motor circuit. Both levers must then be returned to the off-position at the extreme left before another start can be made. The field connections shown by full lines are those of a shunt motor; the series field of a compound-wound motor is connected as shown by the broken lines, the direct circuit between the series-field terminals *e* and *f* then being open.

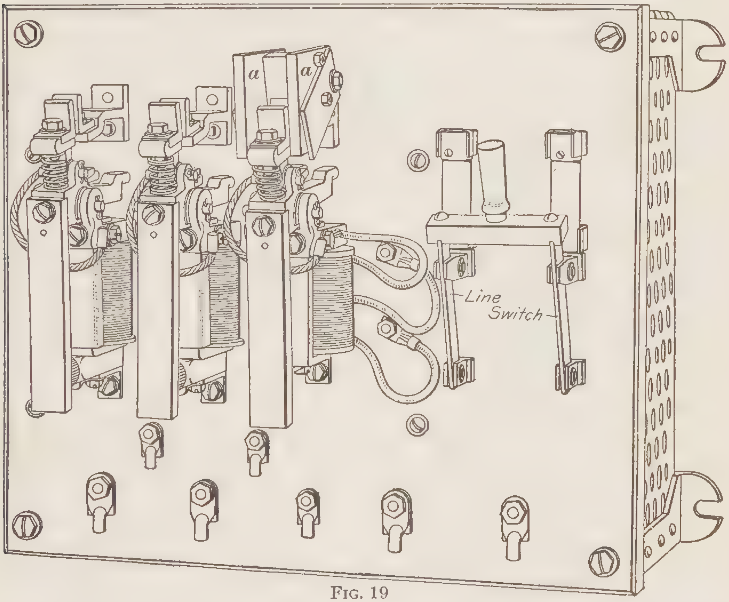


FIG. 19

43. Panels with both low-voltage release devices and circuit-breakers, as shown in Fig. 17, or with fuses afford full protection against injury by voltage failures and overloads. The choice between fuses and circuit-breakers depends on the service conditions. Fuses are better where overloads are rare or of such brief duration as to work no injury; circuit breakers are preferable where injurious overloads may occur frequently and the continued replacement of fuses would be troublesome.

44. Starting and Stopping a Motor.—Starting a motor with a starter like that shown in Figs. 15, 16, or 17 is accomplished by first closing the line switch and then moving the starting lever over the row of resistance contacts, frequently called *steps* and *points*. The movement should be slow enough to allow the motor speed to accelerate smoothly. On the point at the extreme right, the lever is held by the low-voltage retaining magnet; this is the point on which the lever remains while the motor is running and is therefore called the *running point.* The lever will not remain at rest on any intermediate point and must not be held there longer than necessary for the speed to pick up.

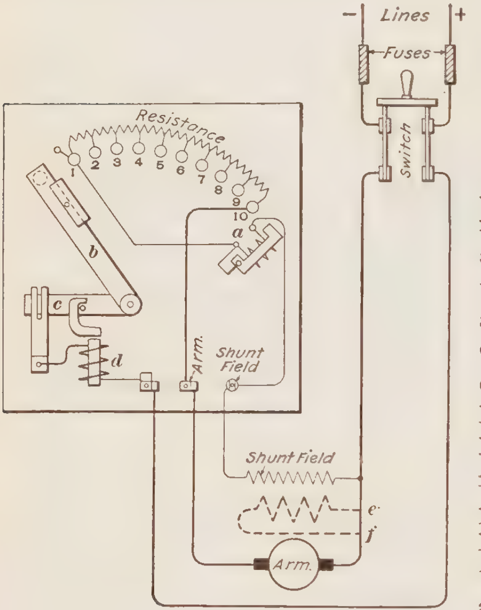


FIG. 20

Starting with a multiple-switch starter, such as is shown in Fig. 18, is accomplished by closing the switches successively, beginning with the switch at the left of the box. They should not be closed more rapidly than the motor speed can accelerate, nor more slowly than is necessary for smooth acceleration.

An automatic starter, such as is shown in Fig. 19, needs no further attention after closing the line switch: the magnetic contactors close successively as the speed accelerates.

45. Stopping a motor with any of the starters illustrated is generally best accomplished by opening the line switch or circuit-breaker. Such a circuit-opening device should be a part of every motor installation, as indicated in Fig. 20. On opening this switch the motor speed will decrease until the magnets release the switching devices, which will automatically return to position for the succeeding start.

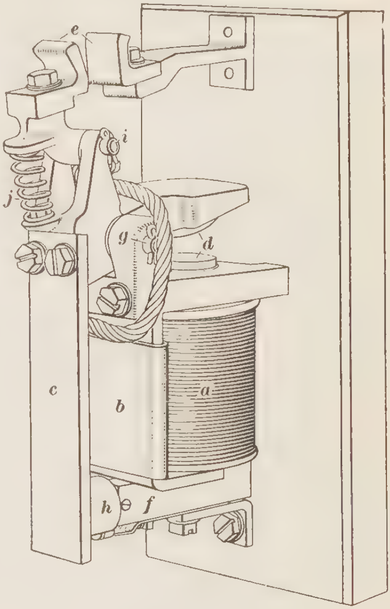


FIG. 21

46. Automatic Starting Switches.—The operation of the automatic starter shown in Fig. 19 may be explained by reference to Fig. 21, which illustrates one of the switch units, or contactors. These contactors are made especially for starters, and are variously called *series switches*, *series contactors*, and *magnetic lock-out switches*, the last name referring to the method by which their operation is delayed. The magnet coil *a* is series-wound, that is, it carries the armature current, and the switch is so made that it cannot close while the current through this coil exceeds the predetermined value for which adjustments are made.

Two return paths are provided for the magnetic flux outside the magnet core, one through a *magnetic shunt b* and the other through a vertical iron strip *c* attached to the moving element. The magnetic pull across the air gap *d* tends to close the contacts *e*, while the magnetic pull between the lower end of the iron strip *c* and the pole *f* tends to hold the contacts open. A copper band around the magnetic shunt *b* prevents sudden changes of flux, and the cross-section of this shunt is so small that it becomes highly saturated when the magnetizing force is high.

When the current in the coil exceeds the adjustment, enough flux passes through the iron strip *c* to hold its lower end against the pole *f*. But when the current decreases, the flux through the strip *c* becomes less until the pull across gap *d* becomes superior, and the moving element turns on its pivot *g*, closing the contacts *e* and causing the lower end of the strip *c* to swing outwards.

47. A self-locking nurled nut *h* serves to adjust the length of the gap *d* and thus adjusts the current at which the switch closes. The contacts *e*, in closing, touch first near the tips and then rock back toward the heels by turning the arm to which the moving contact is attached around the pivot *i*. In thus turning, the spring *j* is compressed, so that when the contacts open, the rocking motion is reversed and the circuit is opened at the tips of the contacts.

48. Several designs of series contactors, or switches, in addition to the one described are on the market. When several of these contactors are properly grouped in a starter and adjusted, they will close automatically, as soon as the line switch is closed, in the sequence and time necessary to bring the motor up to speed. In this case, the delay in operation of each switch is entirely owing to the current in the circuit.

49. Fig. 22 shows a **shunt contactor,** or **shunt-magnet switch,** the word *shunt* indicating that the magnet coil is connected across the circuit. The main contacts *a*, shown between the arc shields, are of the rocking type, as explained in Art. **47.** Shunt contactors do not operate on the magnetic lock-out principle previously explained, but interlocking contacts *b* are added when necessary to control the circuit of the coil operating the next switch in a group that forms a starter or controller.

The control current of shunt switches is only enough to excite the magnets and is so small that an ordinary snap switch or push-button switch can be used for controlling it, and can be located at any convenient point near or remote from the starter. For example, such a starter can be located near its motor and controlled by means of a small hand-operated switch some distance away. Closing the switch causes the contactors to close and start the motor; opening the switch causes the contactors to open and stop the motor.

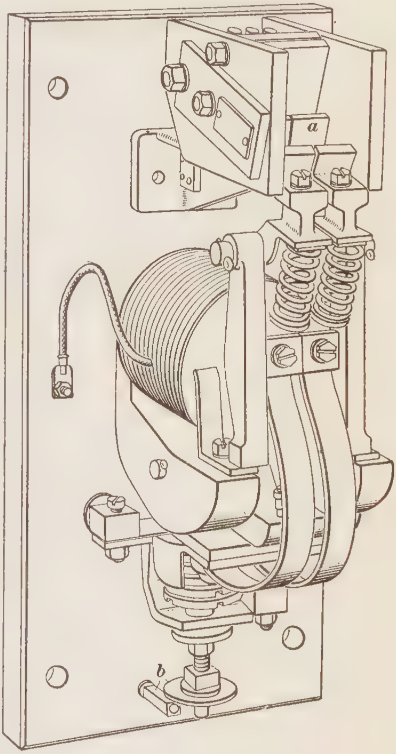


FIG. 22

50. With motors driving pumps and compressors, the control switch can be arranged so as to be operated by the movement of a float or a pressure gauge. The entire operation of the starter and motor can thus be made automatic, starting, for example, when the liquid in a tank reaches a predetermined level and stopping at another level. By this plan, the level of water in a tank or a reservoir or the pressure of air or gas in a receptacle can be maintained automatically within fixed limits without the continual presence of an attendant.

51. Connections of Automatic Starter.—Fig. 23 shows the connections of an automatic starter with one shunt contactor *1* and two series contactors *2* and *3*. The shunt contactor can be controlled manually or automatically from a distant point by means of the control switch, and the closing of the shunt contactor is followed by the closing of the series contactors, each being delayed until the motor current falls to a safe value, as previously explained.

Closing the control switch places the coil *a* of the shunt contactor directly across the circuit, causing contactor *1* to close. The shunt field is thus fully excited and the armature circuit is closed through the starting resistance and series coil *b*. The motor starts, and as soon as the motor current falls to a safe value contactor *2* closes, short-circuiting one section of starting resistance and placing coil *c* in circuit. The motor speed continues to accelerate until contactor *3* closes, short-circuiting the remaining section of starting resistance and bringing the motor to full speed.

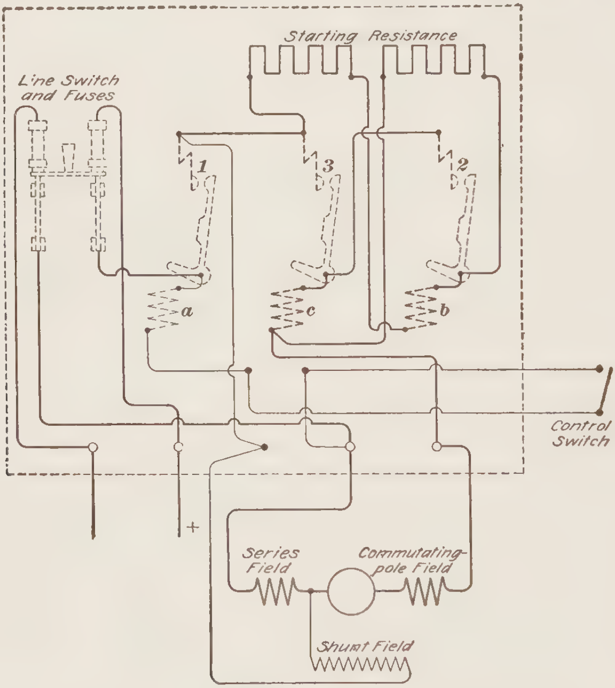


FIG. 23

The delay in the operation of contactors *2* and *3* thus depends entirely on the motor current. A motor starting with a light load is accelerated more rapidly than a motor starting with a heavy load.

52. The **starting resistor** for use with any device intended only for starting a motor is selected to carry the starting current during only the short period required to accelerate the motor speed. In most cases, these resistors would be overheated and possibly ruined if left in circuit long; that is, if too much time is taken to start. The time required to start a motor depends on its size and on the torque required at the start and during acceleration. A large motor usually requires more time than a small motor. Ten seconds is enough for fairly small motors starting with low torque; ½ minute is enough for most motors; while a few large motors under unfavorable starting conditions may require from 1 to 1½ minutes. In starting any motor with a manually operated starter, the successive steps of resistance should be cut out promptly as the motor speed accelerates.

MOTOR-SPEED REGULATION

53. Changes in the speed of a direct-current motor can be effected by changing the impressed voltage at the motor brushes or by changing the field strength of the motor. In either case, the motor speed automatically changes enough to keep the difference between the impressed volts and the counter volts at the value necessary for the torque. Regulation by varying the impressed volts may be called *armature control*, and regulation by varying the field strength *field control.*

54. Armature Control.—External resistance in series with the armature circuit serves to reduce the voltage at the brushes according to the formula of Art. **36,** namely, $e=E-IR$. If it is assumed that the field remains constant at full normal strength, the speed varies with the impressed volts e. Since the brush volts e cannot exceed the line volts E, all speed regulation by armature control is downwards from the full rated speed. The maximum speed occurs when e is nearest E; that is, when $IR=0$, or when no resistance is in the external circuit. The speed of any direct-current motor, whatever its class, can be regulated by armature control.

55. If the current in a direct-current armature remains constant while the resistance in the armature circuit is increased, the speed decreases; or, if the resistance remains constant while the current is increased, the speed decreases. With resistance in the armature circuit, therefore, the motor speed will not remain constant if the load conditions vary. For example, if the motor is operating a machine tool, every time the cutting tool comes to a hard spot or a deeper cut in the metal the motor will slow down very much more if its armature circuit includes regulating resistance than it would otherwise; when the tool has passed the hard spot or comes to a lighter cut, the speed increases again.

56. Since speed reduction by armature control is proportional to the voltage drop in the regulating resistance, it is accompanied by a corresponding loss of efficiency; for example, at 25-per-cent. speed reduction, 25 per cent. of the energy taken from the line is dissipated as heat in the resistance. This method of speed regulation is therefore inefficient, especially where full-load torque is required at reduced speeds.

57. Speed regulation by armature control is suitable for applications where the torque required to drive the load decreases rapidly as the speed is reduced and is constant at each speed; for example, the operation of fans, blowers, and some classes of pumps, which require very much less torque at low speeds than at high speeds. This method is also suitable where operation at reduced speeds is intermittent or brief, as on cranes and hoists. It is not recommended for cases in which the speed should remain practically constant at each adjustment while the torque may vary, as in operating many machine tools.

58. Field Control.—The counter voltage generated by a motor armature depends on the speed and on the strength of the magnetic field. The winding on a motor armature being fixed, the speed must change with each change of field strength in order to maintain the relation $I=\frac{e-e'}{r}$ (Art. 34). The normal rated speed of the motor being with full field strength, and any decrease of field strength being accompanied by an increase of speed, all speed regulation by field control is above the normal rated speed. The speed at any adjustment remains practically constant with all changes of load. The principal application of this method of control is to motors driving machine tools, for which field control is generally the ideal method.

59. The field strength of a motor can be weakened by reducing the excitation or by increasing the reluctance of the magnetic circuit. The former method is most commonly employed, the shunt-field current being reduced by inserting resistance in the field circuit. Since this resistance carries only the shunt-field current, which is comparatively small, the losses in the resistance are small.

60. Speed adjustment by field control is therefore usually confined to shunt motors, though the speed of some compound motors can be adjusted within comparatively narrow limits by regulating the shunt-field current. In general, caution should be used in operating a compound motor with a weakened shunt field, because the effect of the series field causes wide variations of speed with varying load; dangerously high speed would result if the load were removed from a compound motor when operating with a very weak shunt field. Not all shunt motors will commutate well when the field is weakened; and, therefore, only those of suitable design are susceptible of speed adjustment by this method.

61. Speed adjustment by changing the reluctance of the magnetic circuit is successfully employed, two methods being in use. In one, the central part of each field pole is arranged to be withdrawn an adjustable distance; a system of screws operated by a hand wheel enables the operator to withdraw these cores and thus weaken the field flux until the desired increased speed is obtained. By the other method, the armature is withdrawn endwise from the field by means of a screw adjustment until the speed has increased to the desired point; the armature and pole faces are slightly cone-shaped, so that the air gap increases more rapidly than in proportion to the movement of the armature; that is, a 10-per-cent. movement of the armature means more than a 10-per-cent. increase of the air gap. Thus, wide speed adjustment is obtained with small movement.

SPEED-REGULATING DEVICES

62. Speed-regulating devices are the same in appearance and in general design as motor-starting devices, the chief difference being that the contacts and resistors of regulating devices are generally larger, better to withstand the more severe service imposed on them.

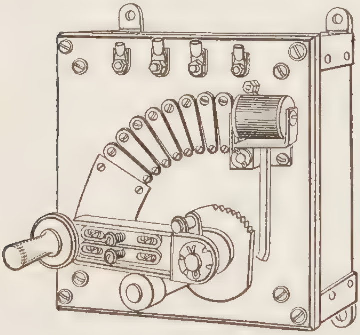


FIG. 24

63. Face-plate regulators of the box type, as shown in Fig. 24, are for adjusting resistances in the armature circuits for continuous operation. The moving contact can be left on any point, from which it can be readily moved to any other point by the operator and from which it is automatically released on failure of voltage; in case of such release, the handle returns promptly to the off-position. With such a regulator, the motor operates with varying speed on any point except full speed; that is, the speed increases when the load decreases.

64. Regulators of the type shown in Fig. 24 are much used with motors operating fans and blowers and in some cases with printing-press motors; such regulators are rarely used with machine-tool motors. The resistor must be designed for the service in each case, more resistance of lower carrying capacity being necessary for fans and blowers than for most other classes of service. The power delivered by the shaft of a motor to a fan or a blower varies nearly in direct proportion to the cube of the speed; hence, at reduced speed, the current is small and the resistance must be high to give the necessary voltage drop. Regulators for fan-and-blower service and for printing-press service have been standardized.

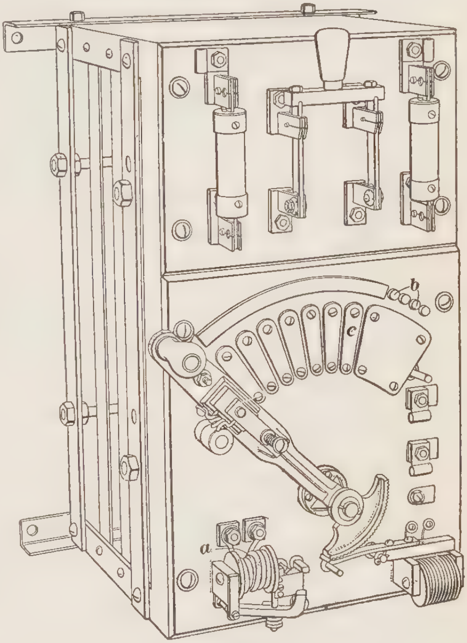


FIG. 25

65. A speed regulator of the universal type, Fig. 25, is similar in principle to that just described, but with a line switch and fuses, an overload release coil *a*, and four points *b* for field control. The operating lever is moved over the stationary contacts from left to right, as before, and can be moved on to the field contact points if speeds above normal rated speed of the motor are desired. The arm can be left on any point and will be automatically released in case of voltage failure, when it returns to the off-position. The rated motor speed is obtained when the arm is over the armature contact *c* and higher than rated speed is obtained when the arm is over any of the field contacts *b*. This type of controller is also standard for fans, blowers, and printing-press service.

66. Face-Plate Field Controller.—Fig. 26 shows a face-plate starting and controlling device for speed regulation entirely by field control. The operating lever is double, one part carrying the moving armature contact and the other the moving field contact. In starting, both levers are moved to the right by means of one handle. At the extreme right, or full-speed position, the armature contact lever *a* is held by the retaining magnet, and the field contact lever *b* is moved back until the desired increased speed is obtained. The field-contact lever will remain on any point, but on failure of voltage the other lever is released and both move back to the off-position.

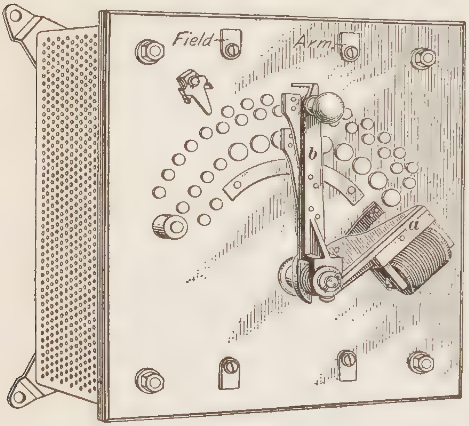


Fig. 26

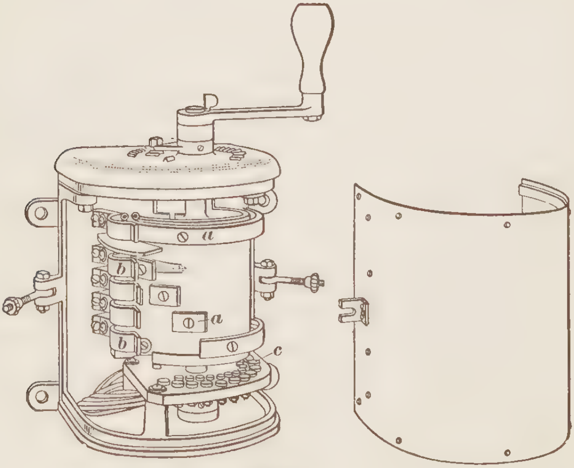


Fig. 27

67. The device just described is used principally with machine-tool motors, but is applicable to any motor where

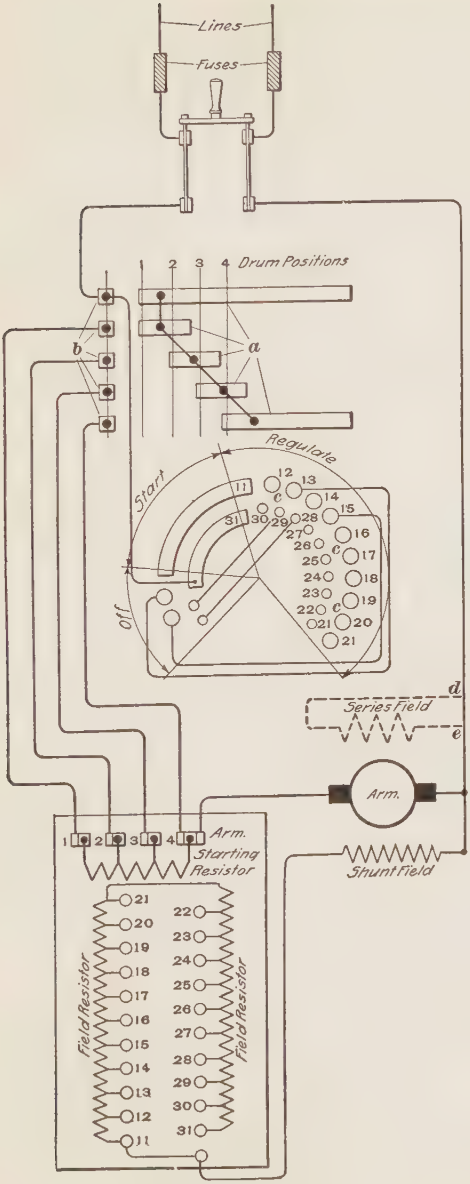


FIG. 28

wide speed adjustment above rated speed is desired. Provision can be made for both armature control and field control in regulators of this type.

68. Machine-Tool Controller. Fig. 27 shows a drum-type controller with the cover removed. This controller is designed for starting machine-tool motors and adjusting their speed by field regulation. Copper segments *a* attached to a drum slide under stationary fingers *b* when the drum is turned, and a contact arm also slides over the buttons *c*. The drum segments change the connections of the armature, and the contact arm changes those of the field coils.

69. A connection diagram of this drum-type controller is shown in Fig. 28. The drum segments are indicated at *a*, the stationary fingers at *b*, and the field-contact buttons at *c*. The line switch and the starting resistor are connected with the fingers, and the field resistor with the buttons; no circuits are connected with the segments until they slide under the fingers. The central part of the figure shows the circular arrangement of the field contacts, while the lower part is a development showing how the steps of resistance are cut into or out of circuit. The similarity of numbers indicate the connections, which are not represented by lines in the diagram.

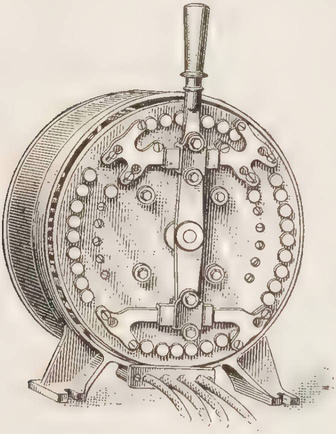


FIG. 29

As the drum is turned, the drum segments *a* slide successively under the fingers *b* to positions *1*, *2*, *3*, and *4*, while the field-contact brush short-circuits segments *11* and *31*, giving full shunt-field strength. In position *1*, all the starting resistor is in circuit; in position *2*, one step is cut out; in position *3*, two steps are cut out; and in position *4*, all the starting resistor is out of circuit. At this point the motor runs with full shunt-field strength, but further rotation of the drum cuts the field resistor into circuit step by step, weakening the shunt field and increasing the speed to the desired point. The connections of the series field of a compound motor are shown by dotted lines; when a series field is used, the main circuit between terminals *d* and *e* is open.

70. Crane Controllers.—Figs. 29 and 30 show typical crane controllers for motors of 5 and 30 horsepower, respectively. All speed regulation is by armature control, and in each case the motor can be operated in either direction, depending on the way in which the handle is moved from the *off-position*, or the position in which the motor circuit is open. Both controllers are shown so arranged that the operators must stand near them, but each can be arranged for operating from a distance by means of ropes or rods. Thus, the smaller controller can be operated by ropes from the floor underneath the crane, and the larger one can be mounted outside the crane cage and operated by means of a bell-crank and a connecting-rod.

71. The larger controller, Fig. 30, has two pair of moving contacts, those of each pair being connected in parallel. A magnetic blow-out coil, Fig. 31, mounted near each contact and connected in series with it sets up a strong magnetic field between the poles *a* directly through the space where arcs form between the moving and the stationary contacts; the arcs are thereby promptly disrupted, thus minimizing injury to the contacts.

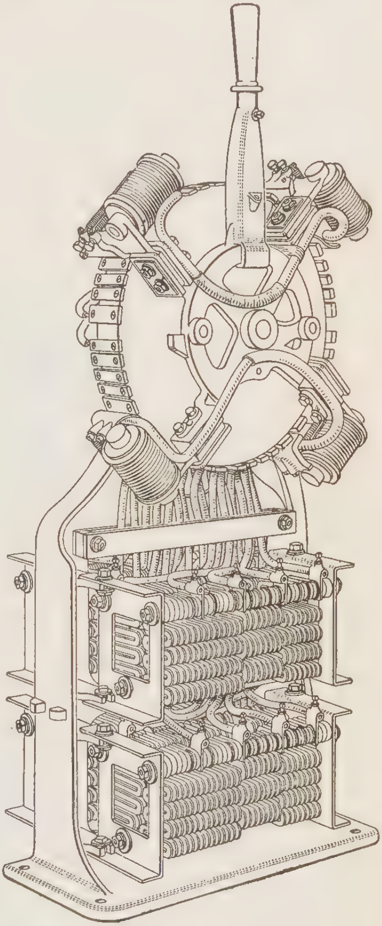


Fig. 30

72. Drum Controllers. Drum controllers arranged for adjusting armature-control resistance only are much used in connection with heavy crane and hoist service, as well as for railway work. The upper portion of Fig. 32 shows external connections of such a controller, with motors having different types of field windings. The lower portion of the figure shows the internal connections. The stationary fingers in the controller are represented by the circles in the center; fingers R_5, R_4, etc. are connected with resistor terminals similarly lettered, and the other terminals with the armature and the line. The moving drum segments are represented by small rectangles, and the drum positions, or steps, are indicated by the vertical dotted lines *1*, *2*, *3*, *4*, *5* through these rectangles. Blow-out coils *a* are provided where necessary to prevent destructive arcing.

73. Turning the drum either way from off-position moves segments under the fingers in a way to complete a circuit through the motor and to cut out the control resistance step by step until all is out on the fifth step. The direction of current in the motor armature and, consequently, the direction of rotation of the motor depends on the direction in which the drum is turned.

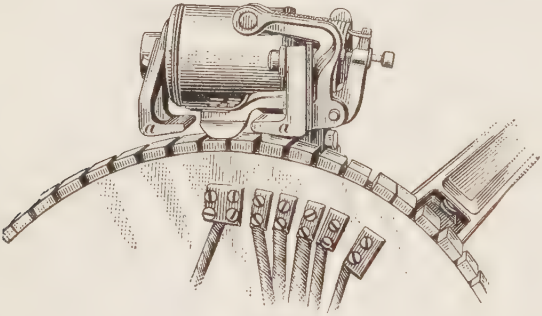


FIG. 31

74. Automatic Controllers.—Automatic controllers composed of magnetically operated switches, or contactors, are made for many kinds of industrial service. Shunt contactors are generally used for speed-control purposes, and the interlocking contacts are so interconnected that each switch controls the operation of the next switch in the series. By means of these switches and suitable safety devices, an operator with a simple master controller can exercise perfect control over large motors in difficult processes where starting, stopping, and reversing are frequently required. The *master controller* is a simple switching device to control the exciting current of the magnets on the contactors.

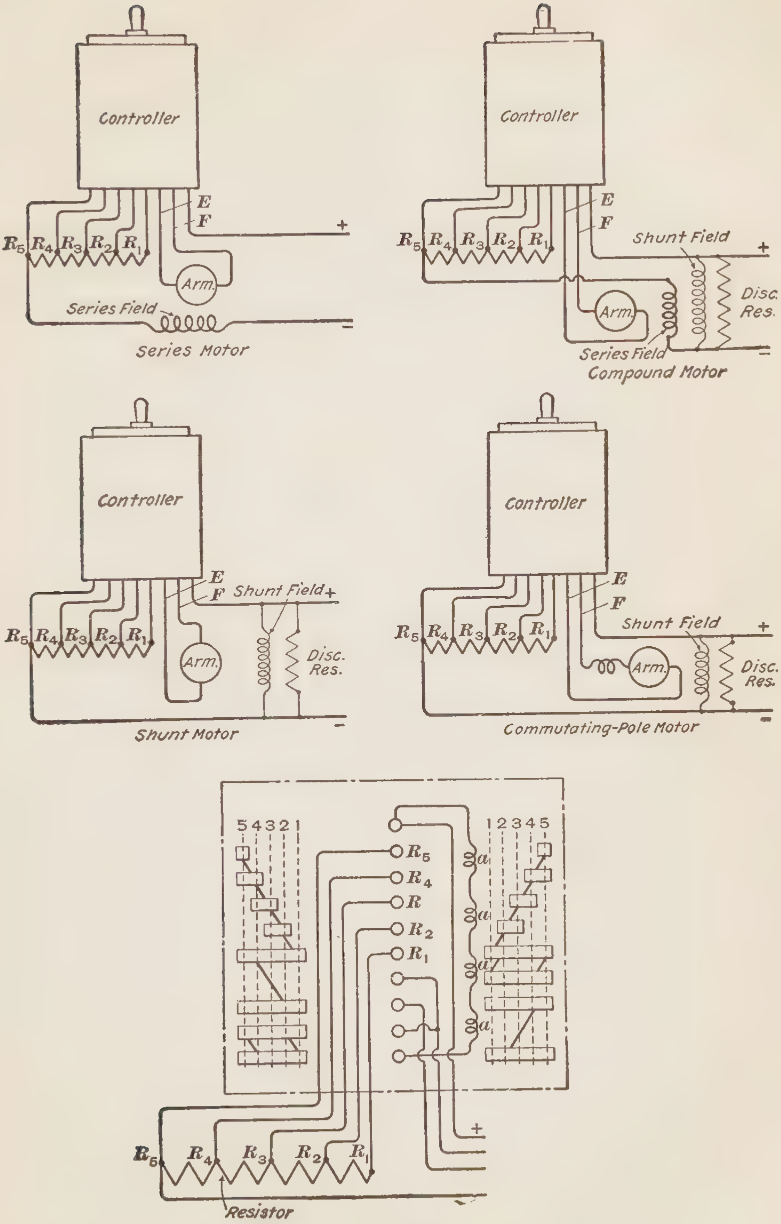

Controller
E
F
R_5 R_4 R_3 R_2 R_1
+
Arm.
Series Field
−
Series Motor
Controller
E
F
R_5 R_4 R_3 R_2 R_1
Shunt Field
+
Arm.
Disc. Res.
Series Field
−
Compound Motor
Controller
E
F
Shunt Field
+
R_5 R_4 R_3 R_2 R_1
Arm.
Disc. Res.
−
Shunt Motor
Controller
E
F
R_5 R_4 R_3 R_2 R_1
Shunt Field
+
Arm.
Disc. Res.
−
Commutating-Pole Motor
5 4 3 2 1
1 2 3 4 5
R_5
R_4
R
R_2
R_1
a
a
a
a
+
R_5 R_4 R_3 R_2 R_1
Resistor

FIG. 32

DIRECT-CURRENT MACHINERY OPERATION

MOTORS AND GENERATORS

INTRODUCTION

PRELIMINARY INVESTIGATION

1. Immediately on taking charge of any electrical machinery, the operator should acquaint himself thoroughly with the nature of the apparatus that will be under his care, including the scheme of connections of each unit, the connections of all of the auxiliaries and operating devices, all of the mechanical details, the normal and safe overload capacities of the equipment, all regular and any special limitations as to the performance of the equipment, and all sources of danger of physical injury. Unless entirely familiar with his plant, the operator may be unable to proceed properly in trying to locate defects in the equipment; he may damage the apparatus by overloading it, or may expose himself to serious personal danger or even to loss of life.

SAFE OPERATING TEMPERATURES

2. It is important that the operator bear in mind and apply in practice certain temperature limitations that experience has indicated to be proper. Temperature limits, often considerably lower than the temperatures at which immediate damage occurs, may, if long continued, cause insulating materials

COPYRIGHTED BY INTERNATIONAL TEXTBOOK COMPANY. ALL RIGHTS RESERVED

to deteriorate gradually, and lead to eventual breakdown, or may so change the magnetic properties of an iron core as to cause it always afterwards to heat seriously in normal operation.

3. Cotton insulation painted with black asphaltum insulating paint as is commonly used for the field coils and armature windings of low-tension machines, is able to stand safely a temperature up to about 195° F. When heated to a higher degree, however, the insulation begins to carbonize, a process that gradually converts insulation into a conductor, thus reducing its insulating and dielectric properties, a condition favorable to breakdown.

4. Generator armature cores are made of a special grade of steel with a high permeability, in order to reduce the hysteresis losses. If, however, the steel is subjected to too high a temperature, even for only an hour or two, its magnetic properties may be so impaired that it will always afterwards have larger hysteresis losses and, consequently, will heat unduly, even under light loads. This effect becomes accumulative in some degree; the greater losses produce higher temperatures, which still further damage the iron. Experience has shown that for the grades of steel now in use for magnetic cores, the temperature limit given in Art. **3** for cotton insulation is about correct.

5. Knife-blade switches so constructed as to depend partly on elasticity, or spring effect, of the copper parts for tightness of electric contact should never be heated to the point at which the temper of the copper is reduced. Operation at a lower temperature may be carried on without injury. As commercial copper differs so much in quality, it is impossible to name the critical temperature at which the temper is drawn, but, for many kinds of switches in use, it is about 167° F.

6. Bare copper used for conductors in which the joints are secured by more reliable means than the spring effect, as, for instance, by bolts, may be much hotter without injury, and a temperature of 212° to 230° F. would not be dangerous. However, it is important to observe whether or not the high temperature in causing expansion of the copper may produce a mechanical strain at any of the joints, a condition that might cause poor contact and excessive heating at the joint.

7. Bearings of machines should not be operated at temperatures that would cause the Babbitt to soften, but if the bearings are kept properly oiled there is not much danger of this trouble. Babbitt bearings may be safely run at 160° F., and bronze bearings still hotter. If the hand can be held on the outside of a bearing without great discomfort, the temperature is usually safe. It should be borne in mind that the internal temperature of a bearing is much hotter than the external, and the sensation experienced by placing the hand on the outside of the bearing should be only a guide to enable the operator to judge as to the internal condition.

OPERATION OF DIRECT-CURRENT MOTORS

SHUNT MOTORS

8. A direct-current motor, when set up and connected, should, before being started, be given a critical inspection, with a view to determining its mechanical and electrical condition. The armature must rotate freely in its bearings, all the electric contacts must be good, the machine anchored securely in its position, the connections correct, and the brushes well seated and under proper tension on the commutator.

Direct-current motors larger than about $\frac{1}{8}$ horsepower are, in almost all cases, started at a suitably reduced voltage, in order to avoid too great a current, which might injure the windings. The most common method of obtaining the reduced pressure for starting is by means of a starting box, or rheostat, inserted in the armature circuit. The procedure in starting a motor so equipped is as follows:

1. See that the starting resistance is all in circuit, that is, the arm of the rheostat is in the starting position.
2. Close the main armature switch (line switch) or circuit-breaker.

3. Slowly move the arm of the starting rheostat from its initial position, point by point, until it has reached the limit of travel and all the resistance is cut out. If, instead of a simple starting box, the starting resistance is also used to control the speed, stop cutting out resistance when the desired speed has been reached.

4. If the starting box, or speed controller, has an automatic low-voltage release, see that the rheostat arm is securely held in place by the retaining magnet.

5. Examine the motor to see if the bearings are getting sufficient oil and if the commutation is practically sparkless.

If the motor fails to start on the first step, the rheostat arm should be moved promptly to the second step and, if necessary, to the third step. If the motor fails to start on the third step, the line switch should be opened at once, the rheostat arm allowed to go to the off-position, and an inspection begun for faulty connections, overload, etc.

The foregoing instructions do not, of course, apply to motors equipped with automatic starters.

9. The time required for speeding up the armature of a direct-current motor is dependent on the size and weight of the rotating mass. The armature of a 1-horsepower motor should ordinarily not require more than about 15 seconds for bringing it from rest to full speed. If, however, there is mechanically connected to the armature, either by direct coupling or by belt, a rotating body having a large amount of weight at a considerable distance from the center of rotation, the time of acceleration should be considerably increased. Failure to observe this precaution may result either in blowing the fuse in circuit with the motor or, possibly, in damaging the motor armature. Twenty-five seconds is ordinarily sufficient for speeding up a motor of from 25 to 75 horsepower if there is no heavy connected load, and motors up to 500 horsepower may be brought up to speed in from 60 to 70 seconds.

If there is an ammeter in the motor circuit, its indications will be the best possible guide as to the proper rate of cutting out starting resistance. If the starting rheostat has a considerable number of contact points, as, for instance, ten or fifteen, the rheostat arm may be moved just fast enough to prevent the current from becoming greater than about 80 per cent. of the initial rush of starting current. If the starting rheostat has only a relatively small number of contact points, as, for instance, from three to six, the arm or switch blade should be held at each point until the current, as indicated by the ammeter, has practically stopped decreasing in value, at which time another section of resistance should be cut out.

When ammeters are installed, they should be frequently observed, as their indications enable the operator to know at once whether or not the motor is taking too large a current. As a rule, such instruments are not provided in connection with small motors and only infrequently with moderate-size ones.

10. During the operation of the motor, routine inspections should be made to see that no mechanical or electrical trouble is developing.

The field circuit of a shunt motor or the shunt-field circuit of a compound-wound motor should never be opened, even for an instant, while current is being supplied to the armature, as the result may be such a dangerous increase in speed as to cause the rotating parts to be seriously damaged. In some such cases, the centrifugal force is sufficient to cause the commutator to fly to pieces and the conductors of the armature winding to fly out of their slots; this constitutes one of the dangers of physical injury incident to improper operation of shunt- and compound-wound motors. It is customary to protect motors against such a result by inserting current interrupting, or cut-out, devices in the motor-armature circuit. When the current in the field circuit is interrupted, the magnetic flux cut by the armature conductors is very much reduced and the counter electromotive force of the motor is also reduced, thus permitting a larger current in the armature. If the current-interrupting devices, either fuses or circuit-breakers, operate quickly enough, damage is avoided, but in many cases the acceleration is so rapid that the time element of fuse or circuit-breaker allows a dangerous speed to be reached before the circuit is opened.

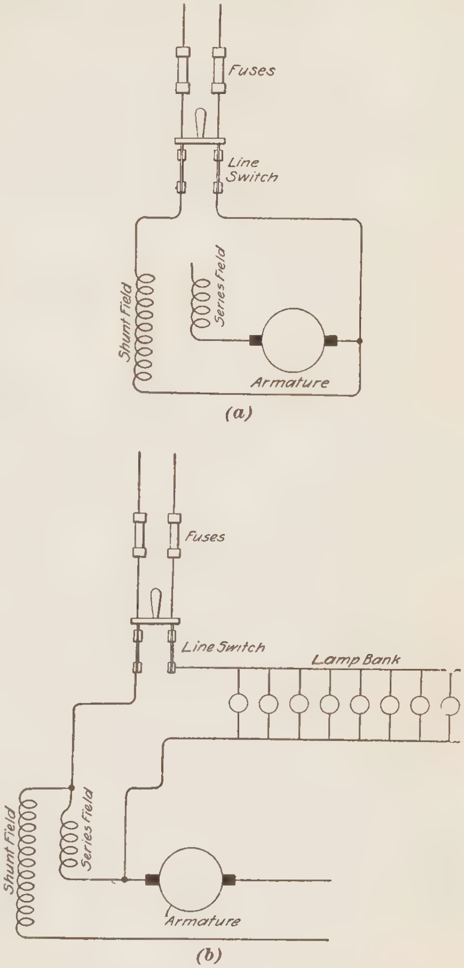
Speed control of both compound and shunt motors is sometimes effected by the use of a variable resistance in the armature circuit. The operator should examine this resistance often enough to satisfy himself that it is not overheating, and that there are developing, in resistance conductors, contact arm, or contact buttons, no faults that could result in opening the circuit.

Before shutting down the motor, the operator should assure himself that the bearings are not heating unduly. Overheated Babbitt may *seize*, or, as it is frequently expressed, *freeze*, when cooled, and thus render it impossible to turn the armature when cold. This remark applies with equal force to all rotating machinery.

11. The usual procedure in shutting down a motor is to simply open the main switch or the circuit-breaker. When this is done, all automatic devices return to their normal starting positions, so that if the switch is again closed no harm will result. After the motor is at rest, the operator should, however, carefully inspect the automatic devices, in order to see that they have operated properly. The arm of a starting box provided with automatic release should never be forced back to stop a motor; that type of rheostat is not designed for interrupting the current. Some types of combined starting and speed-control rheostats are made to open the motor circuit; but even with these, it is advisable to open the line switch, if the motor is to remain idle for some time.

With most drum-type controllers used with machine-tool equipment, a quick stop can be made by moving the handle quickly to the first running notch, holding it there for a moment, and then moving it to the off-position. An emergency stop can be made by moving the handle to the first running notch, letting it rest there momentarily, and then moving it to the first reversing notch. This last method should never be used except in emergency, and in using it the handle should never be moved beyond the first reversing notch.

12. The direction of rotation of the motor armature can be reversed by reversing the direction of the current in either the armature or the field circuit, but not in both. If the motor is provided with a reversing switch for this purpose, it is used in the armature circuit. If no reversing switch is provided, the current supply should be cut off and the armature allowed to come to rest. Either the field or the armature terminals may then be reversed according to convenience.



(a)

(b)

FIG. 1

COMPOUND MOTORS

13. Before starting a compound-wound motor for the first time, see that the series and shunt fields agree in direction, unless intended to act differentially. The test for polarity may be made by opening the series field circuit of the motor and sending current through the shunt circuit, as in Fig. 1 (*a*), observing by means of a pocket compass or permanent bar magnet the polarities of the pole pieces, then disconnecting the shunt circuit, sending current through the series circuit in the direction that it will have in operation, and repeating the observation for polarity. Under the latter condition, the field flux will generally be rather weak, even when the current is equal to the full ampere rating of the motor, and it is important that the device used for determining the polarity be fairly sensitive. In testing the series circuit, it is generally necessary to limit the current by a suitable resistance, such as a bank of incandescent lamps, connected as shown in (*b*), remembering that the current is decreased by decreasing the number of lamps turned on in the lamp bank.

14. The methods of procedure followed in the operation of compound-wound motors are similar in some respects to those used for shunt motors.

A compound-wound motor can be started and stopped in the same manner as a shunt motor. Owing to the influence of the series current, the accumulative compound motor has a strong starting torque. Its speed regulation is not so good as that of a shunt motor and, in some cases where good speed regulation is of importance, a special provision is made for shunting the series field after the armature is up to speed. Care must be taken that the special shunt circuit has good contacts and low resistance, or else a considerable current will still pass through the series field. The operator should frequently examine all the contacts in this shunt to see that they are clean and to prevent the development of a resistance that is high as compared with that of the series-winding.

The direction of rotation of the armature should be reversed by changing the direction of current in the armature rather than in the fields, since there are two field windings, each of which would require reversal, thus making the change more difficult and increasing the liability of making wrong connections.

SERIES MOTORS

15. Series-wound motors are generally used for heavy duty where speed regulation is of comparatively little importance. Like shunt motors, they are started with a resistance to limit the flow of current. In many cases, as in electric cars, this starting device is so designed as to perform, also, the duties of a controller for starting, stopping, and reversing the motor and for regulating speed.

A characteristic of series motors is that when running unloaded their speed becomes excessive. This is a dangerous condition and should be avoided by mechanically connecting such motors so that the load can never be entirely removed, as by coupling or gearing to the driven machine.

OPERATION OF DIRECT-CURRENT GENERATORS

PRELIMINARY INSPECTION

16. Generating machinery should not be started for operation without first being subjected to a careful inspection. The condition of the brushes and of all electric contacts that are liable to any considerable change should be determined. Any foreign articles, such as tools, rags, waste, etc., found in or about the machine should be removed. If possible, the operation of the switches in the circuit should be tested.

The condition of the armature switches of a direct-current generator is a matter of considerable importance, as inattention to the contacts may result in serious overheating. If conditions permit, each switch should be tested by being closed and opened several times to see that it works freely. The armature circuit must not be closed to live busses unless the generator is up to speed and has the same voltage and polarity as that of the busses; otherwise, the busses will be virtually short-circuited by the low resistance of the armature circuit.

SHUNT GENERATORS

17. A direct-current generator should be brought up to normal speed gradually. After the armature has begun to rotate, the surface of the commutator should be cleaned by using a cloth or piece of waste slightly moistened with engine or dynamo oil, the commutator passing first under the oily rag and then under a dry cloth to remove most of the oil, leaving the surface clean and slightly moistened with oil. During the latter part of the acceleration, the operator should observe the polarity and potential of the generator.

18. If a direct-current generator is to be paralleled to any other source of potential, it is, of course, necessary that the polarities of both be alike. It is also necessary for the proper indication of measuring instruments that the direction of current is correct.

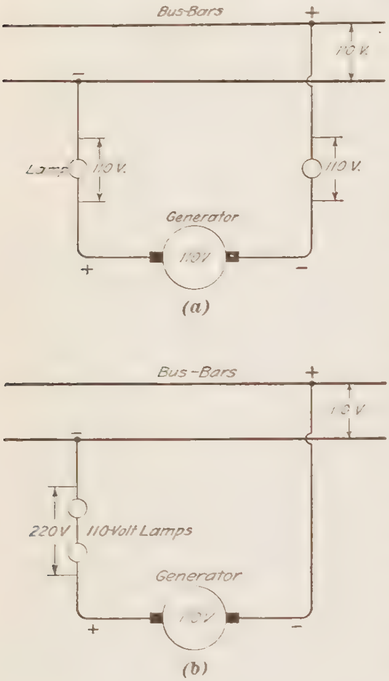


FIG. 2

If a direct-current voltmeter properly connected across the open leads of the generator armature shows a reversed indication, the polarity is incorrect. The voltmeter indication can be checked by allowing the generator to reach full speed and voltage and then switching in series between each armature lead and the live bus to which the lead is intended for connection a sufficient number of incandescent lamps in series to require for full brilliancy the normal voltage of the machine. If the polarity is correct, the lamps will not burn; if it is reversed, they will burn at approximately full brilliancy. The connections for reversed polarity are shown in Fig. 2. For a 110-volt system and 110-volt lamps, the distribution of potential will be as indicated in (*a*) if the polarity of the generator is reversed, and there is no resistance in any of the conductors except the filaments of the incandescent lamps.

The test just described can be varied to suit the facilities at hand. For example, the generator may be closed on one side, or terminal, to the bus on which it is to operate, and all the lamps connected between the other armature lead and its bus connection, as shown in (*b*). If the polarity of the machine is incorrect, as indicated, the lamps will burn at about full brilliancy; if correct, the lamps will not burn.

Instead of the lamps connected as in (*b*), a portable voltmeter capable of taking twice the normal voltage of the generator can be used, connecting it in series between one generator terminal and one bus-bar. If the polarity is correct, the voltmeter indication will be very low—zero, when the pressures on the machine and bus are equal. If the polarity is incorrect, the voltmeter will show the sum of the machine and bus voltages.

19. If, by observation and test, it has been determined that the polarity is reversed, the armature should be brought to rest and the polarity corrected by the safest and most easily applied method available. If a bus to which the generator can be connected is alive and of correct polarity, all the brushes of one polarity (either positive or negative) are raised from the commutator and the main armature switches of the generator are then closed. This will send current through the field windings in such a direction as to produce fields of the proper polarity. After a few seconds, the armature switches are opened, care being taken to open the circuit very slowly by gradually increasing the distance between the knife blade and the contact clip.

When the circuit is broken in this manner, a long, flaming, hissing arc will occur at the switch, but the operator should feel no concern, as the arc is perfectly normal under this condition. It is quite important that the switch be opened slowly, as a quick break would cause to be established in the highly inductive field circuit an electromotive force that might be great enough to damage the insulation. The brushes can be separated from the commutator by putting paper under them.

If inconvenient to raise all of the brushes of one polarity from the commutator, a machine lead can be disconnected between the armature and the tap for the shunt field. This is really the safer method of opening the armature circuit, because, in the first method, if one brush is overlooked, there will be a short circuit. It should be remembered that the object of either method is to establish a current in the field circuit, but not in the armature windings.

20. When the polarity is correct, an observation should be taken to determine the voltage of the generator. If the machine is to be connected to busses not already alive, the voltage may be brought to normal by adjustment of the field rheostat and the armature may then be connected to the busses by means of the armature switches. The voltage is adjusted by varying the current in the field windings; increasing the field current increases the voltage, and vice versa. If a series rheostat is used, the field current is increased by cutting rheostat resistance out of circuit and is decreased by the reverse process. With a shunt rheostat, the field current is increased by increasing the rheostat resistance, and vice versa.

21. If the generator is to be paralleled to some other source of current, as a storage battery or other shunt-wound generators, the voltage must be correct for paralleling. The correct theoretical voltage of the incoming machine depends on a number of factors; but, practically, it may properly be about 2 or 2½ per cent. higher than that of the busses to which the machine is to be connected. This will cause the generator to take a little load as soon as connected, which is not objectionable, if not too great, and is generally desirable for engine-driven units, as it is not advisable to risk motorizing the generator. If the paralleling is done at equal pressures, some variable condition may cause a reversal of current through the generator, thus tending to drive it as a motor.

If the load is steady and the speed regulation of the driving power of the generators is very good, the voltage differences just given can be reduced, and, if conditions are sufficiently reliable, paralleling can be performed at equal voltages. With voltages equal, the generator is brought into service entirely without load, a practice that can be recommended when the conditions are favorable or when a slight motorization of the generator is not objectionable, as, for instance, when the generator is being driven by a direct-current motor.

When putting a generator into service for the first time or when the proper voltage difference for paralleling is not known, it may be regulated to 2 or $2\frac{1}{2}$ per cent. greater than that on the busses to which the machine is to be connected. After paralleling, the rheostat may be adjusted until the load is as small as can be held steadily without danger of reversal. Then, if the armature switches are opened and readings of both the bus and the generator voltages taken, the operator will have obtained, once for all, the most desirable relation between generator and bus voltages for safe and proper paralleling of these particular generators. It must be remembered that not all machines designed for the same voltage require precisely the same relation, as they differ greatly in armature characteristics.

22. After the generator voltage is correctly adjusted and the main armature switches are closed, the operator should at once observe, by the ammeter, the amount of load taken, in order to see that no unusual or dangerous condition exists. The generator may then be put under load by increasing its field strength. If the addition and loading of the generator renders necessary the transfer of load from other similar machines, the reduction of load on the others is obtained by weakening their fields by means of the rheostats. The transfer of load from a storage battery is obtained by reducing its potential by means of the end-cell switches, booster generators, or other means provided for the purpose. The added generator may then be continued in operation and its load regulated by the field rheostat, the field strength being increased to increase the load on the generator and weakened to reduce the load.

Though the operation of shunt machines in parallel is a stable condition, unsafe load conditions can be developed by transferring to one or more machines a larger load than they can carry safely, and this may be done merely by reducing the load on some other unit. This emphasizes the importance of observing the load on each generator every time that a change of adjustment is made in the load on any one. In addition, routine observations of load conditions should be made at frequent intervals, the frequency of inspection depending somewhat on the character of the service.

23. As the load on the station increases, that on each generator is also increased and the station voltage is reduced, thus requiring an increase in the field strength of each machine. If the generators are so designed as to be all alike as to armature characteristics, and if under the lighter load condition all were loaded in the same proportion to capacity, then under the heavier load condition this proportion will continue. If, however, the armature characteristics of the different generators in parallel are unlike, those generators having the largest internal armature drop will take less than their share of the increase of load and those having the lesser internal drop will take more than their share of the increase. If the generators were all alike as to armature characteristics, but not loaded alike in proportion to their capacities, the machines carrying the largest percentage of full load would take less than their proportional part of the increase. This last-named feature tends toward safety of load distribution, and therefore is not liable to develop dangerous load conditions. With unlike characteristics, however, lack of proper attention may result in too much load being thrown on some of the machines.

The reverse conditions as to regulation are, of course, true, but no dangerous load conditions are liable to develop as a result of reduction of load except when the latter gets so light as to interfere with the proper operation of the driving machinery. Some types of steam engines do not work reliably when lightly loaded, and care should be taken not to allow the load to get down to the point where racing or unsatisfactory speed regulation results. Such conditions cause variations in the voltage and impair the service. It is better practice to reduce the number of units in operation until those remaining in service are fairly well loaded, thus improving both the regulation and the economy.

24. If a load equal to several times normal rating is thrown on a shunt generator, its voltage and current will be reduced practically to zero. Under such a condition, although the armature may continue to rotate at full speed, the machine will be without electromotive force until the excessive load is removed. Generally, severe sparking of the brushes occurs at first when a shunt machine is heavily loaded or short-circuited; but the result is usually not destructive, as the drop in voltage causes an immediate reduction of field current, this resulting in a further lessening of the electromotive force, the effect being cumulative until the voltage and current are both zero.

COMPOUND GENERATORS

25. A compound generator is started, accelerated, observed for polarity, and regulated for voltage in the manner described for shunt machines. But the parallel operation of compound-wound generators provided with the same terminal connections as shunt machines is unstable. The proper closing of equalizer switches is a matter of extreme importance in connection with paralleling compound-wound direct-current generators, and the operator should never perform the paralleling operation without first checking the conditions of the equalizer circuit and seeing that the equalizer switch is closed. Failure to do so may result in motorizing one of the generators, reversing its polarity, or, in some cases, so severely overloading the windings of one of the units as to cause serious damage.

Two forms of practice are followed in adjusting the generator pressure before paralleling: 1. The voltage of the incoming machine is adjusted before the equalizer switch is closed. 2. The series-field windings of the running and incoming machines are placed in parallel by means of the equalizer switches and then the voltage is adjusted. Though the first method is, perhaps, the more common, the latter is the better practice, as the compounding effect of its share of the load is produced before the generator is connected, thus permitting a more exact control over the amount of load that will be taken when the paralleling is completed. In the operation of railway generators, on which the load fluctuations are large, one method is practically as good as the other.

26. A change in voltage of a compound generator can be made by means of the field rheostat, as with a shunt machine; but the compounding effect of the series-winding, under the influence of load changes, performs, automatically, much of the regulation that would require manipulation of the rheostat of a shunt machine. If the compounding is properly adjusted, the series field will correctly regulate the voltage for all changes in load within limits, and the compounding effect can be changed to suit the required conditions. That is, adjustment can be made to give a constant voltage at the station busses, or, if preferred, a little overcompounding can be produced to compensate for a drop in voltage in the distribution system and thus to maintain a constant voltage at some designated feeder terminal.

The method of changing the amount of compounding is to change the amount of resistance in the shunt connected in parallel with the series-field winding. If additional compounding is desired, more current is diverted through the series-winding by increasing the resistance of the shunt. If less compounding effect is required, the resistance of the shunt is reduced.

It should not be assumed that once a generator is started and put into service its voltage will require no further observation or adjustment; because, as the windings heat up under the influence of the load current, their resistance increases, which increases the armature drop and also decreases the shunt-field current. For this reason, the operator's attention will be required to see that the proper voltage is maintained.

27. On account of the increase of electromotive force as the load increases, a compound-wound generator is not suitable for operation in parallel with storage batteries, because the voltage of a storage battery, like that of a shunt generator, decreases as the load increases. As soon as a compound generator, paralleled with a storage battery, delivers any current to the battery, the effect is to increase the voltage of the machine, which then sends more current through the battery, the effect becoming accumulative within limits. If the series-field winding is shunted by a heavy copper bar and the series-field circuit is opened, the machine is converted into a shunt generator, which operates well with a storage battery.

SERIES GENERATORS

28. Series-wound generators are used principally for supplying constant direct current for series-arc-lamp circuits. No new installations are being made, and the treatment herein covers only the general principles applicable to all the various types of series generators still in use.

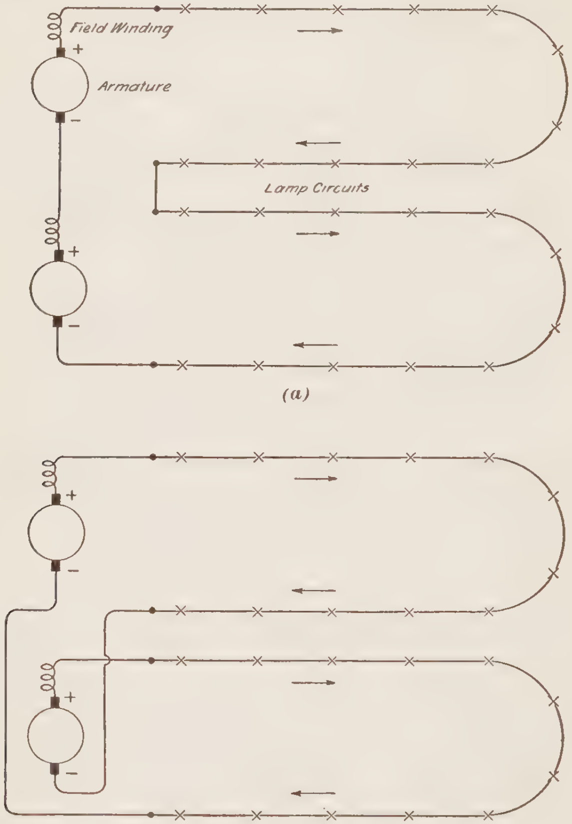
Care must be taken that a series generator is correctly connected to the circuit; a reversal of the polarity of the circuit will cause the arcs to burn upside down.

Usually, one circuit is connected in series with each generator, but two or more circuits can be connected in series with one another and the combination connected to one machine, provided that proper attention is given to polarities and that the total voltage required by all lamps does not exceed the voltage capacity of the generator.

If one generator is underloaded and another in the same station is overloaded, two lamp circuits can be connected in series with the two machines, in order to distribute the load more evenly. One machine is then generally made to generate a constant voltage by making its automatic regulator inoperative, and the other machine, with its regulator, takes care of the load changes. Two forms of connection for this method of distributing a load are shown in Fig. 3, in which arrows are drawn to show current direction. The system shown in (*b*) has the advantage over (*a*) of placing a lower voltage to ground on the generators in case a ground occurs on a lamp circuit.

29. In general, the lamp circuits should, when possible, be connected to the generators before the machines are started. A lamp circuit should never be disconnected unless the voltage of the machine to which the circuit is connected is first reduced to practically zero by short-circuiting the field, the armature, or the machine as a whole.

Sparkless commutation on an arc machine is not to be expected, but for any given type of generator there is a normal spark that the operator should learn by observation, and the cause of any abnormal sparking should be promptly located. Some of the possible causes are faulty brush setting, poor current regulation, or a break in the armature circuit.



(a)

(b)

Fig. 3

FIELD CIRCUITS OF DIRECT-CURRENT GENERATORS

30. In all that has been said so far about the operation of both shunt- and compound-wound generators, the operating practice described is such as should be followed with machines connected without any field switches or other convenient means of opening the field circuit. Direct-current generators operating self-excited should, in general, be connected without switches in the shunt-field circuit.

When field switches are provided, it is, of course, necessary that they be closed before the unit is started, or the machine will not generate. It is even a matter of more importance that while a generator, either shunt or compound, is operating in parallel with any other source of current, its field circuit be kept closed. If the field circuit should be opened, either by mistake or otherwise, the machine having no excitation would stop generating, and the armature, still connected to the other source of current, would be a short circuit of low resistance. If not promptly disconnected from the bus-bars, either by the operator or by automatic protective devices, the other units may be overloaded and damaged. If sufficient capacity in compound generators or in storage batteries is in parallel with the unit with the open-field circuit, its armature will be burned out if not disconnected at once.

31. In some installations, it is desirable to operate generators excited either from the busses or from a special exciting set. In such cases, it is necessary that the precautions before mentioned, are observed with reference to any interruption of the field circuit. When generators operating in parallel are neither self-excited nor excited with current from the load busses, all should be excited from the same exciter set, or some other means should be taken to make impossible the interruption of the field-current supply to a part and not all of the units

EDISON THREE-WIRE SYSTEM

32. All the foregoing instructions on the operation of direct-current generators applies to the Edison three-wire as well as to the two-wire system. Only the system of connections shown in Fig. 4 requires special mention; because, though the operation of the individual generators is not greatly affected, the regulation of the station voltage is more complicated than in the two-wire system.

33. Owing to the importance of maintaining nearly constant voltage at customer's premises, general central-station practice is to regulate the bus-voltages so as to give standard conditions at feeding centers in the distribution system. Observations of voltage at these points are obtained by means of voltmeters connected through voltage wires to the feeder terminals, and the voltages on the two sides of the system at a feeding center are kept equal. It is therefore necessary to regulate together all the generators on each side of the system and to observe carefully the result on the relative voltages of the two sides.

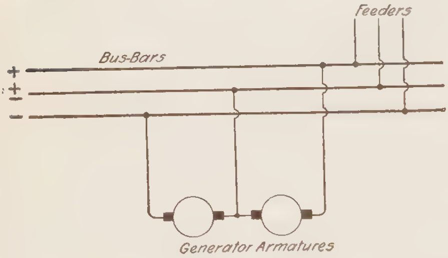


FIG. 4

If the load is perfectly balanced throughout the distributing system, the bus-voltage will be balanced also, but this is a condition that seldom exists, and usually the voltages on the two sides of a three-wire system are unlike at the station. If it is observed that the voltage of one side is becoming too low or too high, correction should be made in the same manner as on a two-wire system; but when regulating voltage on one side, attention must also be given to the other side. High or low voltage on one side may be the result of either one of two possible conditions: Load conditions existing on one side and not on the other may make one side high or low; or the total voltage between the positive and negative sides may be unchanged and the generators on one side may be delivering a higher or lower voltage than those on the other, although the connected load may be the same on each side. The second condition amounts practically to a displacement of the neutral and is to be corrected by regulating the generators on both sides, reducing voltage on the high-voltage side and raising it on the other.

TIRRILL REGULATORS WITH DIRECT-CURRENT GENERATORS

34. If a Tirrill direct-current regulator is used, the procedure in the operation of the form for generators of small or medium size is as follows:

Before connecting the regulator, mark the position of the generator field rheostat arm at which the generator voltage is reduced about 30 per cent. below normal when the regulator is not in operation.

With the generator in operation at normal voltage and either with or without load, and with the main and relay contacts of the regulator closed, close the switches connecting the main control magnets to the direct-current supply (if any such switches are provided), and close, also, the switches connecting the relay contacts to the field rheostat. Turn the rheostat arm gradually to the marked position. The regulator will begin operation as soon as the voltage has been reduced sufficiently to operate the relay, and afterwards will keep the voltage at the value for which the instrument is adjusted.

If one or more generators are in operation with Tirrill regulators, another may be connected in parallel as follows: Equalize the voltage of the incoming machine to that of the units in operation, close the switches connecting the regulator relays to the field rheostat of the incoming generator and close all other regulator switches pertaining to this unit. Close the main armature switches. Equalize the load by means of the equalizing rheostat in the generator that has the most sensitive regulation.

35. In order to take out of service one of several generators in parallel, reduce its load, open its armature switches, and then open the regulator switches corresponding to this machine.

To take the regulator out of service, when one generator is in operation, turn the arm of the generator field rheostat toward the strong field position. When the relays stop vibrating, open the switch that connects them to the field rheostat. Voltage regulation may then be carried on by means of the generator field rheostat.

In order to avoid unequal burning of the main and relay contacts of the regulator, the current through them should be reversed about every 6 hours of operation. This is done by means of the small reversing switches on the bottom of the regulator panel. If the contacts burn unevenly or develop tips, they should be evened with a strip of very fine emery paper or cloth. As the contacts are made of precious metal, no more material than necessary should be removed.

36. Large direct-current generators to be regulated with Tirrill regulators must be excited by a special exciting generator, a particular form of regulator being used. The relays shunt the field rheostat of the exciter generator instead of the main generator rheostat. The regulator has a potential coil supplied from the main generator, load busses, or back feed from the feeding center, as preferred. If a switch is installed in circuit with this potential coil, it must be closed before starting the regulator. The procedure is as follows: With the generator in service at normal voltage and the regulator main and relay contacts closed, close the switches connecting the potential coil to the busses or generator or back feed from distributing center, as the case may be, and close the switches that connect the relays to the exciter field rheostat. Gradually turn the exciter field rheostat arm to the position that would, under normal conditions, cause the exciter to generate a voltage about 35 per cent. below normal. The regulator will start operating as soon as the generator voltage drops enough to start the relays. Set the main generator field rheostat in a position for a strong field, if this setting can be obtained without disturbing the operation of the regulator.

ROUTINE CARE AND INSPECTION

37. During operation, direct-current generators should be frequently inspected to see that they are working under proper conditions. Bearings should be examined at least every $\frac{1}{2}$ hour to see that a sufficient supply of oil is passing through them to avoid overheating. The commutator should be kept clean, and care should be taken to see that brushes do not get hot.

A hot brush on the commutator indicates a poor contact and relatively high resistance. Frequently, the fault is on another brush, which, unable to carry its proper proportion of the load, overloads the brush in parallel with it, though the latter has good contact and relatively low resistance. Not only the brushes that are overheating should be inspected, but the condition of the others, also, should be observed. If only one brush on a stud is heating, the trouble is probably confined to that one.

38. The operator should, as far as possible, observe the temperatures of the generators, at least to the extent of feeling the field coils and placing his hand in the air discharge from the armature. He should always be on the alert for any odor of burning insulation or for any unusual noise. Any of these conditions requires his immediate and continuous attention until the cause and its seriousness are determined and the fault remedied. A generator that is apparently burning out or smoking from overload should be relieved and shut down as quickly as possible. Whether it shall be disconnected before another is started to take its load, or continued in operation with the risk of further damage until relieved, depends on the character of the service and the extent to which the trouble has already progressed. These matters are for the operator to decide and a specific rule of general application cannot be given. A generator that breaks down while operating in parallel with other sources of current may short-circuit the latter and cause serious damage; therefore, it should be separated from other units as quickly as possible.

CARE AND MAINTENANCE

GENERAL INSTRUCTIONS

INSTALLATION

39. Large machines, too heavy to handle complete, are shipped in sections, which are assembled at their destination, usually by the erecting department of the manufacturer, and always according to the manufacturer's instructions. The following general remarks on installation apply more particularly to machines of small or moderate size shipped completely assembled.

During unpacking and handling, machines or parts should be carefully protected from severe shocks or blows. The eye-bolts, or bails, when provided, should be used for lifting the machines, if possible. Great care should be exercised in handling castings during extremely cold weather, as they are then easily broken.

40. The location of an electrical machine cannot always be selected with a view to the best results in operation. The ideal location is clean, dry, well ventilated, easily accessible, and in plain sight. A machine should not be placed where it will be exposed to moisture; provision should be made for protection against drippings from steam or water pipes, or from the roof or wall of a tunnel or a mine. The atmosphere surrounding a machine should not be so warm that the maximum temperature of the machine, while operating, will exceed the limit for normal running set by the manufacturers. Freedom of the atmosphere from acid fumes is essential.

41. Solid masonry foundations are best for electrical machines, but timber is often used for supporting machines of the smaller sizes. Masonry foundations naturally are not used for installations of electrical machinery on board of ships. In any case, the foundation should be solid enough to prevent vibration. If it is desirable to insulate a machine frame from the ground, the slide rails or bedplate must be bolted to dry timbers, which are, in turn, firmly fastened to the masonry foundation. The timbers should be treated with some kind of moisture-repelling compound, and they should be countersunk where they receive the bolts that hold the machine bed. The heads of these bolts should be covered with insulating compound. The wood or iron uprights or stringers for mounting motors on ceilings or walls should be heavy and rigidly anchored to prevent vibration.

42. Before a machine is firmly fastened to its foundation, the driving and driven shafts must be lined up. The pulleys of a belt drive must be in line, so that the belt will run true. The gears of a gear drive must mesh properly. The two shafts of a direct drive must be in line before the coupling is made. The alinement of pulleys can sometimes be tested by installing the belt temporarily and turning the pulleys by hand; if the alinement is poor, the belt will tend to run off.

Motors in industrial service are frequently mounted on a wall or a ceiling. If such mounting is intended, the manufacturers should be so informed when the order is placed. Some types of motors are provided with bearing brackets, or housings, that can be turned through an angle of 90° for wall mounting and through 180° for ceiling mounting. Sometimes the motor is fastened to its bedplate or its slide rails, as the case may be, before it is hoisted into position; in other cases, the slide rails are mounted in place before the motor is hoisted to its position on wall or ceiling; the method followed depends on the conditions.

Vertical motors must be mounted so that the shaft does not incline from the vertical by an angle greater than the limit set by the manufacturers. Also, especial care must be taken that the driven shaft is properly alined with the motor shaft. Vertical motors should be very securely bolted to their foundations.

43. The windings of electrical machines must be dry before full voltage is applied. Machines that have been stored in unheated buildings or have been long in transit should have their windings thoroughly dried before they are put into service.

A machine can be conveniently dried by sending through its windings a current large enough to raise the temperature to about 160° F., but not over 185° F. The temperature should be raised gradually through several hours, the time depending on the size of the machine, and should be kept as nearly uniform as possible throughout the windings. At no time during the drying process should the temperature of the windings be allowed to drop to that of the surrounding air, because the moisture would then condense on the coils. A generator can be dried by its own current, the armature circuit being short-circuited beyond the ammeters and the field excitation being reduced. Field-exciting current from a separate source is preferable. Both the field and armature voltage should at first be kept low, because damp insulation is easily broken down. Some means must be provided for controlling the current, and the temperature of the machine should be watched closely, in order to prevent the interior of any winding from becoming too hot.

Small motors can be dried in ovens. If large machines must be dried by external heat, sheet-iron compartments can be built around them and the interiors heated by charcoal stoves or steam pipes; the latter must be free from leaks. The drying of electrical machinery by any process is very slow. It may take several days to dry a large machine.

BEARINGS

44. When first starting a machine, the bearings should be watched carefully to see that they are receiving proper lubrication. If possible, a new motor should be run for 1 or 2 hours on light load to make sure that the bearings will work without undue heating.

Most bearing troubles are due to insufficient lubrication, caused by poor lubricant, lack of lubricant, dirty lubricant, failure of oil rings to revolve, clogged oil grooves, or poor grade of waste in waste-packed bearings. Oil that is too thin will not stick to the rings or chains of self-oiling bearings in sufficient quantity to be drawn up onto the shaft, and the bearing will heat due to lack of oil. In large bearings, where the distance that the oil must be lifted by ring or chain is considerable, the oil must be thicker than that used for small bearings. Sometimes oil rings, especially those of small bearings, do not revolve properly; this fault, though rare, should be remedied at once.

45. The first symptom of poor lubrication is heating of the bearing. A motor or generator bearing is usually safe if it operates at a constant temperature below the boiling point of water, but a rapid rise of temperature toward this limit is indicative of danger. An overheated bearing will be hot to the touch and will give off the odor, and perhaps the smoke, of burning oil.

Other causes of heated bearings are poor bearing surfaces, which may be caused by careless handling or by dirt in the lubricant, journal cap too tight, bent shaft, unbalanced rotating parts, bearing out of line, or excessive belt tension.

When an overheated bearing is observed, an abundant supply of fresh clean lubricant should be supplied to it. If this treatment does not afford relief in a reasonable time, part or all of the load should be removed from the machine, which should be kept rotating slowly until the bearing cools; otherwise, the bearing will freeze, or set. When the bearing cools sufficiently, the machine can be stopped and a search made for the trouble. An inexperienced person should not attempt to repair bearings.

TROUBLES OF DIRECT-CURRENT MACHINES

POOR COMMUTATION

46. Under unsatisfactory performance of direct-current machines may be included poor commutation, excessive heating of electrical parts, and noisy operation.

Poor commutation may result from overload, improper position (lag or lead) of brushes, hard or high-resistance brushes, rough commutator bars, high mica on the commutator, grounds, short circuits, or open circuits in the armature, high-resistance connections between the armature coils and the commutator, poor brush contact, inaccurate brush spacing, uneven air gap, or weak magnetic field.

47. The symptom of overloading is general overheating of the armature and commutator accompanied by sparking, which can be reduced, but not stopped, by properly advancing the brush position. Overloading may also be recognized by comparing the ammeter reading with the name-plate rating of the machine.

48. For satisfactory performance, the brushes of a direct-current machine must be at the neutral points of commutation. On most standard machines without commutating, or regulating, poles, the no-load neutral points are in line with the centers of the main poles, but the points change positions as the load varies. On generators, especially those that have large armature reaction, the brushes must be set forwards in the direction of rotation as the load increases; on motors, the brushes are set backwards. The amount of change must be determined by experiment for each machine. If the load on the machine is of a more or less fluctuating nature and attention cannot be given to the brush position, the brushes should be left in the position that gives the best average commutation for all loads at normal voltage.

49. Brushes that are too hard or have too high a specific resistance may be unable to carry the current at the contact surface. Some generators require brushes that are harder and of higher resistance than can be used satisfactorily with others. In such cases, some experimenting is usually necessary to determine the proper kind of brushes to use. Brushes should not be changed, however, until it is certain that the cause of faulty commutation is not elsewhere.

50. Eccentricity of the commutator, high bars, low bars, flats, and high mica are all defects that necessitate resurfacing. Eccentricity may be detected by a regular rise and fall of the brushes in their holders, the frequency corresponding to the speed. The commutator should be put into a lathe and turned concentric to the shaft center, or, if preferable, the lathe tool may be carried on a portable holder mounted on the generator frame and the commutator turned in place.

High bars, if not too high, may sometimes be cut down by using carborundum. Sandpaper or carborundum paper is best for cutting down projecting mica. Emery paper or emery cloth should not be used for this purpose. Low bars and flats are best treated by turning down the commutator with a tool.

51. Short circuits, grounds, reversed coils, open circuits, and high-resistance contacts in armatures cause firing, or sparking. If the commutator shows burning or blackening on certain bars and inspection shows no high or low bars or flats, some one of the foregoing troubles should be looked for. Short-circuited armature coils will be indicated by their excessive overheating. Open-circuited coils will not overheat, but will cause severe firing at the commutator bars, as will also high resistance in a coil or at the connection between the coil and the commutator. The commutator bar will usually be found to be burned more on one edge than on the other. The most probable place for the high resistance to occur is at the end joints or in the commutator connection.

52. If the curvature of the brush does not properly fit that of the commutator, the effective brush surface is reduced, causing on the active surface a high current density accompanied by overheating and firing. This condition may be found by inspection. The brushes should be resurfaced with a strip of sandpaper drawn back and forth under the brush while in place. In sandpapering brushes, care must be taken to hold the smooth side of the sandpaper down on the surface of the commutator on both sides of the brush. If this is not done, one or both edges of the brush will be rounded off, and it will have contact only at the middle.

53. If the brush-holder studs are not properly spaced around the commutator, when the brushes on one stud are set for a non-sparking position those on some other stud will spark because they are not in the neutral commutating plane. The remedy is to set the brushes in the positions, found by experiment, where the largest number of rows are not sparking and to mark the places. This having been done, the brushes are shifted forwards or back, as may be required, to positions where no sparking is obtained on other rows, thus experimenting to find how much it is necessary to move each row forwards or back to secure sparkless commutation. The generator should then be shut down and the brush-holder studs respaced accordingly.

Another method in considerable use for checking the spacing of brushes is to stop the machine and wrap a strip of paper close around the commutator under the brushes and to mark the paper at the front edge of a brush in each stud. If these marks are not equidistant the brush-holder studs should be respaced.

54. Unequal air gaps between pole faces and armature will cause poor commutation, producing an effect similar to unequal brush spacing. The air gaps can be measured by a long tapering wedge, which is inserted into the gaps in a direction parallel to the armature slots. If the air gaps are found to be unequal, they should be corrected by recentering the armature among the field poles. On some machines, the bearing housings are specially arranged for centering the armature; on others, the bearings must be relined.

55. As a cause of poor commutation a weak field is usually quickly recognized, as it also results in low voltage in a generator and high speed in a motor. If, however, the sparking is confined to one or two rows of brushes, the trouble may be caused by a local weak field due to a high reluctance in the magnetic circuit caused by the pole piece not being tightly secured to the field yoke.

EXCESSIVE HEATING

56. Excessive heating, another form of poor performance of electrical machinery, may result from errors in design or construction, defective condition of the apparatus, or unfavorable conditions of operation.

57. Electrical machinery is usually subjected to severe test by the manufacturer so that it is very infrequent that defective apparatus is sold. Errors in design or construction include use of the wrong kind of steel for magnetic cores, copper conductors too small or with too low a specific resistance, and faulty workmanship.

58. Excessive heating of armatures may be caused by poor insulation resulting either from dampness or from carbonization of the insulating fabric by previous overheating due to overloads. A test for low insulation resistance may be made with a high resistance voltmeter. There will usually be little difficulty in determining whether or not low insulation results from dampness, as in this case the armature will steam slightly after being shut down. If due to carbonization of the insulating fabric, the insulation will be brittle and weak instead of tough and strong.

Excessive temperature rise in the core of an armature may be a result of severe overheating due to an overload at some previous time. In such cases, the remedy is to rebuild or to replace the core. If large eddy currents are generated in the core, they may be due to the destruction of the coating of insulating japan on the laminations. If this is the case, it will be sufficient to disassemble the core and rejapan the laminations, after which they may be restacked and the winding replaced. This is a very expensive process, and can usually be done only by the manufacturers, or by repair men equipped for such work. Short-circuited armature coils may cause severe local heating which, if not stopped in time, will cause the coils to be burned out.

59. The most common unfavorable condition causing overheating is overload. If it is not possible to reduce the load, every effort should be made to improve the surrounding conditions and to remove as much of the heat as possible by better ventilation. Doors and windows should be open to establish a good circulation of air, and if a fan is available, even of the ordinary desk variety, cool air should be blown on the overheated part.

Unfavorable surrounding conditions include closed doors or windows, close proximity of other heavily loaded machines, uncovered steam pipes, radiation from steam engines, and poor circulation of air. The operator should be on the alert for conditions of this nature and do whatever is possible to remove them.

NOISY OPERATION

60. Noisy operation may be caused by features of design and manufacture, such as high magnetic densities, form and number of armature teeth, etc., or it may be due to mechanical defects that require attention. Imperfections of design are beyond remedy by the operator.

Brushes loose in their holders or working on a rough commutator are common sources of noise; they should be readjusted or the commutator resurfaced. Armature rubbing against field poles, rotating fields touching armature cores, and induction-motor rotors rubbing against their stators are also mechanical defects indicated by noise. In most of such cases, the noises are barely noticeable at first, as the rubbing is generally slight in the beginning. As it is important that they be found as quickly as possible, the operator should carefully investigate the cause of every unusual sound.

LOW VOLTAGE OF GENERATOR

61. The service of a generator is unsatisfactory when its voltage is too low. The causes of low voltage are overload, low speed, improper setting of brushes, armature defects, and weak magnetic field. Overloading and wrong position of brushes are recognized by the symptoms given in Arts. **47** and **48**, respectively.

62. Armature defects, such as open-circuited or reversed coils, lower the generator voltage by reducing the effective number of conductors cutting the magnetic field. Such defects are easily recognized by the kind of sparking on the commutator. Sparking caused by improper brush position or overload is continuous; that due to armature defects is either confined to the commutator bars connected to the affected coils or, if continuous, is more severe on those bars.

63. Low speed can be detected by means of a tachometer, or speed counter. If the generator is driven by a belt, the speed of the driving pulley and also that of the generator should be taken, in order to determine whether or not there is any slip of the belt. In most cases, belt slip can be recognized by a sharp screeching sound, which may be continuous or intermittent. In some cases, belt slipping is stopped by so moving the generator as to tighten the belt, but as this increases the tendency to heat the bearings, it is not always the best procedure. If possible, the belt friction on the pulleys should be increased; and it sometimes occurs that this friction is reduced by oil getting on the working surface.

64. A weak magnetic field may be due to improper position of the rheostat contact-arm, causing too much resistance to be included in the field circuit; and it may be caused by a poor magnetic contact causing high reluctance in the magnetic circuit. In order to test the field circuit, turn the rheostat arm to the position for strong field excitation, that is, so as to cut the rheostat resistance out of circuit, and by means of a portable voltmeter measure the fall of potential across the entire field winding, making connections at the field-terminal lugs on the machine frame. If the potential drop across the field winding is less than the machine voltage indicated by the switchboard voltmeter, test each part of the field leads separately in order to locate the defect. If the potential drop in the winding is equal to the generated voltage, test each coil separately, taking care to include in each case the connection to the adjacent coil. If the fall of potential across any coil is found to be very much higher than it is on the others, the high resistance is between the points over which the large pressure drop occurs. The connection should be carefully examined, as it will be much more probable that the high resistance is in one of the connections between coils than inside the coil itself.

FAILURE OF MOTOR TO OPERATE

65. The mechanical troubles that may cause a motor to stop or fail to start are severe overload, bent shaft, tight bearings, contact between armature and field poles, brush holders jammed, or excessive friction due to any cause.

A motor of small or moderate size that can be disconnected from its load can be examined for mechanical trouble by turning it by hand. If it turns harder than it should, mechanical trouble is indicated. If resistance to turning is greater at one point than at others, it may be caused by a bent shaft or rubbing contact between armature and field or between rotor and stator. If the resistance to turning is uniform, or if the rotating parts cannot be turned at all, the bearings may be tight owing to the bearing cap having been screwed down too firmly or to seizing, or freezing, caused by overheating of the Babbitt, due to lack of oil.

Severe overload may cause the fuse in the supply circuit to blow and thus shut down the motor, or, if the load is heavy enough, may prevent the motor from starting. The remedy is, of course, to reduce the load to within proper limits.

66. The more common electrical troubles that cause failure of operation of a motor are open circuit in the supply, open field circuit, and wrong connections.

An open circuit in the supply may be caused by a melted fuse, or by a broken wire or connection; the brushes may not be in contact with the commutator; or the current may be shut off at the generating station or at a break in the line.

67. If the field circuit is open, the motor will not start; and if very much of the starting resistance is cut out of circuit, the motor fuse will blow. If from any cause the field circuit is opened while the motor is in operation, the armature speed will increase and the current will become very great; the armature will probably be destroyed if the fuse does not blow. If the motor is under heavy load, the mechanical drag will prevent a rapid acceleration, a heavy current will result, and the fuse will blow or the circuit-breaker will open. A break in the field circuit is indicated by the failure of any pole piece to attract a small piece of soft iron when the field terminals are connected to the source of supply.

68. A break in a field circuit will usually be found, by inspection, to be in one of the leads or connections. If inspection fails to discover the break, the field coils should be tested with a voltmeter, as follows: Connect the field terminals to the source of supply, leaving the armature circuit open. Connect one terminal of the voltmeter to one of the field terminals, making sure that the polarity is correct. To the other terminal of the instrument connect an insulated wire bared for a short distance from the free end, so as to make a contact point. Touch this point successively to the junctions between the field coils. When the voltmeter shows a deflection, the last coil included by the voltmeter connections is the defective one. The voltmeter must have a range at least equal to the full voltage of the supply.

An open-circuited field coil can also be located by the following method: Connect the field terminals to the source of supply, leaving the armature circuit open, as before. Short-circuit each field coil in succession, each time testing the field poles for magnetism with a small piece of soft iron. When the defective coil has been short-circuited, the field poles will strongly attract the iron.

69. The directions given in Arts. **67** and **68** have reference to a shunt-field circuit; a break in the conductors or connections of a series-field winding is equivalent to a break in the supply circuit.

70. Wrong connections on motors are uncommon, but among those to be looked for are: Motor armature in series with retaining magnetic coil of starting box; shunt field in series with armature; shunt field connected on wrong side of starting resistance; wrong terminal of the motor connected to the starting box; and part of the field coils reversed.

If the armature is in series with the coil of the retaining magnet, an attempt to start the motor will result in burning out the coil. This error will result from interchanging the field and armature leads at the connection to the starting box.

If the shunt field of a motor is connected in series with the armature, the current from the latter will be so limited as to prevent the motor from reaching any considerable speed.

If the shunt field is connected on the wrong side of the starting resistance, the motor will be weakly excited when starting current is applied and may start badly or not at all. If it starts, the field will be strengthened as starting resistance is cut out and the motor will run slower than normal.

If the armature connection of the starting box is made to the motor terminal to which the shunt field is connected, the fields will be practically unexcited, and the motor may not start, or, if it does, will not speed up, but the fuse will blow or the circuit-breaker will open.

If a part of the field coils are reversed, the error may be found by exciting the fields and using a pocket compass, holding it near the pole pieces in such position as to indicate the polarity of the magnets. If a pocket compass is not at hand, a piece of soft iron bar or a nail may be held so as to bridge across between two adjacent pole pieces. If the bar of iron or steel is held firmly to both pole pieces the poles are of opposite polarity; if it is repelled from one they are alike.

FAILURE OF GENERATOR TO OPERATE

71. A direct-current generator may fail to generate because of wrong connections, open field circuit, too weak residual magnetism, severe overload, reversed polarity, or poor brush contact.

Wrong connections include such errors as connecting shunt field in series with armature, or reversing part of the field coils. An open circuit in the field circuit will result in no excitation and consequently no generation.

A shunt generator will not build up its voltage if started under a heavy load. It builds up its voltage best on open circuit. Also, in operation, it drops its voltage if a certain critical load is exceeded.

If a generator fails to excite itself, the operator should examine all connections, try a temporarily increased pressure on the brushes, examine the field rheostat for a burn-out or a broken coil, test the field coils for open circuit, and check up the position of the brushes. If nothing wrong is found with the connections or the windings, it may be necessary to excite the field from an outside source of energy in order to restore the residual magnetism.

72. If the residual magnetism is too weak or is completely destroyed, the poles will not attract pieces of soft iron held in such position as to bridge across two adjacent poles.

If the machine is compound wound and another machine is in operation, close the equalizer connections of both generators and close to the bus the armature switch of the same polarity as the equalizer connection of the unexcited generator. A part of the output of the running machine will then pass through the series-winding of the unit that will not generate. If the machine is then run at full speed, the generator will pick up.

If the machine is shunt wound and current from another machine or a storage battery is available, pass current from the running machine in proper direction through the fields but not through the armature. The procedure is as follows: Open the armature circuit by disconnecting one terminal of the armature leads between the shunt field tap and the armature; close the main armature switches to the busses to which the running machine or the storage battery is connected; first strengthen and then weaken the field excitation by means of the field rheostat; open the armature switches, taking care to open the first one slowly, drawing out a long arc, to avoid a high inductive electromotive force in the field winding; reconnect the armature terminal and start in the usual way.

73. If no other generator or storage battery is available, disconnect one terminal of the shunt-field circuit and connect it to one terminal of a few primary cells in series. Connect the other terminal of the battery to the remaining shunt-field terminal. Use cells with large current capacity and take care to connect the positive terminal of the battery to the positive end of the field circuit. Connect a portable voltmeter between the disconnected terminal of the shunt-field winding and the binding post from which it was disconnected. The instrument will then indicate the voltage of the batteries while exciting the fields. Cut out of circuit all resistance in the field rheostat, and start the machine. When the voltage of the generator is equal to that of the batteries, which will be indicated by the voltmeter reading being reduced to zero, reconnect the shunt-field terminal and immediately disconnect the battery. If the voltmeter reading cannot be reduced to zero, it is an indication that more exciting current is required; either more cells should be connected in series to secure higher potential or additional cells should be connected in parallel with those already in circuit, in order to obtain a larger current capacity. High-voltage machines, 500 volts, require more cells than do those of lower potential.

74. Reversed polarity of a generator will be indicated by reversed readings of the main voltmeter. It may be corrected by either of the first two methods (Art. 72) given for exciting a machine that has lost its residual magnetism. If no other generator or storage battery is available, the third method (Art. 73) may be used, but in this case about twice as many cells will be needed as for mere excitation.

DANGERS OF PHYSICAL INJURY

DANGERS FROM MECHANICAL SOURCES

75. The operator should be especially careful to avoid contact with moving belts, gears, rotating armatures, or fields, flywheels, etc. When working in the neighborhood of rotating machinery, he should avoid proximity to live circuits, or to static discharge from belts, either of which may cause an involuntary movement that may bring about a dangerous contact with the belt or with gears or other rotating parts.

Bursting of rotating parts may cause pieces to be thrown about with great force. Such bursting may, in some cases, be caused by an error in operation, as, for example, the opening of the field circuit of a direct-current motor while the armature is still connected. Under this condition the acceleration is generally very rapid, and only a few seconds may be required for the armatures to reach dangerous speeds.

DANGER FROM ELECTRICAL SOURCES

76. The extent of danger due to electric causes depends somewhat on the character of the apparatus, being greater when the voltage is high and when the safeguards are inadequate. It is not practicable to name any particular voltage as a safety limit, as a voltage that might be harmless to one individual might be fatal to another. Again, a voltage that would be harmless if applied across the hand, might cause instant death if so applied that the current would pass through the heart or the brain. In addition to the danger of injury due to electric shock is that due to electric burns, of which there are two kinds, flesh and contact burns. For these reasons two safe rules are: Never allow any part of the body to become a part of an electric circuit; and never expose any part of the body to even close proximity to an electrical arc.

77. In order to avoid accidentally causing a part of the body to become a part of a circuit, the operator should be careful, when obliged to touch live parts, to stand in such a manner that he touches them with only one part of the body. If any portion of the machine circuit is grounded, the ground is also a live part, and contact with it when handling the machine is to be avoided. Thus is indicated the advisability of using an insulated platform, stool, rubber mat, rubber shoes, or rubber gloves.

A dry wooden floor is ordinarily good insulation for potentials up to 250 volts; dry white pine is better than the hardwood. A $\frac{1}{4}$-inch to $\frac{1}{2}$-inch pure-rubber mat, if clean and dry, is good protection up to about 1,000 volts. For pressures exceeding

1,000 volts, nothing less safe than a stool with good glass or porcelain insulators should be used, and the safer practice is not to touch live parts carrying such voltages.

78. In order to avoid the possibility of current through some part of the body, it is advisable, whenever live circuits must be handled, to do as much as possible of the work with one hand. The other hand should be kept on the side of the body away from live parts, and care should be taken that the knees do not touch the machine frame while work is being done with the hands.

No work on high-tension equipment should be done without permission from the person in charge of such matters, as it is nearly always necessary to disconnect the equipment from the system. The cleaning and inspection of high-tension equipment is usually done at regular intervals under the supervision of the chief operator, who takes the necessary precautions to render the work safe.

79. Electric burns may result from blowing fuses, accidental short circuits, breaking heavy currents with air-break switches, or causing current to pass through a part of the body. In breaking heavy currents with air-break switches, if the voltage tending to maintain the arc is high, the hands and face should be protected against the effects of the arc. Where there is a back feed, as in the case of a parallel system, this condition is not likely to be serious.

The operator should always remember that the source of danger is invisible, that it must be guarded against by continual vigilance, and that, as he becomes proficient in the operation of electrical machinery, there is danger of a growing tendency toward carelessness due to familiarity.

TESTS FOR FAULTS IN ELECTRICAL CIRCUITS

TESTING DEVICES

80. A **magneto-bell,** Fig. 5, is a small, hand-power, alternating-current generator connected to a bell similar to the call bell on a telephone. The armature is driven at high speed by means of gears *g* and *p*. The test leads are attached to terminals *t* and *t′*.

A magneto-bell is used by applying the test leads to the circuit to be tested and turning the crank of the generator. If the circuit is complete and within the resistance through which the magneto is designed to ring, the bell will ring. The failure of the bell to ring indicates that the circuit between the magneto terminals is open, or of a resistance too great to pass enough current to ring the bell. If the generator turns very hard when the bell rings, the indication is that the resistance of the circuit is low.

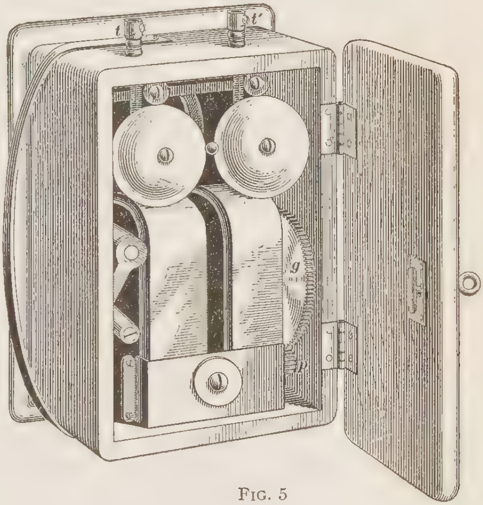


FIG. 5

A magneto-bell should not be used for an insulation test between the frame and armature of a large machine, between coil and core of a large transformer, between the sheath and the conductors of a long cable, or on any other circuit of considerable electrostatic capacity. Such a circuit may act as a condenser, receiving current enough to cause the bell to ring, and thus indicate a complete conducting circuit between the terminals of the magneto, even though the insulation is practically perfect.

81. A **test-lamp outfit** consists simply of a long extension cord terminating at one end in an attachment plug or other device for connecting the outfit to a source of current supply and at the other end to an incandescent-lamp socket or a series of sockets. One of the conductors of the extension cord is cut and the ends thus made are bared for making electric connection to the circuit to be tested. Enough incandescent lamps must be connected in series to burn properly when supplied with the test voltage available.

In using the outfit, the test circuit is connected to the source of current supply and the two free ends of the test leads are applied to the circuit to be tested. The lighting of the lamp indicates that the circuit is complete. If alternating current is supplied to the test circuit. the same limitations apply to the use of the lamp outfit as to the use of the magneto-bell.

82. A telephone receiver connected in series with one or two primary cells (dry cells, if in good condition, are preferable), serves as a convenient testing outfit. The terminals of the series combination are applied to the circuit to be tested. The telephone receiver is held to the ear and the contact of one of the test terminals with the circuit under test is made and broken repeatedly. If the circuit under test is complete, each make or break of the contact will cause a click in the telephone receiver; if the circuit is open, no click will be heard. The device is subject to the same limitations as the magneto-bell.

83. A voltmeter connected in series with the circuit to be tested and a source of electromotive force can be used for qualitative tests. The range of the voltmeter should be at least great enough to accommodate the total voltage impressed on the circuit. When properly connected, a deflection of the voltmeter needle indicates a complete circuit; no deflection shows an open circuit; a relatively small deflection indicates a high-resistance circuit. A direct-current voltmeter can be used in a wide field of testing work.

84. A detector galvanometer is an inexpensive portable instrument that is designed for testing the condition of circuits. The instrument is connected in series with a battery, and the terminals of the series combination are applied to the circuit to be tested. If the needle remains at rest under these conditions, either no circuit or one of very high resistance exists. A deflection of the needle indicates a circuit, the amplitude of the deflection being, roughly, inversely proportional to the resistance. The galvanometer is not subject to the limitations encountered in the use of the magneto-bell, the telephone receiver, or a voltmeter on alternating current.

CONTINUITY TEST

85. A continuity test is made to determine whether or not there is a break in a conductor. The terminals of the test circuit are applied to the extremities of the conductor and the indications of the testing device are observed. In some cases, it may be necessary, in order to reach the remote end of a conductor under test, to use as part of the test circuit another conductor known to be continuous.

In order to test a transmission line for continuity, the line conductors must be connected together at the distant end. The test terminals are then applied to the station end of the line. If the transmission line is long, only direct current should be used for testing for continuity; in tests on short lines, a magneto-bell is convenient. The term *transmission line*, as used in this connection, includes underground cables.

86. If the line tested is found to be open-circuited, the faulty section is located by applying the continuity test to different parts of the circuit until the break is located. For example, assume that a test on an overhead line shows it to be open-circuited. The tester proceeds along the line, applying the test terminals at intervals. As soon as the tester has passed the break, the testing device will give indications of a closed circuit. The tester then knows that the fault is in the part of the line between the places where the last two tests were made, and he proceeds to locate the break by inspection.

Sometimes the procedure is varied somewhat. Thus, the tester goes first to a point about midway between the ends of the line and applies the test terminals. If the test indicates an open circuit, he knows that the fault is in the half of the line beyond; if the test indicates continuity, the break is in the other half. The tester thus at once eliminates one-half the line as a possible container of the fault. A few more tests on the faulty half of the line will probably locate the break within reasonable limits.

TEST FOR CROSSES AND GROUNDS

87. A test for a cross or ground is made to determine whether or not a conducting path exists between two objects, two circuits, or two parts of circuits that should be insulated from each other. The test terminals are applied to the two objects or circuits, and the existence of the fault will be indicated by the action of the testing device. If the test lamp, when used, burns very dimly, the indication is that the cross or ground is of small current-carrying capacity and moderately high resistance.

In general, a test for a leak should be made with a device, such as a magneto, voltmeter, galvanometer, or telephone receiver, requiring only a small current to give an indication. In testing for a short circuit, a test lamp is generally more satisfactory.

DIFFERENTIAL TEST

88. It is sometimes necessary to locate, by a test at the station, a ground occurring several miles away on a series-arc-lamp circuit. For this purpose, a crude form of Wheatstone bridge is used. A series of incandescent lamps is connected across the terminals of the arc-lamp circuit, as shown, roughly, in Fig. 6. Generally, the arc lamps require about 50 volts each, and incandescent lamps rated at about 50 volts are therefore preferable; 110-volt lamps can be used, however, if 50-volt lamps are not available. In any case, the number of incandescent lamps in series should be equal to the number of arc lamps in circuit.

The electromotive force of the generator is consumed in sending currents through the parallel circuits of arc lamps and test lamps. From the positive terminal of the generator to the ground *a*, Fig. 6, on the arc circuit, there is a drop in potential of approximately 50 volts per lamp, or, in this case, 6×50=300 volts. If the movable arm *b* of the testing device is moved from contact *c* toward contact *d*, a point will be reached where the potential drop in the test-lamp circuit is the same as that between the positive terminal of the arc circuit and the ground. There will then be no current through the detector galvanometer *e* and its needle will rest at the zero position. On further movement of the arm *b*, the galvanometer indication will be reversed. The number of test lamps between the positive terminal of the circuit and the test point at which the deflection of the galvanometer needle is zero is equal to the number of arc lamps between the positive terminal of the circuit and the ground. In this case, the number is six.

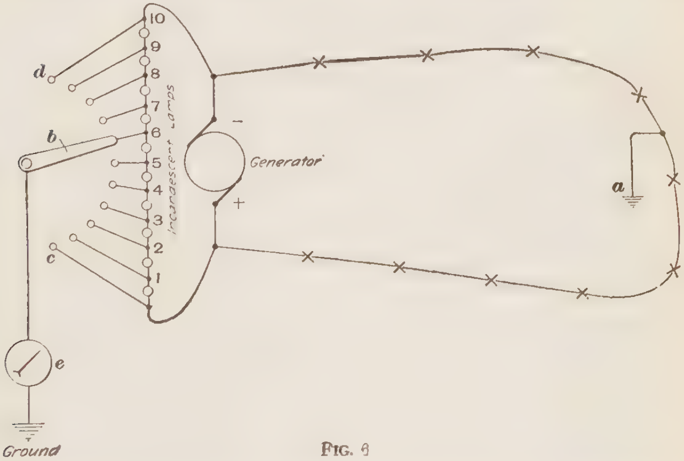


FIG. 6

A voltmeter can be used in place of the galvanometer, but it must be capable of indicating at least half the total voltage of the circuit, and the test must then begin with the arm *b* on the middle contact so that only half the voltage can be applied to the voltmeter. A double-throw voltmeter is preferable; if a single-throw instrument is used, care should be taken not to connect it so as to cause reversed deflection.

TESTS FOR DEFECTS IN DIRECT-CURRENT ARMATURES

89. Faults in direct-current armatures are conveniently located by what is known as the *bar-to-bar test*. Suitable contacts *A* and *B*, Fig. 7, are clamped to opposite sides of the commutator. Current from the mains, or bus-bars, *E* is led to the commutator through a lamp bank *LB*. A movable contact piece, or *crab*, *C* provided with two spring contacts so spaced as to rest on adjacent bars, is connected to a galvanometer or a low-reading direct-current voltmeter *G*.

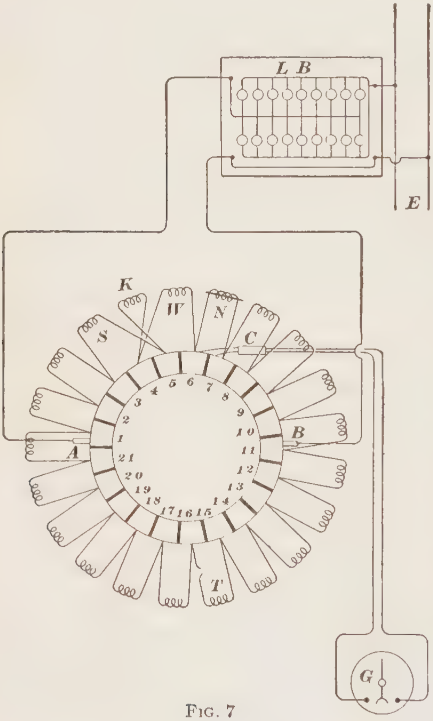


Fig. 7

For the sake of illustration, it is assumed that in coil *N* there is a short circuit, that the commutator leads of coils *S*, *K*, and *W* have been mixed, as shown, and that there is an open circuit in coil *T*. The test is carried out as follows: Adjust the lamp bank until the galvanometer gives an easily readable deflection when the crab *C* is in contact with bars connected with what are supposed to be good coils. This serves as a standard deflection, with which to compare other deflections; all but defective coils will give this standard deflection.

When the test contacts rest on bars *3* and *4*, the deflection will be much larger, about double, than the standard deflection, because two coils, instead of one, are connected between these bars. When the contacts rest on bars *4* and *5*, the deflection will be reversed, because the leads are crossed, but will not be greater in amplitude than the standard. Between bars *5* and *6* a large deflection will be obtained, for the same reason that a large deflection is obtained between bars *3* and *4*. Between bars *6* and *7* little or no deflection will be obtained, owing to the short circuit in coil *N*.

As the contact piece is moved around on the lower side of the commutator, no deflection will be obtained until bars *15* and *16* are bridged. There will then be a violent throw of the galvanometer needle, carrying it beyond the scale. When the contact piece is moved on to bars *16* and *17*, there will again be no deflection, thus locating the break in coil *T*. In order that the other coils connected to the bars in the lower half of the commutator can be tested, bars *15* and *16* must be connected together by a piece of wire.

If any of the coils has poor connections with the commutator bars, the effect will be the same as though the coil had a higher resistance than normal, and the galvanometer deflection will therefore be greater than the standard deflection.

RESISTANCE MEASUREMENTS

THE GALVANOMETER

ELEMENTARY TYPE

1. Description.—Measurements of resistance are usually made with the aid of instruments that, by the motion of their moving parts, indicate flow of electricity. An instrument that indicates the presence of an extremely small current is the **galvanometer,** a simple form of which is shown in Fig. 1. Inside a vertical coil *a* of wire a magnet *b* is so pivoted as to move a pointer *c* over a scale. This pointer is usually arranged at right angles to the magnet. The coil terminals are connected with the binding posts *d* on the base.

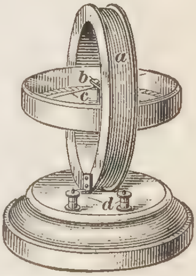


FIG. 1

2. Theory of Action.—With no current in the coil, the magnet of the galvanometer assumes a position pointing north and south, that is, parallel to the earth's magnetic flux. The coil is turned so that its plane is parallel to the direction of the earth's flux. In Fig. 2 (*a*), the coil is shown at *a*; the magnet, at *b*; and the direction of the earth's flux, by the dotted arrows. The plane of the coil, the plane of the magnet, and the direction of the earth's flux are parallel.

When there is current in the coil, the direction of the coil flux in the space near the magnet is as indicated by the dotted arrows in (*b*). If this were the only active flux, the magnet would point at right angles to the plane of the coil.

COPYRIGHTED BY INTERNATIONAL TEXTBOOK COMPANY. ALL RIGHTS RESERVED

The direction of the resultant flux of the earth and the coil fluxes may, in a given case, be as indicated by the dotted arrows in (*c*). The direction, in any case, depends on the relative strengths of the two fluxes. The magnet tends to set itself parallel to the resultant flux.

The deflection of the magnet from its normal position in the earth's magnetic field will therefore depend on the strength of the earth's magnetic flux and on the strength of current in the coil. The greater this current strength the stronger will be the coil flux, the nearer will the resultant flux approach a direction at right angles to the plane of the coil, and the greater will be the deflection of the magnet and pointer.

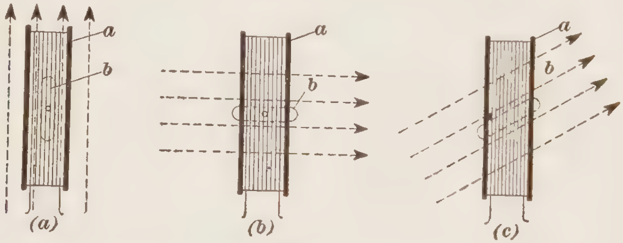


FIG. 2

Reversing the current, reverses the direction of the coil flux, thus deflecting the magnet from its normal, or zero, position in the opposite direction of rotation.

KELVIN GALVANOMETER

3. In the **Kelvin galvanometer,** which is used for the measurement of very small currents, the coil consists of a great number of turns of fine wire, and the moving part of the instrument of a few small pieces of magnetized steel strips suspended by a fine thread. A small mirror also is suspended by this thread.

D'ARSONVAL GALVANOMETER

4. Description.—In the **D'Arsonval galvanometer,** which is extensively used for electrical measurements, the magnetic field in which the moving part turns is provided by means of a permanent magnet. This type of galvanometer can usually be placed without regard to a north or south position, because the earth's magnetic flux has little influence on it.

The moving part consists of a small coil suspended by a thin, flat metal strip. The coil lies between the poles of the permanent magnet and is connected to the circuit on which measurements are to be made.

Fig. 3 shows one type of D'Arsonval galvanometer. In this instrument, the frame forms the permanent magnet and the pole pieces are located at *a* and *b*. Fastened to the back of the instrument case is a stationary iron core *c* that extends inside the movable coil *d* and thus serves to improve the magnetic circuit. A suspension strip *e* supports coil *d* and also carries a small mirror *f*. By means of this strip *e* and the wire *g* the coil *d* is connected to binding posts *h* and *i*.

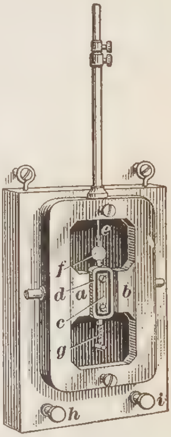


FIG. 3

5. Theory of Action.—In this type of D'Arsonval galvanometer, the suspension strip is so placed that the plane of the coil, in its no-current position, is parallel to the face of the magnet frame. Thus, current in the coil establishes a magnetic flux perpendicular to that of the permanent magnet and tends to deflect the coil until the two fluxes coincide in direction. The resulting movement of the coil twists the flat suspension, and both its resistance to twisting and the weight of the coil acting on it tend to return the coil to the no-current position. These two forces, one tending to turn the coil and the other to prevent it from turning, come to a balance, and the coil comes to rest at such a position that the deflection is proportional to the current in the coil.

Any change in the current causes a corresponding change in deflection. A reversal of the current reverses the direction of the coil flux and the direction of the deflection from zero.

METHODS OF READING DEFLECTIONS

6. A **pointer** and **scale** are often used with a galvanometer of only approximate accuracy to indicate the deflection of the instrument. In many cases such galvanometers simply serve to indicate the presence of a current and its direction. In other cases, the number of divisions on the scale between zero and the final position of the pointer indicates the current value with sufficient accuracy for the purpose intended.

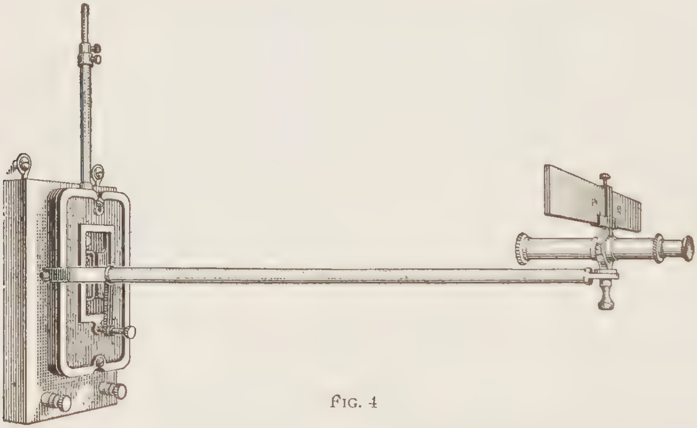
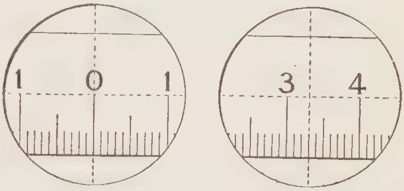


FIG. 4

7. A **telescope** and **scale** are often employed to aid in reading the deflection of the moving system of a galvanometer intended for very accurate measurements. Fig. 4 shows such an arrangement attached to a D'Arsonval galvanometer. On looking through the telescope, the operator sees in the small mirror mounted on the suspension strip of the galvanometer the reflection of a part of the scale. If the mirror turns, a different part of the scale is observed.

8. Before an attempt is made to read a deflection, the telescope and moving element are adjusted with no current in the coil until the vertical cross-hair stretched across the glass at the end of the telescope nearest the galvanometer is in line with the reflection of the zero mark on the scale. In Fig. 5 (*a*), is shown the reflection of the scale in the mirror as viewed through the telescope under conditions of no-current and proper adjustment. The cross-hairs are indicated by the dotted vertical and horizontal lines.

When current is established in the coil, the mirror turns and the markings on the scale appear to move past the vertical cross-hair. With a steady current, the reading is the point on the scale, the reflection of which coincides with the vertical cross-hair when the coil comes to its balanced position. In Fig. 5 (*b*) is shown the reflection of the scale in the mirror as viewed through the telescope when there is a deflection of 33 divisions.



(*a*) (*b*)

FIG. 5

With a momentary current, the general practice is to consider the reading as the point that appears in line with the vertical cross-hair at the extreme end of the throw of the coil. This is the condition when the coil has turned through its maximum angle, and is about to swing back toward its zero position. By this method, a very small turning movement of the coil results in a deflection of several scale divisions. The galvanometer is used to measure very small currents, and therefore, the movable coil is wound with fine wire to obtain the proper coil flux.

AYRTON GALVANOMETER SHUNT

9. Connections.—A large current in the galvanometer causes a deflection that is off the scale and also may destroy the instrument. In order that the current in the galvanometer may be only a small part of the total current in the main circuit that is under test, a *galvanometer shunt* is employed.

The exterior of an **Ayrton shunt** is shown in Fig. 6. It has two line terminals *L*, and two galvanometer terminals *G*.

The connections of this shunt are shown diagrammatically in Fig. 7. The shunt consists of a resistor *0–1*, Fig. 7, all sections of which are connected across the terminals of the galvanometer *a* and some of which can be connected by leads *b c* in series with the circuit from which current is to be taken. The resistor consists of several coils connected between the contact points *1*, *.1*, *.01*, etc. The movable contact *d* is used to adjust the resistance that is included in the main circuit.

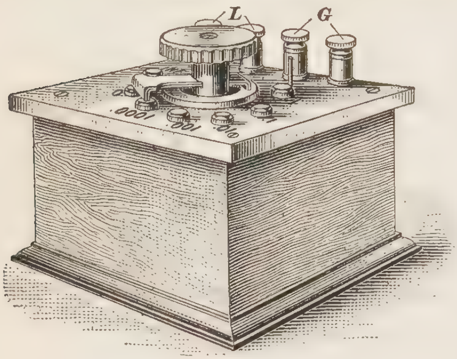


FIG. 6

10. **Theory of Action.**—The main current causes a drop of voltage in the part of the resistor in circuit. For example, with connections as shown in Fig. 7, the resistor sections in the main circuit are those from *0* to *.1*. The current in the galvanometer is equal to the voltage drop between *0* and *.1*, divided by the resistance of the current path *.1–1*–galvanometer–*0*. With this setting of the shunt, the section between *.1* and *1* is not included in the main circuit, but is included in the galvanometer path. If the position of *d* is changed, the relation between the main current and the galvanometer current is altered.

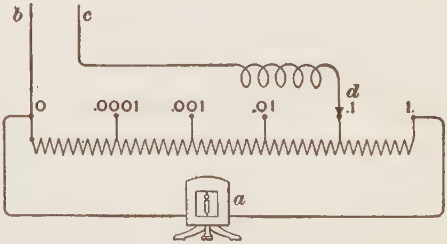


FIG. 7

The marks on the contact points indicate the relative deflections caused by a given current in the main circuit when the movable contact *d* is placed on the various points of the shunt, as compared with the deflection when *d* is on *1*; thus, the deflection with *d* on *.1*, Fig. 7, is one-tenth the deflection obtained with *d* on *1*, or, differently expressed, the deflection when *d* is on *1* is ten times the deflection when *d* is on *.1*, with the same current in the main circuit.

The multiplying powers of the contact points, Figs. 6 and 7, are 1 for point *1*, 10 for point *.1*, 100 for point *.01*, 1,000 for point *.001*, and 10,000 for point *.0001*.

If, when making a test, it is necessary to use one or more points on the shunt to obtain readings on the scale, the deflections should be multiplied by the multiplying power for those points in order that there may be a common base for comparison of current conditions in the main circuit. This base is the equivalent deflections that would have been obtained if it were possible to use point *1* for all tests.

The main circuit is opened by moving the switch arm, Fig. 6, to the point marked *INF*.

Example.—If, when using an Ayrton shunt in connection with a galvanometer, one main current causes a deflection of 50 divisions when the contact *d* is on point *1*, Fig. 7, and another current causes a deflection of 45 divisions with *d* on point *.1*, how do the two currents compare in strength?

Solution.—A deflection of 45 with *d* on *.1* is equivalent to $10\times45=450$ divisions with *d* on *1*. Therefore, the strength of the second current is $\frac{450}{50}$, or 9 times that of the first. Ans.

CONDUCTOR RESISTANCE

WHEATSTONE BRIDGE

11. Connections.—Several methods are available for measuring electrical resistance, one being by means of the **Wheatstone bridge.** Many forms of Wheatstone bridge are in use, but all operate on the general principles explained by means of the diagram shown in Fig. 8.

Known resistances M, N, and P and the unknown resistance X (to be measured) are arranged as shown to form two paths between two points a b of a circuit. A galvanometer is connected between the terminal c of the unknown resistance X in one path and the terminal d of the adjustable resistance P in the other path.

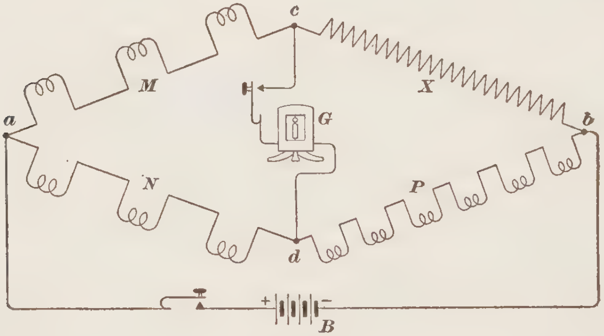


Fig. 8

12. Theory of Action.—The resistances of M, N, and P are adjusted until no deflection of the galvanometer occurs, when the battery key and the galvanometer key are pressed. The bridge is then *balanced.*

No deflection indicates no current in the galvanometer, and there is no current because there is no difference of potential between the terminal points c and d of the galvanometer.

There must be, therefore, the same current I_c in M as in X, and the same current I_d in N as in P, that is, there is no exchange of current from one path to the other through the galvanometer.

The points a and b are common to both paths; therefore, to have no difference of potential between c and d, the drop of potential $M\,I_c$ in M must equal the drop $N\,I_d$ in N, and the drop $X\,I_c$ must equal the drop $P\,I_d$; that is,

$$X\,I_c = P\,I_d$$
$$M\,I_c = N\,I_d$$

By dividing the first equation by the second,

$$\frac{X\,I_c}{M\,I_c} = \frac{P\,I_d}{N\,I_d}, \text{ or } \frac{X}{M} = \frac{P}{N}$$

and

$$X = \frac{M}{N}\,P,$$

which is the fundamental equation of the Wheatstone bridge.

13. Names of Bridge Arms.—The arms M and N, Fig. 8, are called *ratio*, *bridge*, or *balance arms* and they usually have an equal number of resistors of corresponding resistance values. Usually only one resistor in each balance arm is active during a test. The arm P is called the *rheostat arm* and it is provided with a large number of resistors of different resistance values, one or more of which may be active. The resistors are in the form of coils, arranged within a box and connected to terminals on the top of the box for adjusting the resistances of the arms. A switch key in the battery circuit and another in the galvanometer circuit serve to close these circuits when making a test.

14. Description of Dial-Switch Bridge.—In Fig. 9 is shown a Wheatstone bridge set with dial switches for adjusting the resistance in the rheostat arm; the connections of a bridge of this general type are shown in Fig. 10. The resistors in the rheostat arm are in groups, and each group is controlled by means of a dial switch. The coils in each group are connected in series, as are also the groups. Each switch arm serves to connect any number of the coils in its group into the active circuit, the number being indicated by the marking on the contact point on which the arm rests.

Each resistor of a group has the same resistance. The values, .1 ohm, 1 ohm, 10 ohms, 100 ohms, and 1,000 ohms, are marked on the switch arms. The active resistance of each group is found by multiplying the number on the active contact point by the resistance of each active coil of that group. For example, the rheostat arm, Fig. 10, has a resistance of 3,217.7 ohms.

15. The resistance of either of the ratio arms M or N, Fig. 10, is adjusted by placing a metal plug in the hole between the terminal of the resistor selected and either bar M or N. The resistance of each coil is marked on its terminal. Two plugs are shown in place. A 10-ohm resistor is in arm M and a 1,000-ohm resistor in arm N. If, with the resistance adjustment shown for M, N, and P, the bridge is balanced, the value of $X = \frac{M}{N}P = \frac{10}{1,000} \times 3,217.7 = 32.177$ ohms.

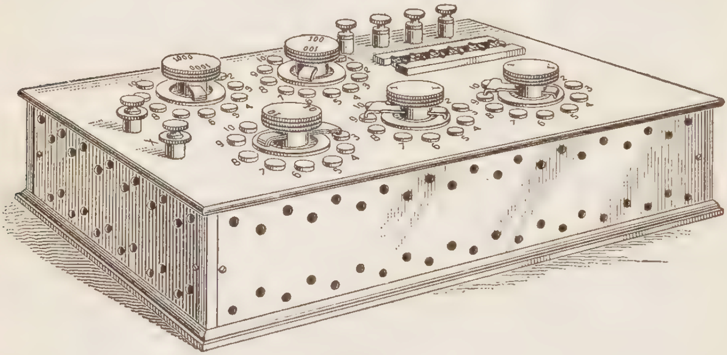


Fig. 9

In Fig. 10, a separate galvanometer and a separate battery are shown. In portable bridges, both of these devices are placed within the case of the bridge set, and a pointer and scale are used to indicate movement of the galvanometer.

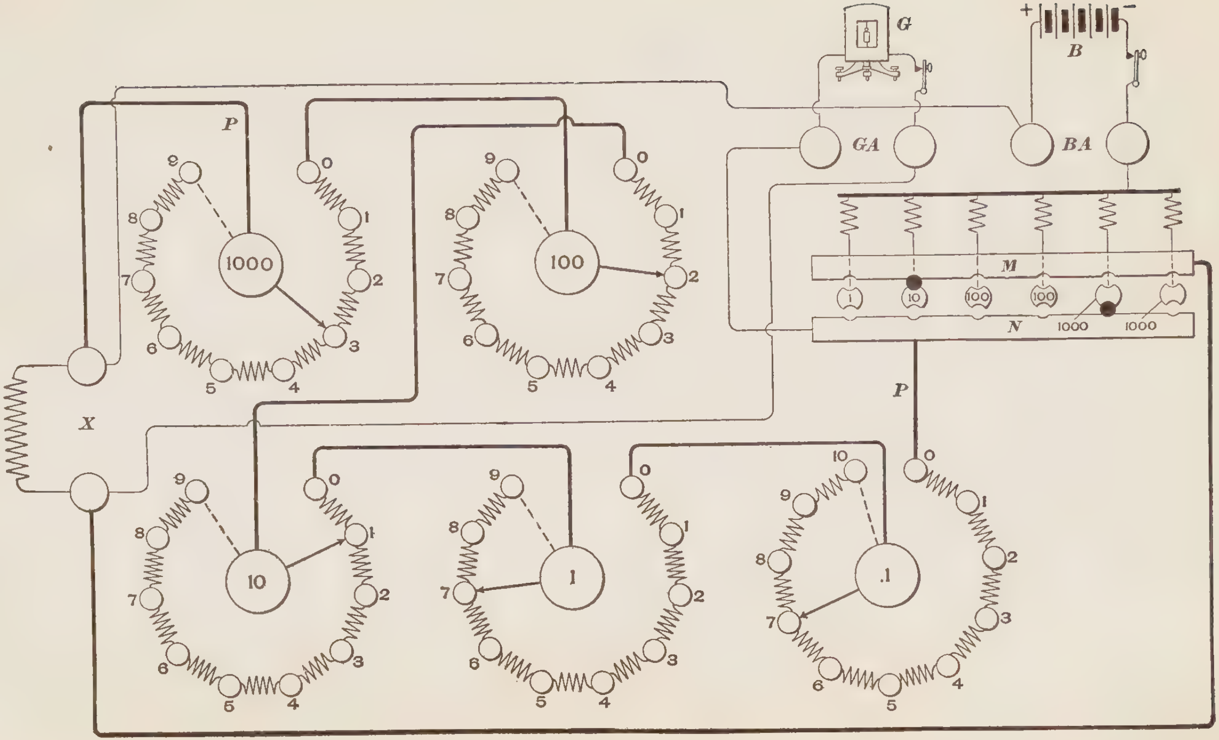

G
+
−
B
GA
BA
M
N
P
P
X
.1

Fig. 10

16. Operation.—The following directions relate to the use of the bridge set shown in Fig. 10, but they apply in general to all Wheatstone bridges:

The wire, coil, or other device to be measured is connected to terminals *X*. If long leads are used to connect the device to the bridge, their resistance should be determined and subtracted from the result of the test on the leads and device. The distant ends of the leads should be connected together when testing the lead resistance.

TABLE I
SETTING THE RATIO ARMS

Unknown Resistance, In Ohms	Resistance, In Ratio Arms		P Equals	X Equals
	M	N		
Below 10	1	1,000	$X \times 1{,}000$	$\frac{P}{1{,}000}$
10 to 100	10	1,000	$X \times 100$	$\frac{P}{100}$
100 to 1,000	100	1,000	$X \times 10$	$\frac{P}{10}$
1,000 to 10,000	1,000	1,000	X	P
10,000 to 100,000	1,000	100	$\frac{X}{10}$	$P \times 10$
100,000 to 1,000,000	1,000	10	$\frac{X}{100}$	$P \times 100$
1,000,000 to 10,000,000	1,000	1	$\frac{X}{1{,}000}$	$P \times 1{,}000$

17. The approximate resistance of the device may be known or it can be determined by a rough test, after which the ratio arms can be adjusted as indicated in Table I.

18. Let it be assumed that the resistance of the device to be measured is known to be approximately 30 ohms. This number lies between 10 and 100; therefore, according to Table I, arm M should have a resistance of 10 ohms, arm N, a resistance of 1,000 ohms, and the rheostat arm P, a resistance of $30 \times 100 = 3{,}000$ ohms, approximately.

In this case the procedure would be as follows: Set the plugs in the ratio arms as shown in Fig. 10; also, set the 1,000 dial switch arm on contact *3* and the other arms on contacts *0*. Next, press the battery key and then the galvanometer key and note to which side of its no-current position the moving element of the galvanometer turns. Release the galvanometer key and then the battery key, and add 500 ohms to the rheostat arm circuit by placing the 100-dial switch arm on contact *5*, making 3,500 ohms in this circuit. Then, note the direction of the deflection of the galvanometer when the keys are pressed, taking care to press the battery key first. If this deflection is opposite in direction to the first deflection, it indicates that 500 ohms was too large an addition.

According to the last column of Table I, the resistance of the device is between $\frac{3{,}000}{100}$ and $\frac{3{,}500}{100}$ ohms, and a comparison of the deflections will indicate which value of resistance is nearer correct.

Adjustments of the dial switches should continue until the deflection is zero, when the keys are pressed. The higher resistance coils should be used for coarse adjustments and the lower resistance coils for fine adjustments. The effect of each change on the deflection indicates whether the change is in the right direction and whether it is too large or too small. A few trials will bring the correct adjustment, and the unknown resistance can then be determined, as indicated in the last column of Table I. If the adjustments are as shown in Fig. 10, the resistance X is $\frac{P}{100} = \frac{3{,}217.7}{100} = 32.17$ ohms.

19. If the resistance of the device is not even approximately known, each ratio arm should be set for 1,000 ohms and the resistance of arm P should be adjusted until two fairly close adjustments that give small deflections in opposite directions are found. Since, with this setting of the ratio arms, $X=P$, the approximate resistance value of X is read directly from the setting of P.

Suppose that with the 1-ohm dial switch on contact *9* (9 ohms) the deflection is one way, and with the 10-ohm switch on point *9* (99 ohms) the deflection is the other way; then, the resistance of X is between 9 ohms and 99 ohms. The ratio arm should then be set according to Table I, and a final test made.

If the even setting of the ratio arms does not produce the approximate value of X, it will be necessary to try the next setting of these arms indicated in Table I, either for higher or lower values of X, as the case demands.

SLIDE-WIRE BRIDGE

20. Connections.—For measuring low resistances, a modified form of the Wheatstone bridge, known as the **slide-wire, or meter, bridge** is often used.

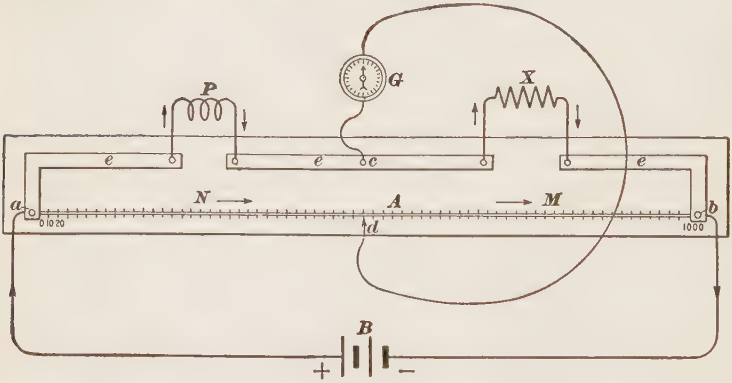


FIG. 11

A diagram of its connections is shown in Fig. 11. The parts are mounted on the board A, and a German-silver wire $a\ b$ of uniform resistance is stretched between large copper blocks, to which the terminals of battery B also are connected. The wire lies over a scale that is divided into 1,000 equal divisions. Point c and slider d form the terminals of the circuit of the galvanometer G.

A switch key (not shown) may be used to close the battery circuit during a test; or, the battery may be connected between *c* and *d* and the galvanometer between *a* and *b*. In the latter case the battery circuit is closed by touching *d* to the slide wire during a test. The object of either method is to limit the time that the battery is active.

The resistance of coil *P* is known, and *X* represents the resistance to be measured. The connecting copper strips *e* have very low resistances. The resistances are connected in the form of a Wheatstone bridge, as is indicated by a comparison of the connections *M*, *N*, *P*, and *X*, Figs. 8 and 11.

21. Names of Arms.—The known resistance *P* forms the *known arm;* the resistance *X* to be measured is the *unknown arm;* and the two arms *a d*, or *N*, and *d b*, or *M*, formed on the wire *a b* by the slider *d*, are the *ratio arms,* which are sometimes called the *proportion arms.*

The value of *P* is usually .1, 1, or about 10 ohms. The resistance coil used in arm *P* should be of about the same resistance as the device to be tested.

22. Operation.—By sliding *d* along the wire *a b*, some point on the wire can be found at which no deflection occurs, thus indicating a balanced condition. As the wire *a b* is of uniform resistance throughout its length, scale divisions can be used instead of resistance values to express the ratio of the arms *M* and *N*. For example, suppose that *P* is 1 ohm and that the balance point of *d* is found to be 400 divisions from *a* and 600 divisions from *b*. Arm *N* is then 400 divisions and arm *M* is 600 divisions. Substituting the known resistance value of *P*, which is 1 ohm, and the known division values of the arms *M* and *N* in the bridge formula $X=\frac{M}{N}\times P$, $X=\frac{600}{400}\times 1$ $=1.5$ ohms.

THE OHMMETER

23. Connections.—The **ohmmeter** is an instrument from whose scale the value, in ohms, of the resistance that is being measured can be read directly. Its principle of operation is similar to that of the slide-wire bridge.

One form of ohmmeter is shown in Fig. 12 (*a*), and its connections are shown diagrammatically in (*b*). By inserting the plug *P* in one of the four terminals marked *Brown*, *Blue*, *Red*, and *Black*, the resistance coil corresponding to that marking is made the known arm. The slide wire is in two lengths from *A* to the metal block *B* and from *B* to *C*. The numbers printed on the scale located below the wire are also in four colors, to agree with the terminals. The slider is shown at *S*, and the terminals for the unknown resistance *X*, at *A* and *D*. Instead of a galvanometer, use is made in this device of a telephone receiver *T*, which is connected between *D* and *S*. A small key switch in the battery circuit is shown at *K*.

The telephone receiver usually consists of two small coils of wire mounted on iron cores, near the front ends of which is placed a thin iron disk. When current passes through the coils, this disk is attracted and a click is heard, provided the receiver is held close to the ear. When no current passes, no sound is made.

The connection of the ohmmeter, as indicated in Fig. 12 (*b*), should be compared with the slide-wire bridge connections shown in Fig. 11. These connections are practically similar.

24. Operation.—To measure an unknown resistance *X*, proceed as follows: First, connect resistance *X* to posts *A* and *D*, Fig. 12. Put plug *P* in the terminal marked *Brown*, place telephone to the ear, press the key *K*, and tap the slider *S* along wire *A B C* until the click of the telephone ceases or is at a minimum. Read the resistance of *X* directly from the brown number under the balance point on wire *A B C*.

If this point is found to be at a place on the scale where close readings are difficult, place the plug in the blue terminal, find the new balance point, and read from the blue scale. If necessary to advance the plug to the red or the black terminal, use the scale markings corresponding in color to that of the active terminal.

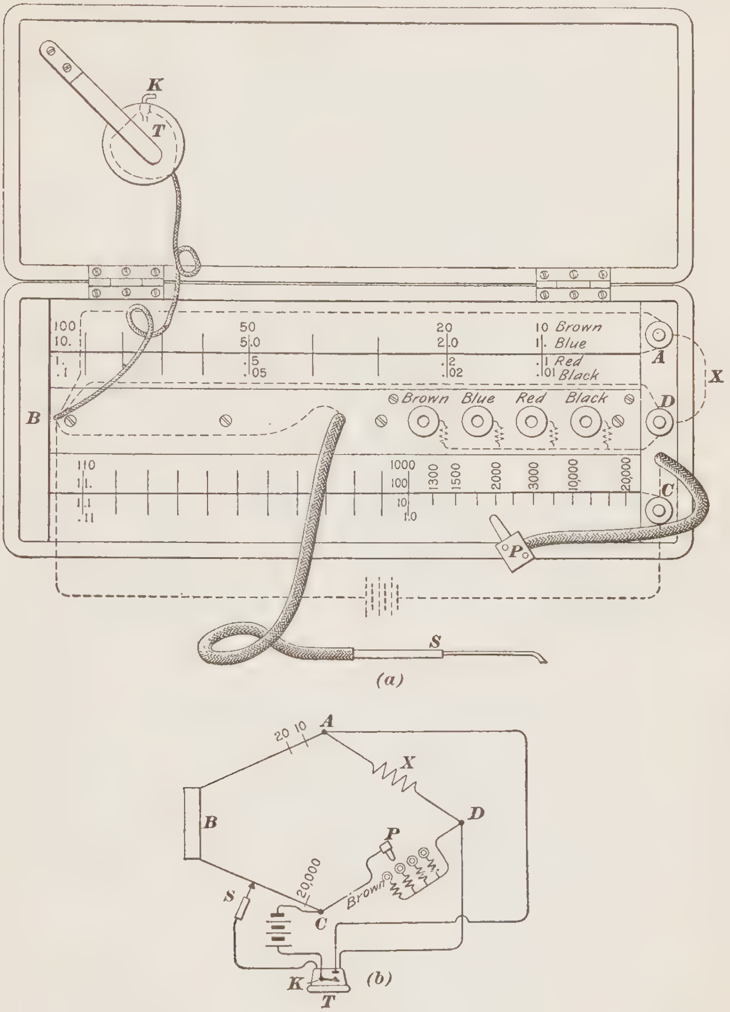


Fig. 12

If there is a short space on the wire *A B C* where no sound is heard, consider that the balance point is at the center of this space.

INSULATION RESISTANCE

GENERAL CONSIDERATIONS

25. Necessity for Insulation Resistance.—In order to confine electricity to intended paths, these paths must be *insulated;* that is, they must be surrounded with materials having a resistance so high that electricity cannot escape through them. Mica, glass, porcelain, rubber, treated cloth, oils, varnish, etc., are some of the materials used; some in the forms of covers for the conductors and others in the form of supports for either bare or insulated conductors.

26. Length and Cross-Sectional Area of Path Through Insulation.—The resistance of the insulation depends on the material, on the length of the leakage path through it, and on the cross-sectional area of this path. The length of the leakage path in the case of a submarine cable is the distance between the conductor and the metal covering, or sheath, placed outside of the insulation, and which is in contact with the water. Since the current can leak from all points on the conductor to the sheath, the cross-sectional area of the leakage path in the cable is equal to the mean circumference of the insulation multiplied by the length of the cable. With a long cable, this area is enormous and explains why, even with good insulating material, an appreciable leakage current may be present.

THE MEGGER

27. The **megger,** Fig. 13, is a device used to indicate directly the insulation resistance of cables, motor and generator windings, line circuits, etc. When the resistance to be measured is connected to the binding posts on the side of the case and the crank is turned, the pointer moves over the scale, shown near the lifted cover to a position that indicates the value of the resistance.

28. Description.—In Fig. 14 is shown the megger with its case removed. A small direct-current generator *a* operated through gearing by means of the crank *b* can be made to produce an electromotive force of from 100 to 1,000 volts, depending on the winding of the armature and on the speed. Four permanent bar magnets, two of which are shown at *c* furnish the necessary magnetic flux for the generator at one end and the flux in which the moving element of the measuring instrument at the other end of the megger turns. A pointer *d* is attached to the moving element.

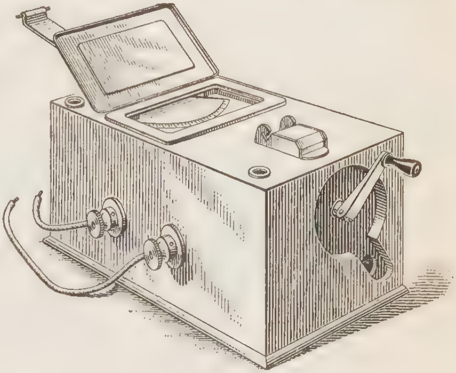


FIG. 13

29. Connections.—In Fig. 15 are shown the internal and external connections for testing the insulation of a cable. The direct-current generator is shown at *a*, and the measuring instrument of the D'Arsonval type at the left. The moving element of the instrument consists of three coils *b*, *c*, and *d* attached to a shaft that moves the pointer. Coils *b* and *c* and a resistor *e* are in series directly across the generator terminals. They carry a current proportional to the generator electromotive force and independent of the condition of the insulation.

The two coils *b* and *c* are rigidly attached to each other in one plane, and this plane is at a small angle from the plane of coil *d*. The outer side of coil *c* projects so far beyond the field of the permanent magnet as to be little affected by it, and the inner side of coil *b* is shielded by the iron core *f*. Coil *b* is wound in the opposite direction from coil *c*, so that if there is a stray magnetic flux from, for instance, a near-by generator, the turning action of one coil that it may set up, is compensated by the opposite turning action of the other coil. The parts of coils *b* and *c* that are active in moving the pointer are the sides between core *f* and the upper pole piece.

Coil *d* is in series with the insulation that is being measured and the resistor *g*, and therefore carries the same current as the insulation.

30. Theory of Action.—Two paths are available for the generator current—one through coils *b* and *c* and the resistor *e* and the other through coil *d*, resistor *g*, and the insulation.

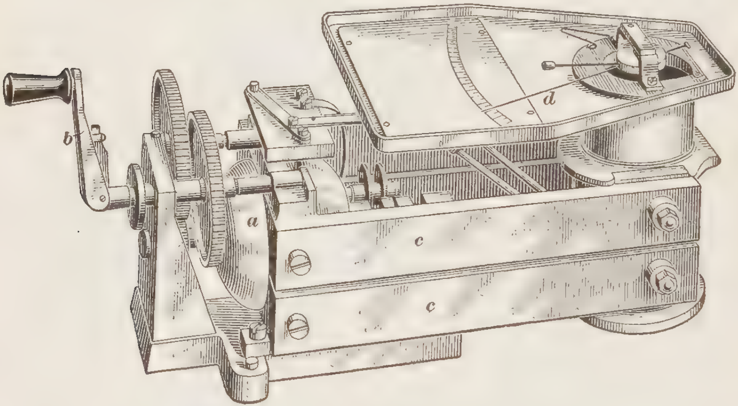


Fig. 14

The current in coils *b* and *c* tends to turn the moving element in a counter-clock wise direction.

With little or no current in coil *d*, the current in coils *b* and *c* will move the pointer to the *INF.* position on the scale. This indicates that the insulation resistance is *infinitely high*.

When the insulation is not perfect and there is current through it and coil *d*, the turning force of this coil is increased and the pointer will take up some intermediate position between *0* and *INF.* At this position of the moving element, the turning force of coils *b* and *c* is balanced by the turning force of coil *d*. The poorer the insulation, the greater the current through coil *d*, and the nearer *0* the deflection becomes. There are no control springs on the moving element, the final position depending on the balance of the opposing turning movements.

31. Constant-Pressure Megger.—When the circuit under test contains a considerable amount of electrostatic capacity, the testing electromotive force should remain constant, as otherwise steady readings will not be obtained. In the constant-pressure megger, the generator is driven through a friction clutch that releases by centrifugal action if the normal handle speed of 100 revolutions per minute is exceeded.

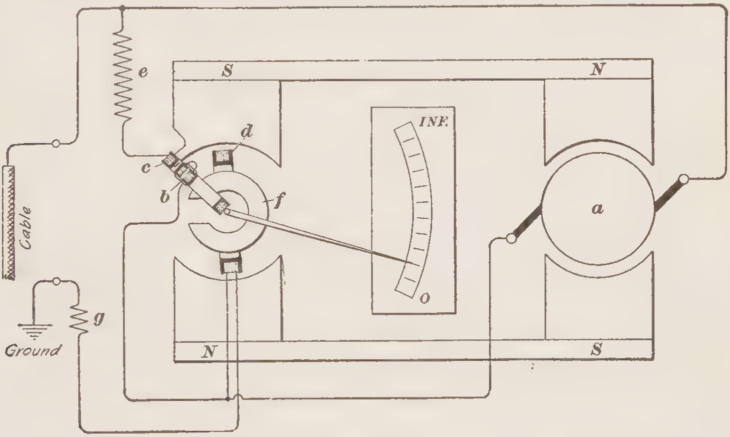


FIG. 15

32. Variable-Pressure Megger.—In the variable-pressure megger, the friction clutch is omitted and the electromotive force generated by the armature may be varied by turning the handle at different speeds. When the insulation resistance to be measured is high and little electrostatic capacity is present, the higher electromotive force that may be generated in the variable-pressure megger is sometimes desirable in order to force an appreciable current through the insulation.

33. Insulation Resistance per Mile From Reading of Megger.—The readings of the megger give the insulation resistance for the entire length of cable under test. If the insulation resistance per mile is desired, the megger reading should be multiplied by the length of the cable in miles.

DIRECT-CURRENT MEASURING INSTRUMENTS

DESCRIPTION OF INSTRUMENTS

GENERAL CLASSIFICATION

1. Instruments are available for the measurement of practically all electrical quantities. These instruments may be classified according to: (1) the kind of current in the circuits on which they are used, as *direct-current* or *alternating-current instruments;* (2) the service for which they are intended, as *switchboard* or *portable instruments;* (3) their principles of operation, in direct-current work, as *moving-iron*, *D'Arsonval*, *electrodynamometer*, *electrostatic*, and *hot-wire*, *instruments;* and (4) the methods of showing the results of measurements, as *indicating*, *recording*, or *integrating instruments*. Instruments of the integrating type are described in *Watt-Hour Meters*.

MOVING-IRON INSTRUMENTS

2. **Repulsion Movement.**—Fig. 1 shows a **Weston eclipse direct-current ammeter** of the switchboard type. This instrument depends for its operation on the magnetic repulsion between a stationary vane *a* of soft iron and a movable vane *b* of soft iron, both of which are placed within the fixed magnetizing coil *c* and are thereby magnetized in the same direction when current is established in the coil. Like

COPYRIGHTED BY INTERNATIONAL TEXTBOOK COMPANY. ALL RIGHTS RESERVED

poles repel each other, and this action causes a movement of the movable vane *b*, its shaft, and the attached pointer *d*.

The movement is damped, or retarded, by a vane *e* that is attached to the shaft of the instrument and turns in a covered chamber *f*. There is but little clearance between the edges of the vane and the sides of the chamber. Two small vent holes in the cover of the chamber, only one of which is shown, allow the air to pass in or out, according to the movements of the vane. By this means the pointer comes to rest at its proper deflection almost immediately. The instrument is said to be *dead-beat*, and this feature materially assists the rapidity with which measurements are taken.

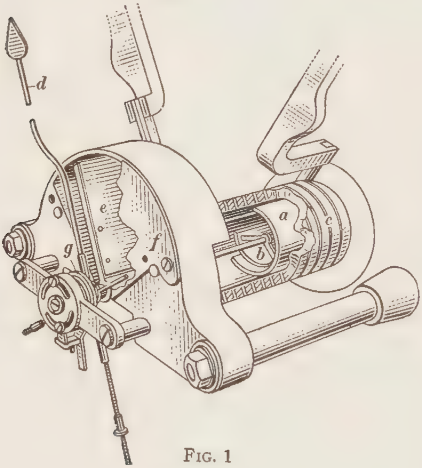


FIG. 1

A spiral control spring *g* opposes the torque acting on the shaft and returns the moving element to zero position when the torque ceases. The coil *c* carries the line current, because it is connected directly in series with one of the line wires. The reading position of the pointer is at the point of balance of the magnetic forces and the force of the control spring.

3. Inclined-Coil Instrument.—The **Thomson inclined-coil ammeter**, also of the moving-iron type, is shown in Fig. 2. This instrument has a stationary coil, shown in section at *c*, mounted on a frame *s* and inclined 45° to a vertical shaft through its center. Within the coil, and mounted on the shaft at an angle of 45° to it, are one or more soft iron vanes *v*. The shaft carries a pointer *p* that moves over a scale *S*.

With no current in the instrument, the pointer is at zero position and the plane of the vane is at an angle to the axis of the coil. When current is established, the vane tends to turn to a position in which its plane is parallel to the coil flux. The long arrows indicate the direction of the coil flux, and the dotted lines near v', the position of the vane when its plane is parallel to the flux. The moving element turns until the magnetic forces are balanced by the force of the control springs a and a'; and the reading is then indicated by the pointer and scale.

In one form of this instrument, the damping effect is due to the eddy currents set up in an aluminum disk mounted on the shaft in such a position that parts of the disk are between the poles of permanent magnets. When the moving element turns, the generator action of the disk and magnets serves to retard the motion and to make the instrument dead-beat.

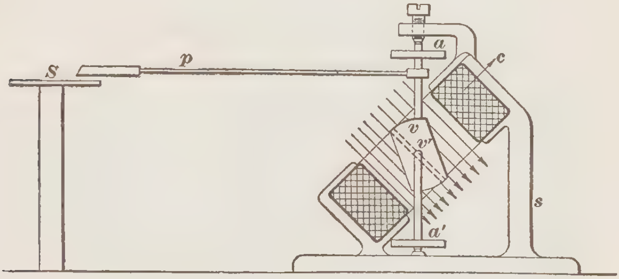


FIG. 2

4. Application of Operating Principle.—The principle of operation just described for the Thomson inclined-coil ammeter is employed in some voltmeters and ammeters for either direct-current or alternating-current service. The coil of the voltmeter consists of many turns of fine wire and is connected across the circuit with or without a resistor in series. The ammeter coil consists of a few turns of large conductor, and for the measurement of small currents it is often connected directly in series with one of the line wires of a circuit. To prevent stray magnetic fluxes of neighboring conductors from affecting the instrument, the working parts are often enclosed in an iron case.

INSTRUMENTS OF THE D'ARSONVAL TYPE

GENERAL DISCUSSION

5. Construction.—The D'Arsonval principle is used extensively in direct-current instruments for portable and switchboard service because it permits of substantial construction and a high degree of accuracy in measurements. Such instruments are based on the same principle as the galvanometer of the D'Arsonval type, but they are not suitable for alternating-current measurements.

The **Weston direct-current ammeter**, an exterior view of which is shown in Fig. 3, is an instrument of this type. Fig. 4 shows some of its working parts, indicating the relative positions of the permanent-magnet pole pieces, the stationary cylindrical core, the brass plate supporting this core, the movable coil, and the control springs.

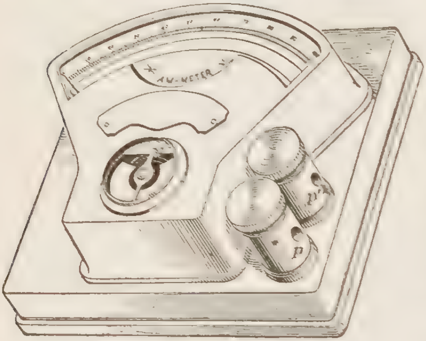


Fig. 3

Fig. 5 shows the magnetic circuit of this instrument. The permanent magnet A has soft-iron pole pieces P fastened to it by screws S, and the stationary soft-iron core C is supported by a screw M passing through a brass plate B. Since the diameter of the core is smaller than the bore of the pole pieces, two narrow air gaps are formed, and in these gaps are placed the vertical sides of the movable coils, Fig. 4. The coil is thus subjected to the action of a nearly uniform magnetic flux, which passes from pole to pole across the air gaps and the core.

The movable part of this instrument is shown in Fig. 6. It consists of a rectangular coil C of fine wire wound on a bobbin, either of aluminum or copper, that is suspended vertically between two delicate jeweled bearings. Two flat control springs *S* oppose the tendency of the coil to rotate, and they also serve to conduct current to and from the suspended coil. A thin aluminum pointer *P* attached to the coil moves over a scale as the coil turns, thus indicating the deflection.

6. Operation.—In instruments of the D'Arsonval type, current in the movable coil reacts with the flux from the magnet and tends to turn the coil to a position in which its flux will coincide in direction with the flux of the permanent magnet. The coil will come to rest at a position in which the torque due to the magnetic forces is balanced by the torsion of the control springs. The scale is so divided and marked as to read in either amperes or volts.

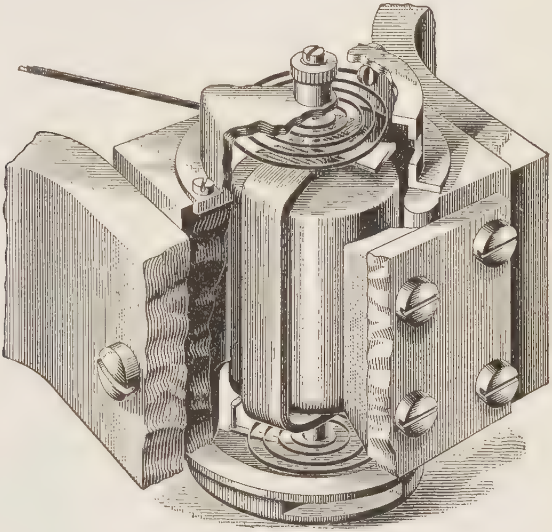


FIG. 4

The strength of the magnetic flux in the air gaps is so nearly uniform that the deflection is closely proportional to the current in the coil, thus making the scale division of very nearly uniform length, as shown in Fig. 7. This scale is from a 150-ampere ammeter and is three-fourths full size.

7. Parallax.—In order to secure great accuracy of observation, a mirror is often set in the scale card, as indicated in Fig. 3. Thus, if the observer stands over the instrument in such a position that the pointer either completely obscures its image in the mirror or appears to lie midway in the image, error due to *parallax*, or reading of the deflection sidewise, is avoided, because the line of vision then coincides with the pointer and its image in the mirror.

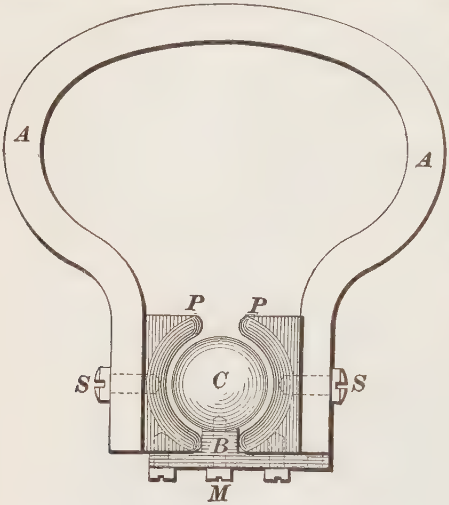


Fig. 5

8. Method of Making the Instrument Dead-Beat.—When the metal bobbin on which the coil is wound moves across the magnetic flux, the electromotive force thereby set up causes current in the bobbin opposite in direction to the current in the coil. Since the current in the coil causes the motion, the current in the bobbin opposes the motion; and, as this tendency exists only when the bobbin is moving, it has the effect of retarding or dampening the motion of the needle, thus bringing it quickly to rest at the proper point. The friction of the coil pivots on the bearings is so slight that it has practically no effect on the position that the needle will take. This is shown by the fact that the needle after having been deflected by a current will respond to very minute variations in that current; that is, the instruments are very *sensitive*.

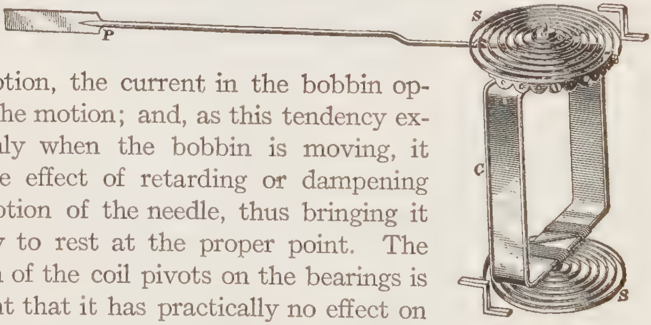


Fig. 6

9. Similarity in Construction of Ammeters and Voltmeters.—The general features of construction are practically the same for direct-current ammeters and voltmeters of the D'Arsonval type. If the instrument is designed for a voltmeter, a high resistance, located in the back of the case, is connected in series with the movable coil, which is wound with a large number of turns of very fine wire. If it is designed for an ammeter, the coil has fewer turns of coarser wire, and for all coils except those intended for very small currents the coil is connected in parallel with a short, thick piece of copper or some alloy, called an *ammeter shunt*, so that only a small part of the current being measured passes through the coil. The shunt of an ammeter intended for moderate current is often placed in the case of the instrument.

10. The joint resistance of the shunt and the coil of an ammeter is extremely low, and the resistance of a voltmeter coil is high; therefore, very little energy is required to operate either instrument. For example, a 15-ampere Weston ammeter has an internal resistance of .0022 ohm; when measuring a 10-ampere current, the drop ($I\ R$) is .022 volt, and the power ($I\ E$) = .22 watt, or about $\frac{1}{3400}$ horsepower. Again, the resistance of a 150-volt voltmeter is about 18,000 ohms. When measuring 110 volts, the instrument takes $\frac{110}{18,000} = .0061$ ampere, and the power is $.0061 \times 110 = .671$ watt, nearly, or about $\frac{1}{1100}$ horsepower.

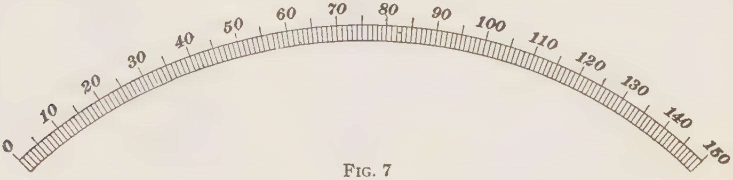


FIG. 7

The moving coil usually consists of fine copper wire, but the resistance wire in a voltmeter and, in some cases, the shunt of an ammeter are made of an alloy that changes very little in resistance if the temperature of the alloy changes; thus, moderate changes in the temperature of either the instrument or the ammeter shunt do not perceptibly affect the accuracy of the readings.

11. Astatic Instruments.—The magnetic flux set up by current in a neighboring conductor may affect the accuracy of an instrument unless means are taken to counteract the influence of such stray fluxes. In any case, it is advisable, if possible, to keep the instruments away from conductors carrying large currents. An instrument is called *astatic* when the working parts are so arranged that stray magnetic fluxes have practically no effect on the readings.

An astatic arrangement is indicated in a conventional manner in Fig. 8. The instruments usually have two movable coils *a* and *b* that are connected in series and are attached to a single shaft. Each of these coils is placed in a magnetic field, but the fluxes of these fields are in opposite directions; also, the two movable coils are so wound and connected that the direction of current in coil *a* is opposite that in coil *b*; therefore, the influence of a current in the coils is to swing them in a common rotative direction about the axis represented by the vertical rod. The flux of each coil agrees in direction with that of the flux between the nearest pair of poles. Therefore, the coils tend to be drawn into the spaces

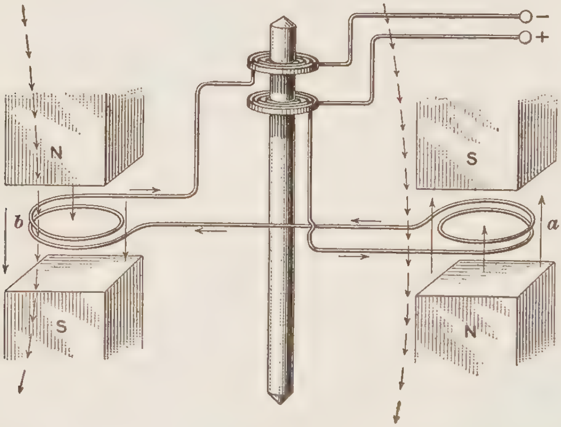


FIG. 8

between the poles from their zero, or no-current, position outside of these spaces. In Fig. 8 the coils are shown between the poles in maximum-current position.

12. If a stray magnetic flux, however, passes through both coils in the same direction, as indicated by the curved dotted lines in Fig. 8, the interaction between the stray flux and the coil fluxes tends to turn the coils in opposite directions. The effect of the stray field on the rotation of the moving element is therefore practically neutralized.

Astatic instruments are much used on switchboards that must necessarily be located in proximity of large current-carrying conductors. In some cases, additional protection from stray fluxes is provided by means of an iron instrument case.

13. In some types of astatic instruments, the magnetic field in which the moving element turns is provided by permanent magnets, electromagnets, or solenoids. In case electromagnets or solenoids are used, the error due to variations in the exciting current of the electromagnet or solenoid is reduced by mounting small pieces of iron on the shaft of the moving element in such positions that when the coils are moving toward the spaces between the poles, the iron pieces are being forced away from the poles. This action tends to retard the motion of the coils. If the exciting current and the flux decrease, the torque on the moving coils decreases, but at the same time the retarding effect of the iron pieces is lessened. If the exciting current and the flux increase, the coil torque increases, and so does the retarding effect of the iron pieces.

AMMETERS AND SHUNTS

14. Connections.—In Fig. 9 are shown the connections of a high-reading ammeter *a* and its shunt *b*. The shunt is connected in the circuit, and the ammeter is connected with the shunt terminals. Care should be taken that the positive terminal of the ammeter, usually marked +, is connected to the positive terminal of the shunt, which is at the end of the shunt through which current enters. The current through the ammeter is proportional to the voltage drop in the shunt, and this drop is proportional to the current in the circuit. An instrument that indicates current strength when connected with the terminals of such a shunt is essentially a *millivoltmeter* with a scale reading in amperes.

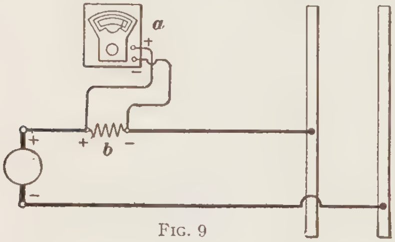


FIG. 9

Fig. 10 shows such a shunt consisting of terminals *a*, and flat strips *b*. Flexible leads *c* serve to connect the shunt with the instrument. All such shunts have low resistance and large radiating surface, and are made of an alloy, the resistance of which remains practically constant at all working temperatures.

Each shunt is made for use with a given instrument; therefore, care should be taken to see that the shunt number is the same as that of the instrument with which it is used. The connecting wires supplied with the shunt should be used; any change in their length or the substitution of other wires may affect the accuracy of the readings.

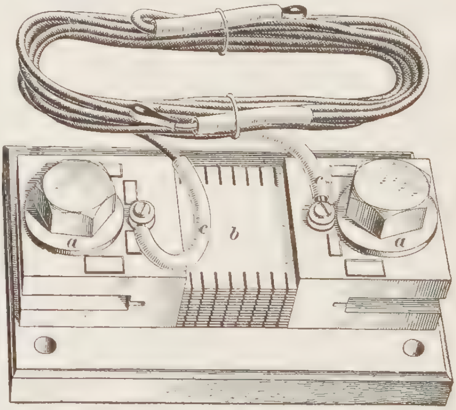


FIG. 10

15. Increasing the Range of an Ammeter.—In many cases a standard millivoltmeter is used as an ammeter; then, one or more shunts are adapted for use with the instrument, so that the range of the instrument may be extended to suit the conditions of work. A millivoltmeter thus used may be considered as an ammeter. However, the readings of the deflections on the scale of the instrument must be multiplied by a constant, the value of which depends upon the shunt used in order to obtain the true current values.

16. In Fig. 11 are shown the connections of a millivoltmeter and an external shunt of three ranges. One set of terminals connects the line to the shunt, and another set, the shunt to the millivoltmeter. The range of the instrument, as connected, is from 0 to 75 amperes. If the scale on the instrument is laid out so as to be direct reading for a full deflection of 15 amperes, the reading in this case must be multiplied by five in order to obtain the true current values. With this shunt box and this millivoltmeter, the range of the currents that may be measured is from 0 to 15, 0 to 75, and 0 to 150 amperes. The constants are 1, 5, and 10.

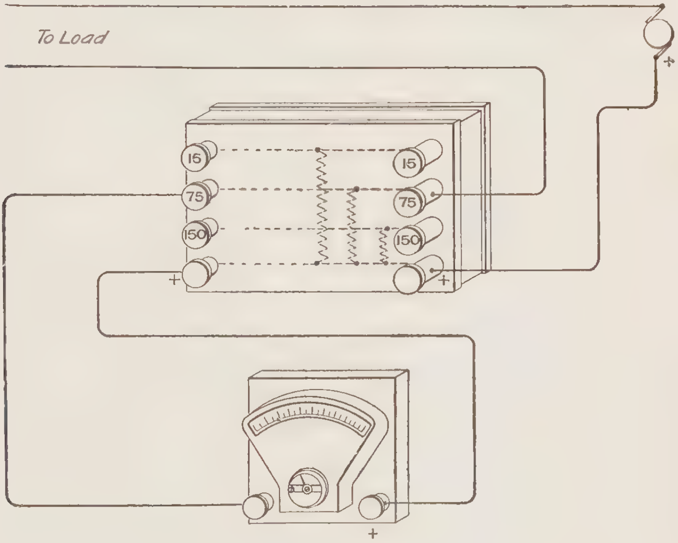


FIG. 11

VOLTMETERS

17. Single-Scale Voltmeters.—Voltmeters that are provided with a single scale intended for only one range of values usually have two binding posts for connecting the instrument across the circuit. In order to have the current through the coil in the right direction, the binding post marked + is connected to the positive line wire and the other binding post is connected to the negative line wire. A small push button switch is often provided in the portable instrument, so that the voltmeter circuit may be closed only when a measurement is being taken.

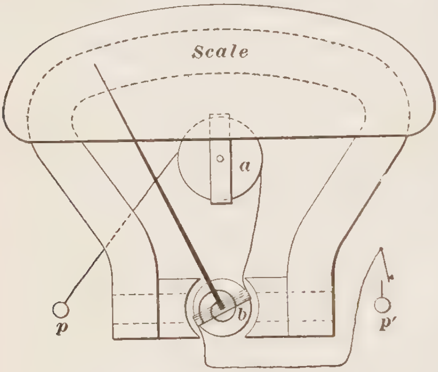


FIG. 12

Fig. 12 shows the working parts of a **Weston direct-current voltmeter** of the single-scale type. One end of resistance coil *a* is connected to the negative binding post *p* and the other, to the movable coil *b*. The other end of coil *b* is connected through a push button switch to the positive post *p*′.

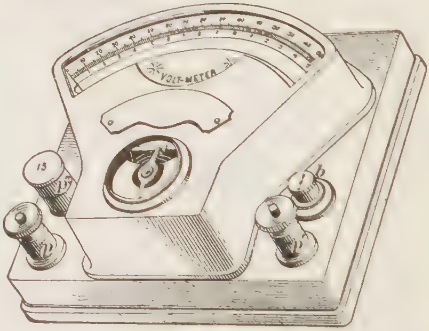


FIG. 13

18. Double-Scale Voltmeters.—In the case of voltmeters, the scale of which is intended to indicate two ranges of values, three binding posts are provided. The values of the upper scale are usually some even multiple of the values of the lower scale. Fig. 13 shows a double-scale voltmeter of maximum readings of 150 and 15 volts, binding posts p' and p being used with the 150-volt scale, and posts p' and p'' with the 15-volt scale. The post p' on the right is usually the positive terminal; both posts p and p'' are negative terminals.

19. The internal connections of a double-scale voltmeter are shown in Fig. 14. The resistance of the circuit from p' to p'' may be 15,000 ohms for use with the 15-volt scale and that of the circuit from p' to p may be 150,000 ohms for use with the 150-volt scale. The ratio between these resistances being 10, the voltage to cause a given current through the movable coil, and, therefore, a given deflection of the pointer, must be ten times as much when the connections for the large scale are used as when the connections for the small scale are used.

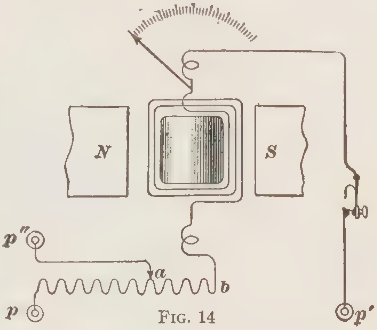


FIG. 14

When using a double-scale voltmeter, care must be taken not to apply too high a voltage to the terminals p' and p'' of the lower resistance circuit. If these terminals are connected to a 125-volt circuit, the instrument may be burned out. The 150-volt terminals p' and p should be used for voltages greater than 15 and up to 150. If the voltage of a circuit is not known approximately, a test should be made with the high-reading scale first. Then, if the reading is less than the highest value of the lower reading scale, that scale should be used for a final test.

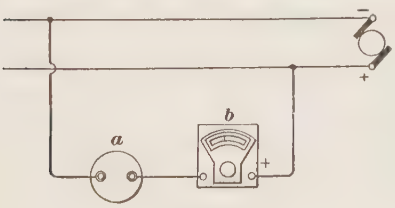


FIG. 15

20. Increasing the Range of a Voltmeter.—A voltmeter can be used to measure voltages much higher than its maximum scale reading by connecting a suitable resistor in series with the instrument across the circuit. Such a resistor is called a *multiplier*. It is connected in the manner indicated in Fig. 15, in which the multiplier is shown at *a* and the voltmeter at *b*. Multipliers are made of such resistances that the scale reading must be multiplied by 2, 5, 10, 20, or 50 to obtain the voltage of the circuit that is under test.

ELECTRODYNAMOMETER INSTRUMENTS

21. In measuring instruments of the electrodynamometer type, the deflections are caused by the interaction of the magnetic fluxes of two or more coils, one of which is movable and is usually placed within a fixed coil or coils. Usually, there is no iron within the coils; therefore, the instruments have but little self-induction and many of them are suitable for either direct-current or alternating-current measurements of watts, volts, or amperes.

When measuring a direct current, readings are taken for both directions of current through the instrument. By this method, the influence of stray magnetic fluxes is eliminated, because if the stray flux acted in unison with the flux of the fixed coils during the first test it would act in opposition during the second test. The mean of the two readings, in such cases, is nearer to the correct value.

WATTMETERS

22. **A Weston indicating wattmeter** is shown in Fig. 16. This instrument is of the switchboard type and is designed for either direct-current or alternating-current measurements. The connections of this wattmeter to a circuit are indicated in Fig. 17.

The fixed coils *a*, Figs. 16 and 17, called the *series*, *current*, or *field coils*, are connected in series with the line wire of the circuit, or, in the case of large current values, to a shunt in the line wire. The magnetic flux produced by the current in the fixed field coils is therefore proportional to the current in the circuit.

The movable coil *b*, called the *potential coil*, is mounted on a shaft that is provided with steel pivot points supported in jewel bearings. Control springs *c* and *d* serve as conductors for current to and from the moving coil; they also keep the pointer at zero when there is no current and oppose the movement of the coil when there is current. The movable coil

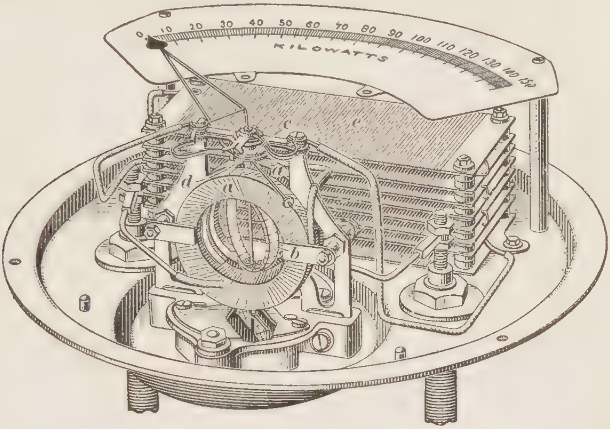


FIG. 16

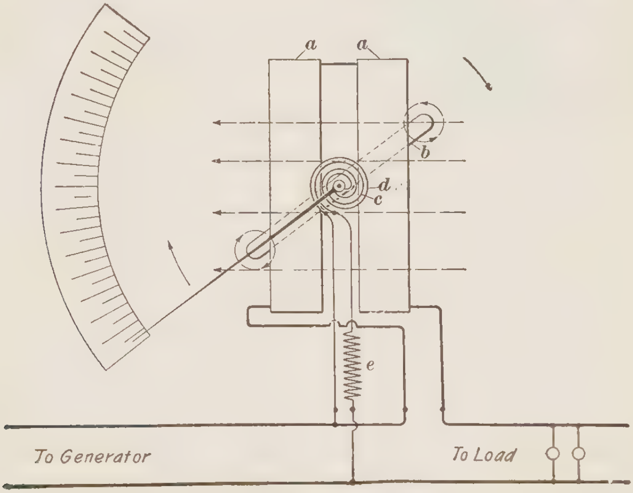


FIG. 17

in series with a high non-inductive resistance *e* is connected across the circuit. The magnetic flux produced by the moving coil is therefore proportional to the difference of potential between the potential-coil terminals of the instrument.

23. The combined action of these two magnetic fluxes, one proportional to the current in the circuit and the other proportional to the voltage, tends to turn the movable coil toward a position in which its flux will coincide in direction with that of the flux of the fixed coils. The total turning effort is therefore proportional to the product of the instantaneous current and voltage, or to the watts. At the point of reading, the torque due to the magnetic action is balanced by the counter torque of the control springs, and the scale is so marked that the pointer indicates the watts corresponding to the load on the circuit.

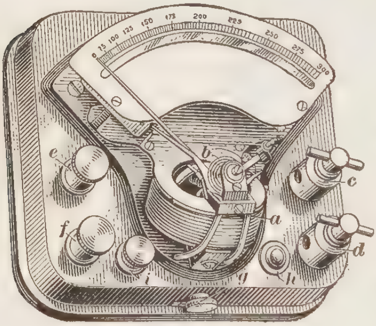


Fig. 18

More watts are required for the potential-coil circuit than for the series-coil circuit; therefore, in order that the instrument will not indicate these watts in addition to the circuit load, the potential coil is connected to the circuit on the generator side of the series-coil connections. The current in the potential coil does not, therefore, pass through the current coil.

24. The **Thomson inclined-coil wattmeter** is shown in Fig. 18. This indicating wattmeter operates on the same general principle as the inclined-coil instrument previously described. In this wattmeter, an inclined fixed coil *a* serves as the current coil and an inclined movable coil *b*, attached to the shaft of the moving element, serves as the potential coil. With current in both coils, the moving element tends to turn so as to make the two coil fluxes coincide in direction, and this movement is balanced by control springs. The extent of the movement is therefore proportional to the product of the current in the two coils, or to the watts in the circuit, and the scale is marked to read watts.

The series-coil terminals of the Thomson wattmeter are shown at *c* and *d*, and the potential-coil circuit terminals at *e* and *f*. A device for clamping the moving system during transportation is operated by the button *g*. An air-damper vane is attached to the lower part of the pointer, and its action tends to make the instrument dead-beat. The button *h* operates a mechanical damper, which consists of a silk thread that can be brought into contact with the top of the pointer. The button *i* is used to complete the potential-coil circuit· the button may be locked in its closed position.

VOLTMETERS AND AMMETERS

25. Voltmeters and ammeters of the electrodynamometer type are constructed so as to be suitable for voltage and current measurements on either direct-current or alternating-current circuits. Their principle of operation is similar to that of the wattmeter. In the case of a voltmeter, the fixed coil, the movable coil, and a resistor are connected in series across the circuit. In the case of an ammeter for small currents, the fixed coil and the movable coil are connected in series, and the instrument is connected in series with one line wire of the circuit. For large currents, the movable coil of some ammeters is connected in parallel with the low-resistance series coil. This allows only a small current in the movable coil.

ELECTROSTATIC INSTRUMENTS

26. Electrostatic voltmeters and other measuring instruments of the electrostatic type, may be used on either direct-current or alternating-current circuits, but they are more commonly used to measure high alternating voltage.

The principle of their action is based on the mutual influence of adjacent charged bodies. Thus, if a fixed plate is connected to one side of a circuit and a movable plate, located near, but not touching the first plate, is connected to the other side of the circuit, dissimilar charges will be impressed upon the plates, and these charges will attract each other. The movable plate, or vane, will move toward the fixed plate, and the extent of this movement indicates the voltage impressed on the instrument.

Electrostatic voltmeters take no current from a direct-current line while the voltage remains unchanged, and but a very small charging current from an alternating-current line. These voltmeters are better suited for high-voltage work, but instruments reading as low as 40 volts can be obtained.

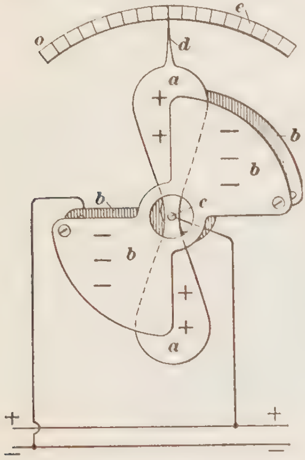


FIG. 19

27. Fig. 19 serves to illustrate the principle of an electrostatic voltmeter. In an instrument for direct-current service, several fixed and movable vanes are provided. The movable vane *a* is connected to the positive side of a direct-current circuit, and the fixed plates *b*, to the negative side. A positive charge is impressed on vane *a*, and a negative charge, on plates *b*. The attraction between these unlike charges draws the movable vane into the space between the fixed plates. The vane is so weighted below pivot *c* that when there is no voltage the pointer is at zero position.

When the instrument is active, the reading position is at the point of balance of the electrostatic forces and the force of gravity acting on the vane, and the latter force tends to return the vane to zero position. In some cases a small spiral spring serves as the controlling element. The pointer *d* indicates the voltage on the scale *e*.

Instruments intended for comparatively low direct voltages are provided with a large number of movable and fixed vanes in order to obtain large charging surfaces. The shaft supporting the movable vanes is suspended by a wire; therefore, a very small turning effort will cause the movable vanes to enter the spaces between the fixed vanes.

HOT-WIRE INSTRUMENTS

THEORY OF ACTION

28. The action of **hot-wire instruments** is based on the heating effect of a current in a conductor and the change of length of the conductor with its change of temperature. The heat developed in a given conductor is proportional to the square of the current, and the length of the conductor varies directly with its temperature.

Fig. 20 illustrates the principle of hot-wire instruments. A wire is kept taut between two fixed points *a* and *b* by a spring *c*. A pointer attached to the wire at *d* moves over a scale marked so as to indicate the current or the voltage that is being measured. An increase of current in the wire causes the wire to lengthen; the spring takes up the slack, and the pointer moves along the scale toward the right. With a steady current, the pointer comes to rest at some point and a reading may be taken. If current decreases, the wire contracts and the pointer is moved toward the left to a new position. Commercial instruments are so arranged that a slight change in length of the conductor causes a considerable change in the position of the pointer.

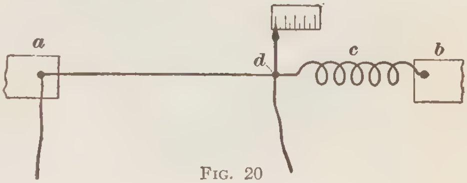


Fig. 20

COMMERCIAL FORMS OF INSTRUMENT

29. Instruments of the hot-wire type are made for measuring current or voltage on either direct-current or alternating-current circuits. An external shunt is employed with a switchboard ammeter having a capacity greater than 5 amperes and an internal resistance for voltmeters reading up to 150 volts.

30. Principle of Operation.—Fig. 21 shows the exterior of a hot-wire switchboard ammeter, and Fig. 22, the working parts. The parts in both illustrations are indicated by the same reference letters. Current in the wire *a b*, Fig. 22, heats it and causes it to expand. To eliminate errors due to changes in room temperature, the wire is supported on a base that expands or contracts at the same rate as the wire when both are subjected to room temperature only. At a point on the wire *a b* is attached a second wire *c*. The other end of this wire *c* is fixed to post *g*. To this second wire is attached a very fine wire *d* that passes around a small pulley *c* and is held taut by a flat spring *f*. The pulley is mounted on a vertical shaft that also carries the pointer *h* and the aluminum damping disk *s*. The shaft is mounted on jewel bearings, so as to turn with the minimum amount of friction.

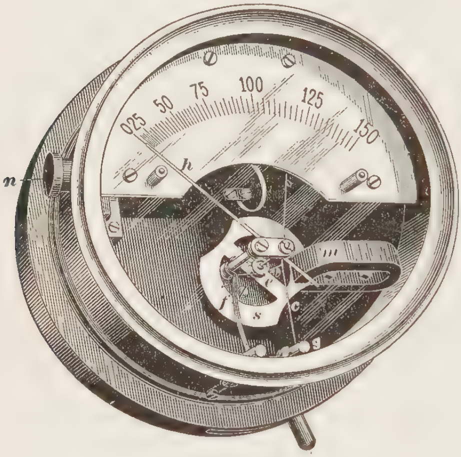


Fig. 21

The wire *a b* is stretched taut when not heated, and a very small expansion causes a considerable sag. The spring *f* takes up the sag in wires *a b* and *c*, and, since wire *d* is wound on pulley *e*, any increase or decrease of sag causes a movement of the pointer. The pointer moves to a position on the scale corresponding to the current measured. The dotted lines show in an exaggerated way the stretched positions of wires *a b* and *c*.

31. Adjustment and Damping.—The knob *n*, Fig. 21, serves to adjust the pointer at the zero point by slightly varying the position of the post *a*, Fig. 22. The movements of the needle are damped by means of the aluminum disk *s*, Figs. 21 and 22, a portion of which turns between the poles of the permanent magnet *m*, Fig. 21, thus making the instrument dead-beat. The handle near the bottom of the case, Fig. 21, operates a switch so that the instrument circuit can be opened when no readings are desired.

32. Arrangement for Large Currents.—When the instrument is used to measure very small currents, the full length of wire *a b*, Fig. 22, carries the entire current that is to be measured. When used for measuring larger currents, however, the wire *a b* is tapped at one or more points by means of flexible silver strips, one of which is shown just below the scale in Fig. 21, and the sections of the wire thus formed are connected in parallel in order that the instrument will have greater current-carrying capacity.

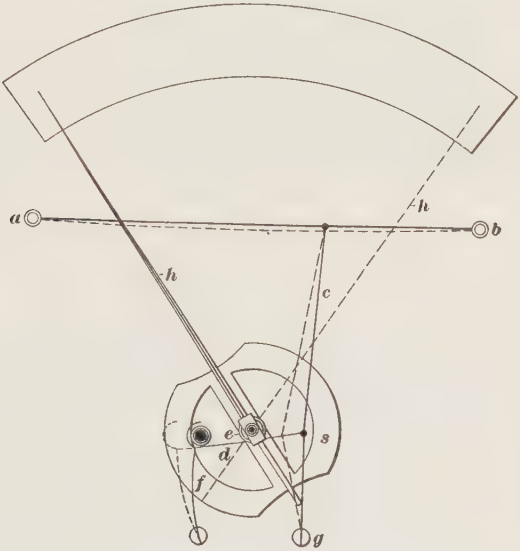


FIG. 22

When still larger currents are to be measured, a shunt of the type shown in Fig. 23 is employed. This shunt consists of low-resistance strips *a* connected to heavy terminals *b*. The shunt is connected in the main circuit, and the ammeter is connected across the shunt by means of flexible cables furnished with the instrument. The cables are attached at points *c*, and it is important to see that the number on the cables corresponds to the number on the instrument; otherwise, the indications will not be correct. The internal resistance of the ammeter is very low, so that a small drop in voltage across the shunt is sufficient to operate the instrument.

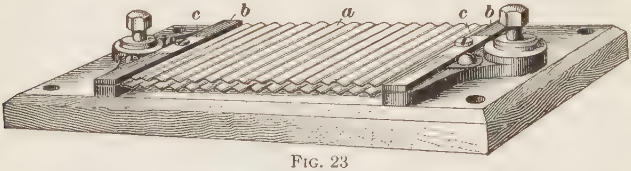


FIG. 23

RECORDING INSTRUMENTS

GENERAL PRINCIPLES AND CLASSIFICATION

33. A **recording instrument** is substantially an indicating instrument with a pointer arranged to trace a line on a paper chart that moves under the pointer at a uniform rate. The chart is marked with two sets of lines, one to represent time and the other to represent amperes, volts, watts, or whatever values the instrument may be designed to record. The line traced by the instrument shows the variation of the recorded values. Such instruments are very useful in power stations to record the variations of the station load.

Recording instruments may be classified as *direct-acting* and *relay*. In the direct-acting instruments, the moving element of the meter makes the record directly on the chart, and in the relay instruments, the moving element operates relay contacts that, in turn, cause a marking device to make the record on the chart.

In some recording instruments, the marking device at the end of the pointer is supplied with ink, and it draws either a continuous or a dotted line on the moving chart. In other instruments, the sharp end of the pointer causes a series of dots to be made on the smoked surface of the chart.

DIRECT-ACTING RECORDER

34. A Bristol recorder with a chart in position is shown in Fig. 24. The measuring instrument, enclosed in a case, is shown at *a*; its pointer, at *b*; and a chart with a smoked surface, at *c*.

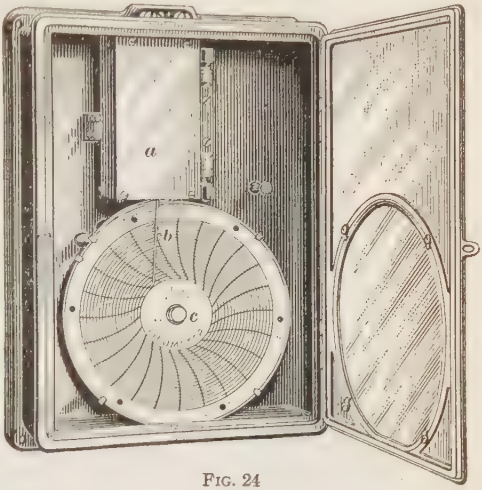


Fig. 24

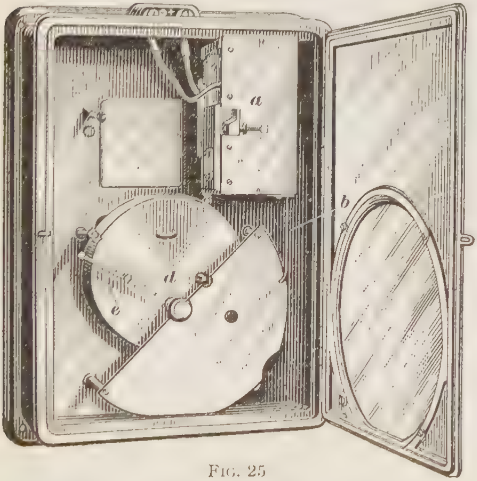


Fig. 25

In Fig. 25 is shown the same recorder with the chart removed; also, the measuring instrument and its case are swung to the right so that the pointer will not interfere with the removal of the chart. Enclosed in the box *d* is a clockwork mechanism that turns the chart and also moves the arm *e* outwards intermittently. This arm forces the smoked surface of the chart against the end of the pointer. The pressure on the chart is applied for only a very small interval of time, however. During the intervening time, the pointer is free from contact with the chart. The line of dots thus formed indicates the values of the quantity measured at very close intervals of time. When the chart is taken from the recorder case, it is dipped into a fixative solution, in order to make the tracing on the smoked surface permanent.

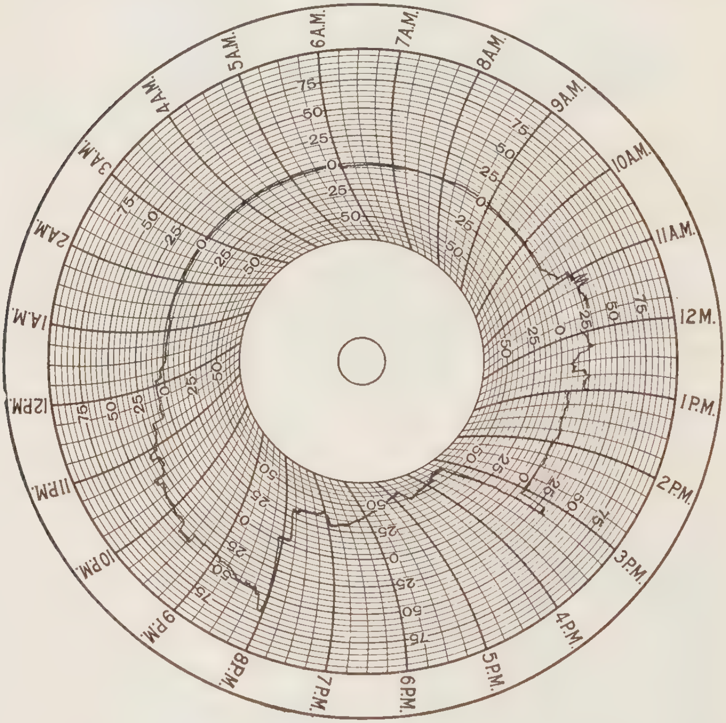


Fig. 26

The 24-hour record of an ammeter chart is illustrated in Fig. 26. It shows no current from 12:15 A. M. to 10:30 A. M. The current is in one direction from 10:30 A. M. to 3:15 P. M.; in the opposite direction from 3:15 P. M. to 8 P. M.; and in the original direction from 8 P. M. to 12:15 A. M.

RELAY RECORDER

35. In Fig. 27 is shown a graphic voltmeter of the relay type that is designed for either direct-current or alternating-current service and arranged to be mounted on a switchboard. The instrument contains two elements, a meter element and a relay element. Fig. 28 shows a diagram of connections. Corresponding parts in both of these illustrations have the same reference letters.

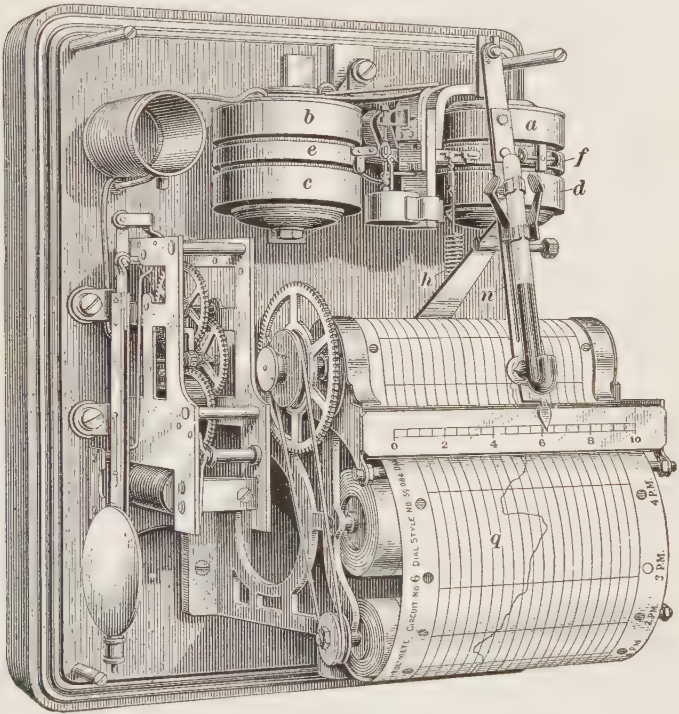


FIG. 27

36. The *meter element* consists of fixed coils *a*, *b*, *c*, and *d* and movable coils *e* and *f*. The movable coils are mounted on a beam that is pivoted at the center so as to tilt easily. All the coils and a resistor *g* are connected in series across the circuit. Current in the coils sets up fluxes that tend to move coil *e* toward coil *c* and coil *f* toward coil *a*. The torque due to the magnetic forces is opposed by the action of a spring *h* connected between the balance arm and a part of the pen-moving mechanism. A contact arm *i* attached to the beam makes connection with a contact point *j* when coil *e* moves downwards, and with point *k* when coil *e* moves upwards.

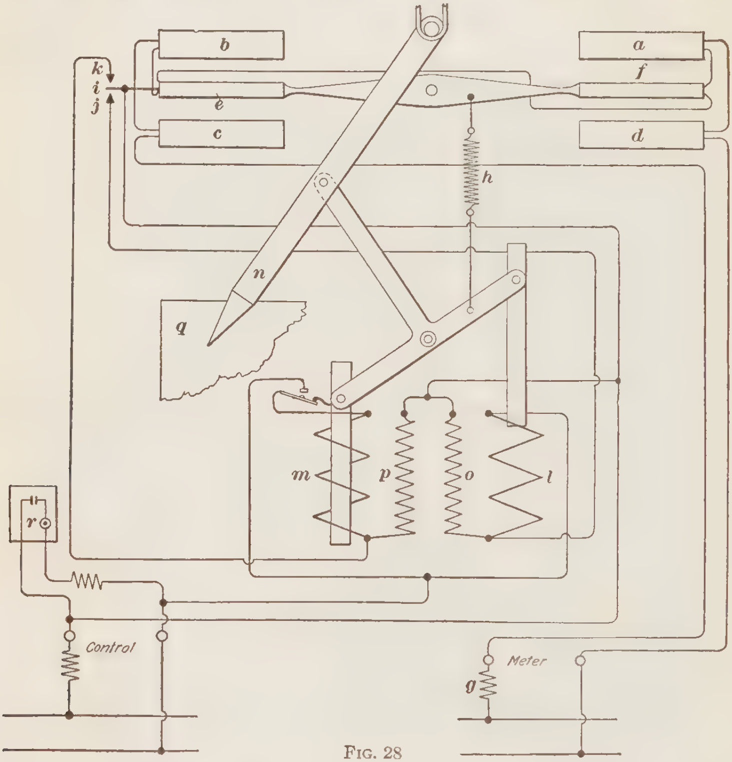


Fig. 28

37. The *relay elements* consist of two coils *l* and *m* that are supplied with current from a separate control circuit. The movable cores of these coils are connected to the pen-moving levers, and the positions of these cores determine the position of the pen. When the instrument is active and the balance arm is in its center position, coil *l* is in series with resistor *o* and coil *m* is in series with resistor *p*. The coils tend to draw their cores downwards, but as the magnetic forces of both coils are now small and balance each other, the pen remains at rest.

When the voltage increases, the current through the coils of the meter element increases, and the balance arm contact *i* makes connection with *j* and short-circuits resistor *o*. The current in coil *l* is thereby increased and its core is drawn down, turning the lever on its pivot and moving the pen arm *n* to the right. This movement puts spring *h* under greater tension and causes the balance arm to assume its central position, thus cutting the resistor *o* again into circuit with *l* and stopping further movement of the pen until the voltage again changes.

When the voltage decreases, the current through the coils of the meter element is decreased, and the spring *h* forces the balance arm to move so that contact *i* makes connection with *k*. Resistor *p* is now short circuited, thus increasing the current in coil *m*, and the pen arm moves to the left. This movement decreases the tension of spring *h*, the balance arm assumes its central position, and the pen arm stays in position until the voltage changes. The pen changes its position only when the voltage changes.

38. At zero position of the relay element, the circuit through coil *m* is opened by the cut-out shown near it. In this position, a projection at the end of the pen lever holds the cut-out open. The current otherwise taken from the control circuit for coil *m* and its resistor *p* is saved. The circuit through coil *l* and resistor *o* is complete, but, with resistor *o* in series, the current in coil *l* is too small to move the pen lever. Coil *l* is the first of the relay coils that becomes active when voltage is impressed on the metered circuit.

The pen is provided with an ink reservoir, and use may be made of a roll of paper that is long enough to record the variations in voltage during several weeks. The paper chart *q* is moved by a mechanism operated by a spring. The spring is rewound by a small motor *r* connected with the control circuit, and the motor is started and stopped automatically by the clock mechanism.

SPECIAL FORMS AND APPLICATIONS OF INSTRUMENTS

DIRECT-READING OHMMETER

39. Description.—The **Weston portable ohmmeter** is an instrument of the D'Arsonval type. In general construction, it is similar to a standard portable voltmeter, but it is provided with a scale reading in ohms instead of in volts. A small storage battery or some other source of constant electromotive force is used with the ohmmeter to supply the current for taking measurements. This type of ohmmeter is used for general resistance measurements ranging from 1 to 1,000 ohms, and it is particularly useful for the resistance measurements of incandescent-lamp filaments.

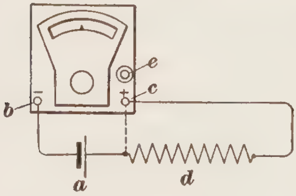


FIG. 29

40. Operation.—When a storage battery *a*, Fig. 29, having the electromotive force for which the Weston ohmmeter is designed is connected directly to terminals *b* and *c*, as indicated by the full and dotted lines, no external resistance being in circuit except the resistance of the battery, the current is sufficient to move the pointer from its no-current position at the extreme left to position *a* at *0* on the lower scale, Fig. 30.

If an external resistance *d*, Fig. 29, is added to the circuit, the current is decreased and the pointer, Fig. 30, takes up a position to the left of *0*. If *d*, Fig. 29, has a resistance of 10 ohms, the pointer rests at position *b*, Fig. 30, and the reading is 10 ohms.

If a resistance of from 50 to 100 ohms is to be measured, the key *e*, Fig. 29, is pressed so as to cut out 50 ohms of the internal resistance of the instrument. If the resistance were just 50 ohms, the pointer would rest at *0*, Fig. 30, on the lower scale, which is the same as 50 on the upper scale. The total resistance of the internal and the external circuit is now the same as the total resistance of the battery and the internal circuit when *e*, Fig. 29, is not pressed and resistance *d* is not in circuit. If the resistance were more than 50 ohms, the pointer would take a position to the left of *0*, Fig. 30, and its value would then be read on the upper scale.

41. Correction-Factor Scale.—In some ohmmeters a correction-factor scale is provided for use when the electromotive force of the source does not cause the pointer to rest at *0*, Fig. 30, when only the source is connected to the ohmmeter. If the pointer rests at position *c* on the correction scale, the correction is + 1 per cent.; if it rests at *d*, the factor is − 1 per cent. In order to apply the correction, the internal resistance of the instrument corresponding to the scale used must be known. This data is usually marked on the instrument base.

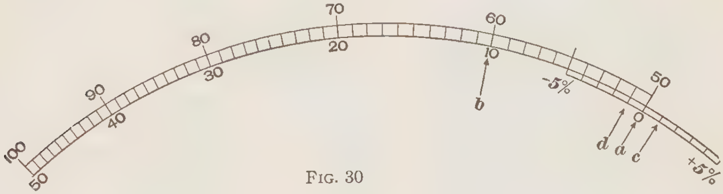


FIG. 30

Let R = true resistance of the device;
R_1 = instrument reading;
$\pm p$ = correction factor, expressed in hundredths;
r = internal resistance of the instrument corresponding to the scale used.

Then,

$$R = R_1 + p\,(R_1 + r)$$

For example, if the test for the correction factor shows −4 per cent. and the resistance reading indicates 75 ohms, the internal resistance of the instrument then being 68 ohms, the true resistance is $R = 75 - .04\,(75 + 68) = 69.3$ ohms, nearly.

When the lower scale of the instrument is in use the internal resistance is $68+50=118$ ohms. If the correction factor is then $+2$ per cent. and the resistance reading indicates 30 ohms, the true resistance is $R=30+.02\ (30+118)=33$ ohms, nearly.

ELECTRIC TACHOMETER

42. In Fig. 31 is shown a type of **portable electric tachometer** for measuring the speed of rotation of shafts. This device consists of a small direct-current magneto generator *a* and a voltmeter *b*, the scale of which is marked to read revolutions per minute. The generator shown has on its shaft a diamond-shaped point that is held firmly against the end of a rotating shaft. The generator speed and the voltage are proportional to the speed of the shaft, and this speed is indicated by the voltmeter.

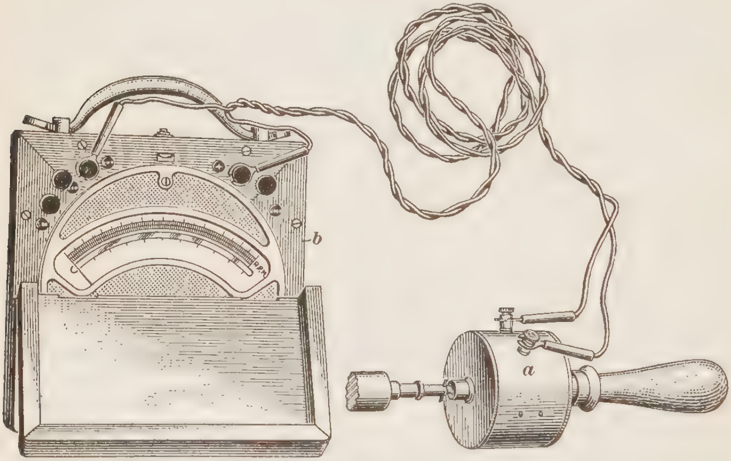


FIG. 31

In *stationary tachometers*, the generators are belted or otherwise connected to the shaft, and the indicators can be mounted on the switchboard or at other convenient places.

To increase the range of readings, a double scale and a divided internal resistance are sometimes provided.

ELECTRIC PYROMETERS

43. Electric pyrometers, whether of the *thermoelectric-couple* or the *resistance* type, are used to indicate temperatures.

44. In the **thermoelectric-couple pyrometer,** the junction of two metals is thrust into the furnace or other source of heat to be measured. The electromotive force generated by the couple is impressed on a millivoltmeter, the scale of which is marked to read in degrees of temperature of the furnace. If the temperature of the furnace rises, the electromotive force is increased and the instrument indicates a higher reading. A lower temperature is indicated by a lower reading.

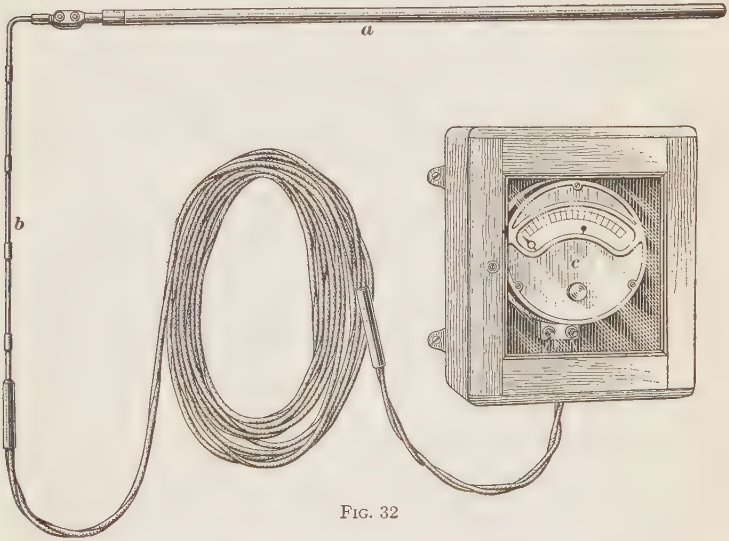


Fig. 32

Fig. 32 shows a pyrometer of this type. The couple is placed within the protecting tube *a*, and an extension piece *b* is connected to the cold ends of the couple. The extension piece of the couple is connected by flexible cable to the indicator *c*.

45. Changes in the room temperature affect the reading of such an instrument to some extent by changing the difference of temperature between the *hot* and *cold* ends of the couple. If the room temperature is subject to change, a *compensator*, Fig. 33, is sometimes connected in the circuit, as indicated at *d*, Fig. 34, in which illustration the reference letters *a*, *b*, and *c* indicate the parts so lettered in Fig. 32. The compensator consists of a bulb of mercury into which extends a loop of fine platinum wire that forms a part of the circuit. If the room temperature rises so as to reduce the difference of temperature between the ends of the couple, the mercury rises and short-circuits more of the platinum loop, thus proportionally reducing the resistance in series with the instrument, keeping the current at practically a constant value for a given furnace temperature, and compensating for the change in the room temperature.

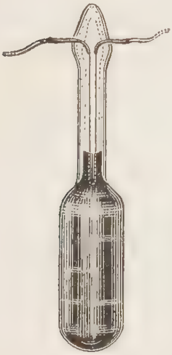


FIG. 33

46. The operation of **resistance pyrometers** is based on the change of resistance in a wire caused by a change in temperature of the wire. A piece of wire, either of platinum or nickel, is placed in a tube that is thrust into the furnace. The changes in the resistance of the wire, due to changes in temperature of the furnace, are indicated in some pyrometers by means of a form of Wheatstone bridge; in others, these changes are indicated by a measuring instrument having two movable coils, current through one being practically steady and current through the other being varied by any change in the resistance of the tube wire. The position of the moving element of the instrument, which indicates degrees of temperature, is changed if the current in one of the coils is varied.

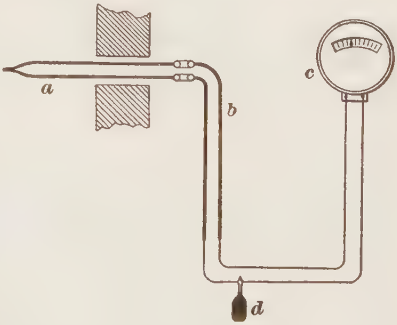


FIG. 34

USE OF INSTRUMENTS FOR MEASUREMENTS

MEASUREMENTS WITH AMMETERS AND VOLTMETERS

47. Measurements of Current, Difference of Potential, and Resistance.—In Fig. 35 is shown a battery *a*, a coil *b*, an ammeter *c*, and a voltmeter *d*. It is desired to find the current in the coil, the voltage impressed on it, and its resistance. The ammeter is connected in series with the coil, so that all the current passing through the coil must pass through the ammeter also. The reading of the ammeter indicates the current. The voltmeter is connected across the terminals of the coil, and its reading indicates the difference of potential, expressed in volts, that is impressed on the coil. The resistance of the coil, expressed in ohms, as determined by Ohm's law, is

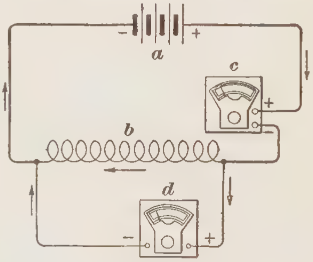


FIG. 35

$$R = \frac{E}{I},$$

in which R = resistance;
E = voltmeter reading;
I = ammeter reading.

For example, if $E = 6$ volts and $I = 1.2$ amperes, the resistance of the coil is 5 ohms.

The ammeter as connected measures the current that passes through the coil and voltmeter in parallel; but the voltmeter resistance is usually so high that the effect of the current in the voltmeter circuit may be neglected. The ammeter connecting wires should be short and of low resistance; the voltmeter connecting wires may be longer and of higher resistance

The resistance of the voltmeter leads is very small compared with the high internal resistance of the instrument.

48. High and Low Resistances.—Ammeters and voltmeters may be employed in the manner just explained to measure a wide range of resistances. When the resistance is high, a low-reading ammeter and a comparatively high-reading voltmeter should be used. When the resistance is low, a high-reading ammeter and a low-reading voltmeter should be used.

This method, sometimes known as the *fall-of-potential method*, is useful in determining the resistance of armatures, of starting resistance sections, and of poor joints between conductors, such as poor contacts on a switch, etc. The voltmeter or millivoltmeter terminals should be applied directly to the terminals of the device under test, and the current should be high enough to obtain readable deflections.

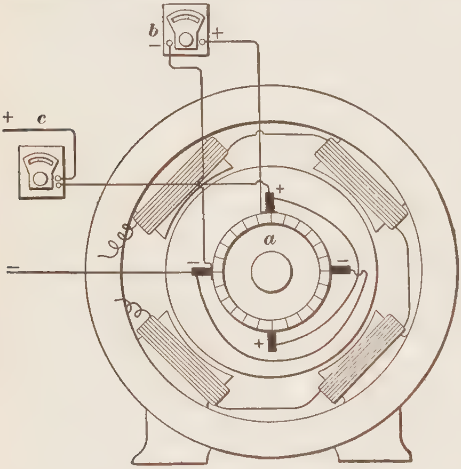


Fig. 36

49. Armature Resistance.—In Fig. 36 is shown the application of the preceding method to a resistance test on an armature *a* of a generator or a motor. The field circuit of the machine is opened so that the armature will not rotate during the test. The terminals of a millivoltmeter *b* are placed under brushes of opposite polarity, and a small current is established in the armature. An ammeter *c* in the armature circuit indicates the value of this current, and a rheostat (not shown) can be used to keep the current within safe limits and to obtain readable deflections.

Several readings should be made with the armature in different positions, and the mean of the readings taken. The millivoltmeter reading divided by the ammeter reading gives the resistance of the armature. This includes the joint resistance of the parallel paths through the armature windings and the commutator, but not the resistance of the brushes.

MEASUREMENTS WITH A VOLTMETER

50. Measurement of Current With a Voltmeter. The current in a simple circuit can be determined by means of a voltmeter and a known resistance. The high-resistance voltmeter *a*, Fig. 37, is connected across the terminals of the known resistance *b*, through which a current passes from a source of electromotive force *c*. The current through the circuit is

$$I=\frac{E}{R},$$

in which, I = current through circuit;
E = reading of voltmeter a;
R = known resistance of b.

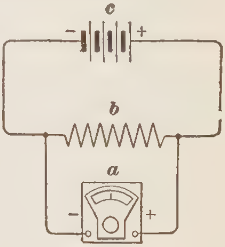


FIG. 37

51. Measurement of Resistance With a Voltmeter. A resistance *a*, Fig. 38, can be measured by means of another resistance *b* of known value and a high-resistance voltmeter *c*. The voltmeter *c* is first connected to the terminals of the unknown resistance *a* and then to the terminals of the known resistance *b*. These resistances are connected in series; therefore, the same current passes through them from the source *d*. The resistances of coils *a* and *b* are proportional to the readings of the voltmeters when connected to the terminals of the coils.

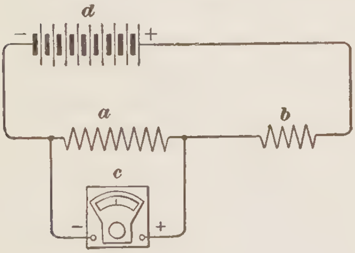


FIG. 38

Let R = unknown resistance a;
R_1 = known resistance b;
E = volts drop through a;
E_1 = volts drop through b,

Then, $R : R_1 = E : E_1$

and
$$R = \frac{R_1 E}{E_1}$$

For example, if the drop in volts E through a is 3.69 volts, the drop E_1 through b 2.6 volts, and the resistance R_1 of b .26 ohm, the resistance of a is

$$R = \frac{.26 \times 3.69}{2.6} = .369 \text{ ohm}$$

52. Measurement of Insulation Resistance of Line Wires With a Voltmeter.—The insulation resistance between either line wire of a two-wire circuit and the ground may be determined roughly by means of a voltmeter. The connections for one test of this kind are indicated in Fig. 39. The voltmeter a is first connected across the circuit, the leads being in positions b and c. The voltmeter is then connected between the negative line wire and the ground, the leads then being at positions c and d. Another connection of the voltmeter is between the positive line wire and the ground, the leads then being in positions b and e. There may be a number of leakage paths from either line wire to the ground. These groups of paths are indicated by f and g. The faults on the line wires and the voltmeter form paths from one side of the circuit to the other side when the voltmeter has one terminal grounded. The readings should be taken as quickly as possible and the voltage of the line should be steady. A good ground connection is made by touching the voltmeter lead to a water pipe or other metal structure that is in contact with the earth.

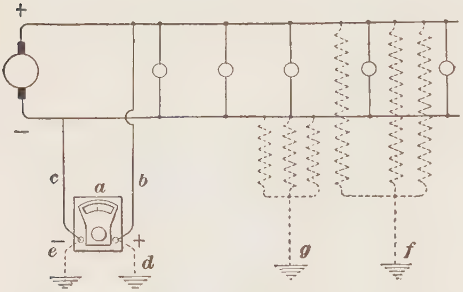


FIG. 39

Let e = reading for first test;
e_1 = reading for second test;
e_2 = reading for third test;
R = insulation resistance between positive line wire and ground;
R_1 = insulation resistance between negative line wire and ground;
r = internal resistance of voltmeter.

Then,
$$R=r\left(\frac{e}{e_1}-1\right) \qquad (1)$$

$$R_1=r\left(\frac{e}{e_2}-1\right) \qquad (2)$$

Example.—When determining the insulation resistance between the two line wires of a circuit and the ground with a voltmeter having an internal resistance of 12,200 ohms, the voltage e between line wires is found to be 113; the voltage e_1 between the negative line wire and the ground, 4; and the voltage e_2 between the positive line and the ground, 1. What is the insulation resistance of: (*a*) the positive line wire? (*b*) the negative line wire?

Solution.—(*a*) The insulation resistance between the positive line wire and the ground is, according to formula **1**,

$$R=12{,}200\left(\frac{113}{4}-1\right)=332{,}450 \text{ ohms. Ans.}$$

(*b*) The insulation resistance between the negative line wire and the ground, is according to formula **2**,

$$R_1=12{,}200\left(\frac{113}{1}-1\right)=1{,}366{,}400 \text{ ohms. Ans.}$$

53. Measurement of Insulation Resistance of a Generator With a Voltmeter.—The insulation resistance of the windings of a generator may be measured with approximate accuracy by the method just described for line wires. The positive terminal of the voltmeter is connected to the positive terminal of the generator (while running), and the other voltmeter terminal is touched to the frame of the machine. If the pointer is not deflected appreciably, the insulation resistance is fairly high. A similar test should be made with the voltmeter connected between the negative generator terminal and the frame. The line circuit should be disconnected while making the test.

Formula 1, Art. 52, should be used in case the insulation resistance is to be calculated. The voltage of the circuit is e, and the voltage between one generator terminal and the frame is e_1.

MEASUREMENT OF POWER

54. Measurement by Voltmeter and Ammeter.—The power, in watts, of a direct-current circuit can be determined by finding the product of the volts across the circuit and the amperes in it.

The connections for any circuit or branch circuit are shown in Fig. 40. The voltmeter *a* across the circuit indicates the volts, and the ammeter *b* in series with the circuit indicates the amperes. The product of the two readings is the power, in watts.

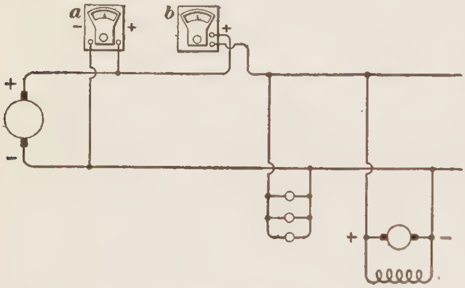


FIG. 40

55. Measurement by Wattmeter.—In Fig. 41 are shown the connections of a wattmeter *a* to measure the power of the main circuit at the generator. The potential-coil terminals should be connected across the circuit on the generator side of the current-coil connections, which are in series with one line wire. The reading of the wattmeter indicates the power required for two branch circuits and line losses.

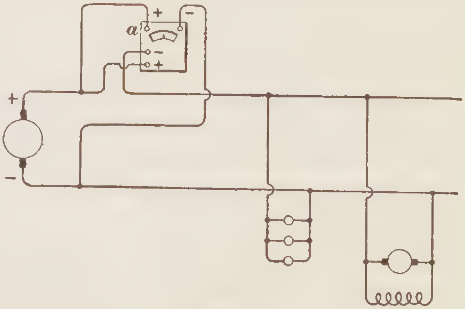


FIG. 41

CALIBRATION OF INSTRUMENTS

CALIBRATION BY COMPARISON OF SIMILAR INSTRUMENTS

56. The **calibration** of a measuring instrument from the operator's point of view, is the process of determining the error, provided there is one, of any of the indications of the pointer on its scale. It is important that all electrical measuring instruments be accurate. Occasional comparison with other instruments known to be accurate is therefore advisable. Any errors in the indications can thereby be detected and corrections thereafter applied. Such comparison requires that the two instruments should be affected by the same values of the quantities measured.

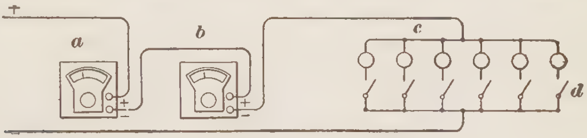


FIG. 42

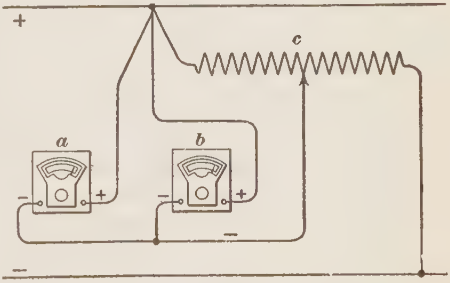


FIG. 43

57. Comparison of Ammeters.—Two ammeters can be compared by connecting them in series in a circuit in which the voltage is steady and the current can be varied. In Fig. 42 the ammeters are shown at *a* and *b* and a lamp bank at *c*; the current is varied by switching lamps off or on by small switches *d* and the ammeters are compared for several readings.

58. Comparison of Voltmeters.—In Fig. 43, the connections for a comparison test of two voltmeters *a* and *b* are shown. A high resistance *c* is connected across the circuit. Two corresponding terminals of the voltmeters are connected together and to one line wire. The two other terminals are connected together and touched to different points on the resistance *c*. The voltmeter should be compared for several different readings of voltages and any errors recorded.

59. Comparison of Wattmeters.—Two wattmeters are compared by connecting both so that they will measure the rate at which energy is expended in the same circuit and then varying the rate so as to make comparisons for several different points on the scales. Fig. 44 indicates the connections of the wattmeter *a* under test, the standard wattmeter *b*, and the lamp bank *c*. If the standard wattmeter *b* is not available, the test of a direct-current wattmeter can be made by means of an accurate ammeter *d* and a voltmeter *e*. The connections of the ammeter and voltmeter are shown by dotted lines. In this case the standard wattmeter *b* and its connections in Fig. 44 should not be considered.

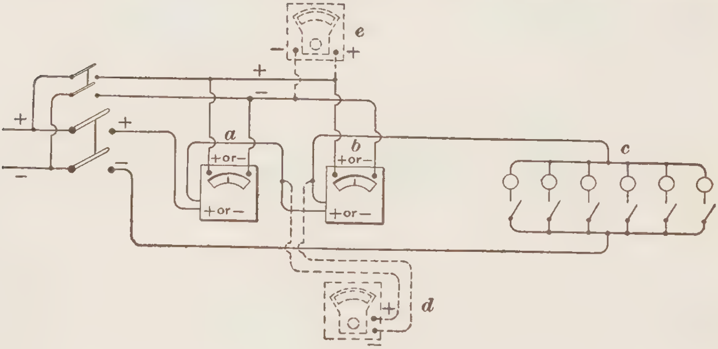


Fig. 44

CALIBRATION TABLES AND CURVES

60. If the calibration test on an instrument shows that the readings are seriously inaccurate, it is usually best to return the instrument to the manufacturer for repair and adjustment.

A table may be prepared showing in one column the readings of the instrument under test, in the next column the readings of the standard instrument, and in the last column the corresponding per cent. error of the scale or the actual number of units that the scale is in error. The table should be kept near the instrument for convenient reference.

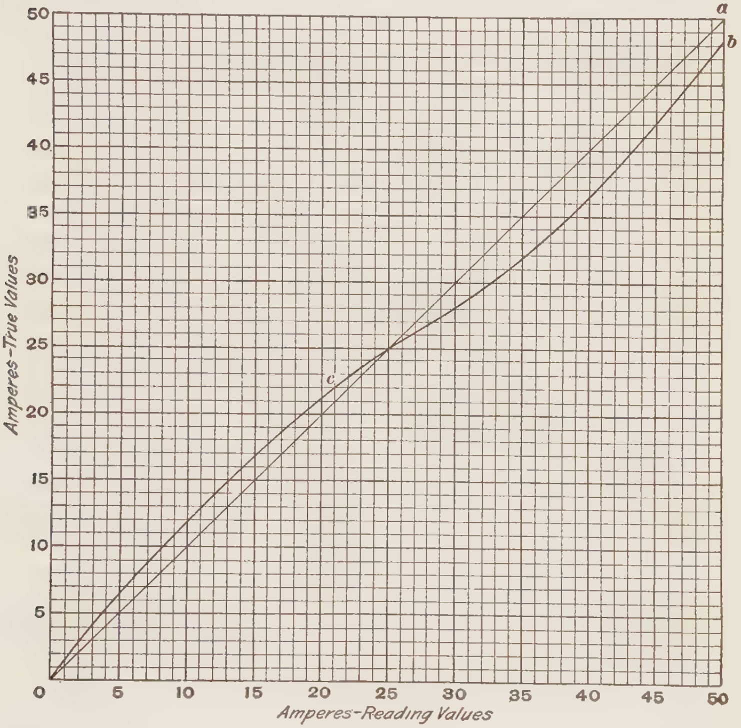


FIG. 45

61. A calibration curve such as that shown in Fig. 45 can be constructed to take the place of the table. The true values are shown in the vertical data line and the readings of the instrument under test in the horizontal data line. If the instrument had been correct, the straight line *0–a* would represent the calibration line. In this case, however, the instrument was not correct and the curved line *0–b* indicates the calibration line. The points on this curve are determined by the intersection of a vertical line through a reading value with a horizontal line through the true value corresponding to that reading value. For example, if the ammeter reading is 21 amperes and the standard, or true, value is 22 amperes, the intersection of the vertical and horizontal lines is at *c*. This forms a point on the curve. Other points can be located and curve *0–b* constructed.

After the curve has been drawn, any true value may be found from any reading value by tracing vertically from the reading value to curve *0–b* and then horizontally to the true value. The vertical distance between any point on the straight line and the curve represents the units that the scale of the instrument is in error at that point. In the case shown, the first part of the scale reads too low and the last part too high. At point *c* the reading is 1 ampere too low.

ALTERNATING CURRENTS

(PART 1)

GENERAL THEORY

REPRESENTATION AND DEFINITIONS

GENERATION OF AN ALTERNATING ELECTROMOTIVE FORCE

1. When a conductor is moved so as to cut across lines of force, or a moving magnetic flux cuts across a stationary conductor, an electromotive force is induced in the conductor. In the case of the moving conductor, if the direction of motion of the conductor and the direction of the lines of force of the flux remain unchanged, the direction of the electromotive force is uniform. In the case of the stationary conductor and the moving flux, if the direction of motion of the flux and the direction of the lines of force making up the flux remain unchanged, the direction of the electromotive force is uniform. With uniform direction of electromotive force applied to a complete circuit, direct current will be maintained in the circuit. But if the direction of the lines of force of the flux periodically reverses while the conductor cuts across the flux or the flux cuts across the conductor, an *alternating electromotive force* is induced in the conductor, and an **alternating current** is established in a closed circuit of which the conductor forms a part. This condition obtains when the conductor moves past

COPYRIGHTED BY INTERNATIONAL TEXTBOOK COMPANY. ALL RIGHTS RESERVED

alternate north and south magnetic poles or poles of alternate polarity move past the stationary conductor.

2. In practically every dynamo-electric machine, alternating voltage is induced in the armature conductors, because the direction of the lines of force that act on the conductors periodically reverses. In direct-current generators, the commutator changes the alternating current in the armature conductors to direct current for the external circuit, while in alternating-current generators, or *alternators,* the current in the external circuit is the same as that in the armature conductors, both alternating.

3. The value of an electromotive force set up in a conductor, between which and a magnetic flux relative motion exists, is at every instant directly proportional to the rate of cutting of the lines; that is, to the number of lines cut by the conductor, or which cut across the conductor, per unit of time. In the modern alternator, the active conductors are usually mounted on the stator and the magnetic poles on the rotor revolve past these stationary conductors. This condition is assumed in the following discussion.

Fig. 1 (*a*) shows an elementary type of alternator. A conductor *a,* one end of which is shown, is mounted in a slot in the stationary steel ring forming the stator. The rotor *b* is provided with a field winding, which, when energized by current from a direct-current generator *c,* will establish a north pole at one side of the rotor and a south pole at the opposite side. The exciting current is transmitted to the field winding through slip rings mounted on the rotor shaft. The flux from the north pole of the rotor enters the stator and returns to the south pole of the rotor along the paths indicated by the dotted lines. The rotor is driven by some type of engine and as the rotor turns, the flux is continually changing in position within the stator; therefore, the rate at which the lines of force cut across conductor *a* changes and, with a two-pole rotor, the direction of the lines of force of the flux affecting the conductor reverses twice during each revolution. The rotor is assumed to be turning at uniform speed and in a clockwise

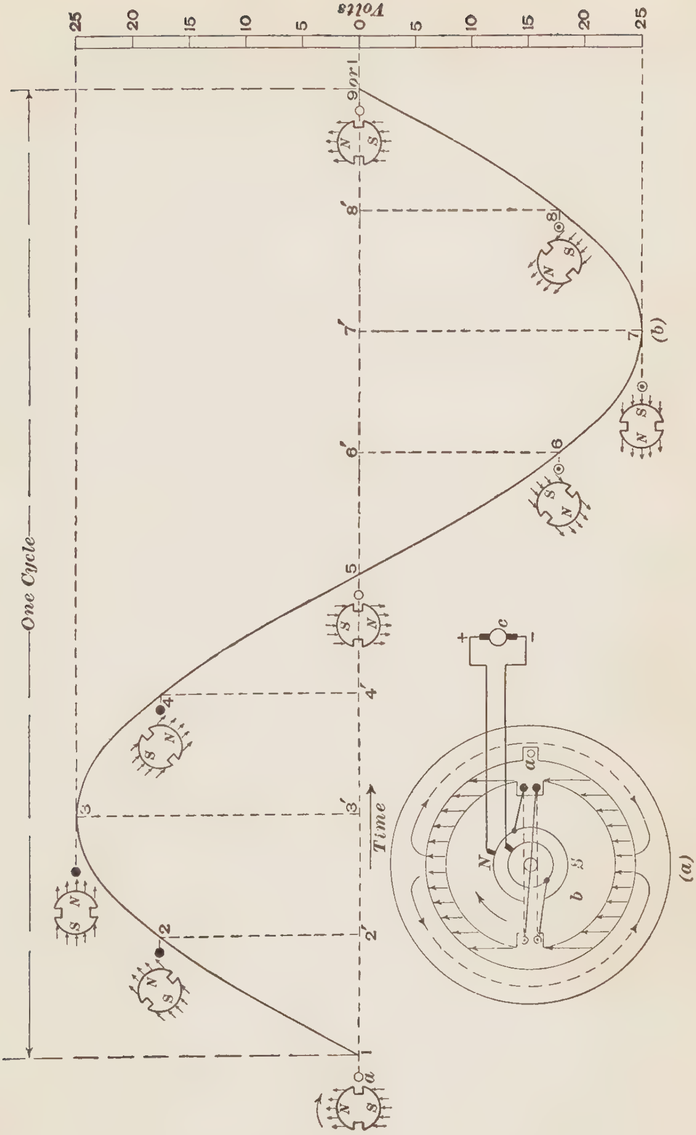

Volts
One Cycle
Time
9 or 1
a
(b)
N
S
+
−
c
a
b
N
S
(a)

FIG. 1

direction. For the purpose of the following elementary theoretical discussion, the lines of force of the flux in the rotor core and in the air gap are assumed to be parallel to the line between the centers of the north and south pole on the rotor; uniform density of the flux is also assumed. In an actual machine of this type the flux distribution will not be so uniform as here indicated.

The curve, Fig. 1 (*b*), represents the rise and fall of the alternating electromotive force generated in conductor *a*, Fig. 1 (*a*), during one revolution of the two-pole rotor. Points *1* to *9*, Fig. 1 (*b*), represent voltage conditions at the end of successive 45° movements of the rotor, and the corresponding relative positions of the poles and of the active conductor are indicated by the small sketches of the alternator. At point *1*, the lines of force in the stator near conductor *a*, views (*a*) and (*b*), are moving in, or parallel to, a vertical plane through the conductor; therefore, these lines *do not cut across* the conductor, and the electromotive force is zero. This is indicated by a circle with a white interior. As the rotor turns, the lines passing between the north pole of the rotor and the stator begin to *cut across the conductor* and an electromotive force is generated in a direction away from the observer, as indicated by the full black circle. At the 45° position, point *2*, view (*b*), the conductor is being cut across by many of the lines, and a considerable electromotive force is generated. At the 90° position, point *3*, the maximum cutting action occurs, since the lines from the north pole cut directly across the conductor, and maximum electromotive force is generated. Further movement of the rotor reduces the rate at which lines of force *cut across* the conductor, and the voltage decreases in value as indicated by the part of the curve through points *3*, *4*, and *5*.

At point *5*, the lines of force in the stator near conductor *a* are again moving in or parallel to a vertical plane through the conductor and, therefore, these lines are not cutting across the conductor. The generated voltage at this point is zero. As the movement continues, the lines passing from the stator to the south pole of the rotor begin to affect the conductor and the electromotive force is reversed as indicated by the dotted circle, because of the reversal in direction of the lines that are now cutting across conductor *a*; that is, the lines are now passing from the stator near *a* to the south pole on the rotor. The rate of cutting increases as the rotor turns through the next 90° of movement and the voltage rises to maximum, as indicated by the part of the curve through points *5*, *6*, and *7*. At the point of maximum voltage *7*, the lines from the stator to the south pole of the rotor cut directly across the conductor. The rate of cutting then decreases and the voltage falls to zero, as indicated by the part of the curve through points *7*, *8*, and *9*. Points *9* and *1* are identical in regard to voltage conditions since the rotor has completed one revolution in passing from *1* to *9* and the rotor and the stator are in the same relative positions at these points.

4. The rate of movement of the rotor, or the time required for it to move through equal angles, can be represented by distances along the *axis of abscissas,* such as *1–2′*, *2′–3′*, etc., Fig. 1 (*b*), and the values of the induced electromotive forces by the *ordinates 2–2′*, *3–3′*, etc.

The induced electromotive force is in one direction while the north pole is adjacent to the conductor, as indicated in positions *1* to *5*, and in the other direction while the south pole is adjacent to the conductor, as indicated in positions *5* to *9*. The first half of the curve *1–5* is, therefore, shown above the axis *1–9* and is called *positive;* the second half *5–9* is shown below the axis and is called *negative*. Assuming that in this particular conductor the maximum electromotive force is 25 volts, the instantaneous value at any point during the revolution can be read off the scale at the right by laying a straightedge against the point and parallel to the horizontal axis *1–9* and noting where it crosses the scale.

5. The curve, Fig. 1 (*b*), is called a **sine curve.** The construction of such a curve is treated later. The electromotive force or the current established by it, in a conductor on a stator within which a rotor is turning at uniform speed and with a *distribution of magnetic flux as stated in the discussion of this problem,* can be represented by a sine curve. The electromotive forces generated in commercial alternators always deviate more or less from a true sine-wave form, owing to irregularities in the distribution of the flux. Calculations in alternating-current work are, however, based on sine curves, and the departure of the electromotive force or current wave from the true sine curve is usually not enough to affect seriously calculations for practical purposes.

The stator of a commercial alternator contains many conductors and these are so connected that the electromotive forces of the individual conductors of a winding are added to form the terminal electromotive force. Alternator windings are treated in another Section.

CYCLE, FREQUENCY, AND ALTERNATION

6. A **cycle** is one complete set of the electromotive forces continually recurring in each conductor of an alternator. One cycle of changes occurs in each conductor while *one pair of poles* are passing by that conductor, and these changes can be represented by a curve, as in Fig. 1 (*b*). In a two-pole machine, 1 cycle is completed per revolution, and in a four-pole machine, 2 cycles per revolution. If the speed of the rotor is uniform, the time taken for the passage of a pair of poles past a conductor on the stator, or for completing 1 cycle, is always the same. The word *period* is ordinarily used to denote the time required for the current or voltage to pass through one cycle.

7. The number of cycles per second is called the **frequency.** Since a two-pole machine completes 1 cycle per revolution, the number of cycles per second, or the frequency, of such a machine, is the same as the number of revolutions per second. A machine with more than two poles completes as many cycles per revolution as there are pairs of poles; hence, the number of cycles per second equals the number ot pairs of poles multiplied by the number of revolutions per second.

Let f = frequency, in cycles per second;
r. p. m. = speed, in revolutions per minute;
n = number of pairs of poles.

Then,

$$f=\frac{\text{r. p. m.}}{60}\times n$$

Frequencies of 25 and 60 cycles are in general use, the first chiefly for power purposes and the second chiefly for lighting, although either of these frequencies can be used for power or lighting. Frequencies lower than 25 are used to some extent, but they are unsatisfactory for lighting because they cause the lights to flicker. In some localities, 30, 40, and 50 cycles are in use, and in some older stations intended for lighting only, frequencies as high as 133 are found.

EXAMPLE.—The rotor of an alternator is driven at a speed of 750 revolutions per minute. What is the frequency of the system if the rotor has four poles?

SOLUTION.—The number of pairs of poles is 4÷2=2, and, according to the formula,

$$f=\frac{750}{60}\times 2=25 \text{ cycles per sec. Ans.}$$

8. An **alternation** is half a cycle; it is therefore represented by one of two half waves representing a cycle. There are two alternations for every cycle. Instead of expressing the frequency of an alternator in cycles per second, it was formerly expressed in alternations per minute. Thus, a frequency of 60 cycles per second, or 3,600 cycles per minute, was expressed as 7,200 alternations per minute.

SPACE-DEGREES AND TIME-DEGREES

9. One of the 360 equal parts into which the circumference of a circle may be divided is an arc of one degree, written 1°; the angle between lines drawn from the extremities of this arc to the center of the circle is an angle of 1°. Fig. 2 shows a semicircle with some of the angular divisions indicated. The angle a between the two radial lines $o\ b$ and $o\ c$ is an angle of 1°, and the arc $b\ c$ is an arc of 1°. The arc $d\ e$ is an arc of 90°,

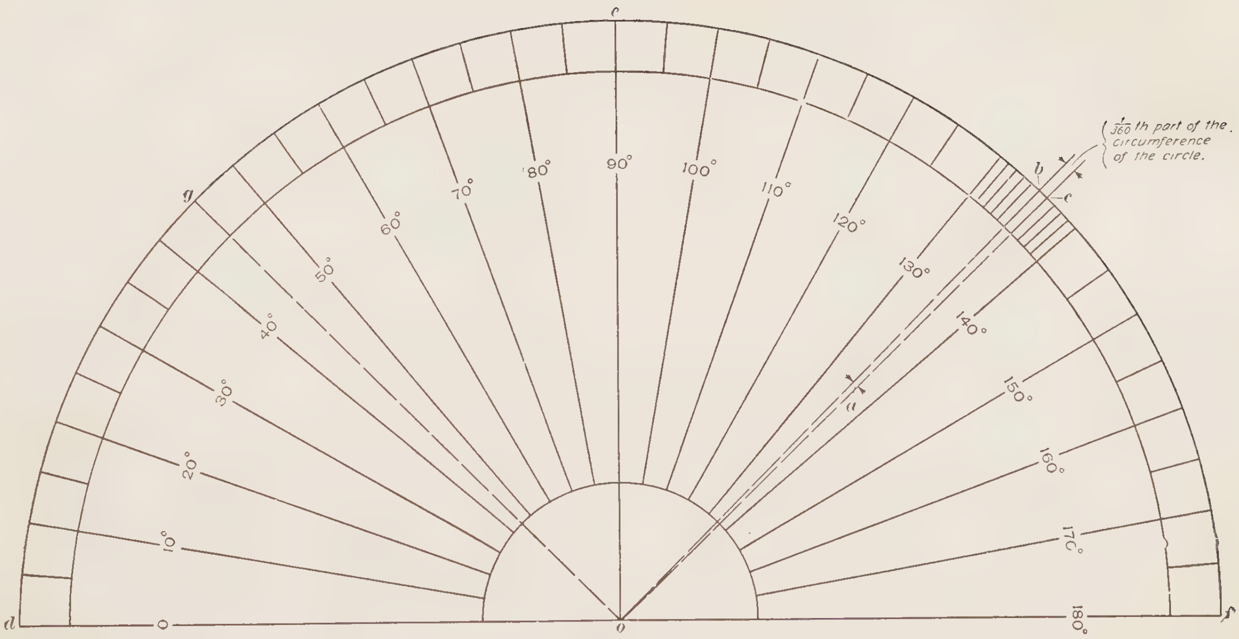

c
g
b
e
$\frac{1}{360}$ th part of the circumference of the circle.
a
0
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°
120°
130°
140°
150°
160°
170°
180°
d
o
f

FIG. 2

and it is said to subtend an angle of 90°; that is, angle *d o e* =90°, or one-fourth circle. Likewise, arc *d e f* is an arc of 180°, subtending an angle of 180°, or one-half circle. These degrees may be called **mechanical space-degrees,** or simply *space-degrees,* when necessary to distinguish them from others used in electrical calculations.

10. The time taken to complete a cycle is arbitrarily assumed to consist of 360 equal intervals called **electrical time-degrees,** or simply *time-degrees.* In Fig. 1 (*b*), time is measured along the horizontal axis, and, therefore, the distance *1–9* represents 360 time-degrees and each division *1–2'*, *2'–3'*, etc., 45 time-degrees. The maximum voltage is reached 90 time-degrees after zero value; and the next zero value, 90 time-degrees after the maximum value.

There is always a difference of 90 time-degrees between the *maximum* value and the *zero* value; also, of 180 time-degrees between the *maximum positive* and the *maximum negative* value; or between two consecutive zero values.

The value of 1 time-degree, in seconds, depends on the frequency. When the frequency is 60, 1 cycle is completed in $\frac{1}{60}$ second. As there are 360 time-degrees per cycle, the value of 1 time-degree in this case is $\frac{1}{360}\times\frac{1}{60}=\frac{1}{21,600}$ second. The value of 1 time-degree when the frequency is 25, is $\frac{1}{360}\times\frac{1}{25}=\frac{1}{9,000}$ second.

11. The movements of the rotating part of an alternator, either of the armature conductors in one type of machine or of the rotating poles in the type shown in Fig. 1, are usually stated in **electrical space-degrees,** or simply *electrical degrees.* The rotor moves through 1 electrical degree during 1 time-degree. Thus, during 1 cycle, the rotor passes through 360 electrical degrees.

With a two-pole rotor, 1 electrical degree is equal to 1 mechanical space-degree, because the rotor must move through 360 mechanical space-degrees in order to complete

1 cycle or 360 electrical degrees. With a four-pole rotor, Fig. 3, 1 electrical degree is equal to ½ mechanical space-degree, because the rotor must move through 180 mechanical space-degrees in order to complete 1 cycle of electrical changes. In an alternator having any number of pairs of poles, 1 mechanical space-degree is equal to as many electrical degrees as there are pairs of poles on the rotor. In Fig. 3 there are 2 pairs of poles, and 1 mechanical space-degree equals 2 electrical degrees. For a four-pole rotor, the relative values of mechanical space-degrees and electrical space-degrees are indicated in Fig. 3.

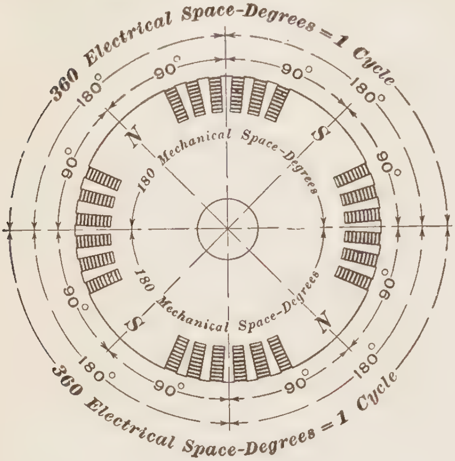


Fig. 3

EXAMPLES FOR PRACTICE

1. A two-pole turbo-alternator is supplying a 60-cycle system. What is the speed of the alternator in revolutions per minute
Ans. 3,600 rev. per min.

2. A two-pole turbo-alternator is running at a speed of 1,500 revolutions per minute. What is the frequency of the system?
Ans. 25 cycles per sec.

3. What length of time, in seconds, corresponds to 90 electrical time-degrees on a 60-cycle system? Ans. $\frac{1}{240}$ sec.

4. In a four-pole alternator, how many mechanical space-degrees are equal to 90 electrical degrees? Ans. 45 mechanical space-degrees

VALUES AND PHASE RELATIONS

VALUES USED IN PRACTICE

12. The many instantaneous values of an alternating current or voltage included in a complete cycle are seldom used in practical calculations and are therefore of little interest. The maximum value, the average value, and the effective value are of practical importance, especially the last named. In the following discussion sine waves are considered.

The *maximum value,* which is the highest value that a voltage reaches during an alternation, indicates the maximum stress to which the insulation of a device is subjected. The *average value,* or the average of the instantaneous values during an alternation, equals the maximum value multiplied by .636. The average value is used in some calculations, but not extensively. The *effective value* equals the maximum value multiplied by the constant .707. The effective value is the one usually meant when referring to alternating current or voltage and is the one used in all further discussions unless otherwise stated.

The relation of the values with reference to a voltage curve based on the sine law is as follows:

$$E_a = .636\, E_m \qquad (1)$$

$$E_e = .707\, E_m \qquad (2)$$

$$E_e = 1.11\, E_a \qquad (3)$$

$$E_m = \frac{E_e}{.707} = 1.414\, E_e \qquad (4)$$

in which E_a = average volts;
E_e = effective volts;
E_m = maximum volts.

For a current curve, substitute I for E in the formulas.

Example.—The effective voltage of an alternating-current circuit is 6,600 volts. What is the maximum voltage of the circuit?

Solution.—According to formula **4**, $E_m = \frac{6,600}{.707} = 9,335$ volts. Ans.

13. Relation of Instantaneous Values of an Alternating Voltage.—In order to understand the derivation of the constants used in expressing the relation between the maximum, average, and effective values of an alternating-voltage or alternating-current curve it is well to study the characteristics of a sine wave. Reference should also be made to the subject of the trigonometric functions of angles in another Section.

In Fig. 4, a magnetic flux of uniform density is indicated by the short equally spaced horizontal arrows. A conductor a, one end of which is shown, is mounted on the end of an arm $a\,b$. The arm is pivoted at b and is rotated through equal angles in equal intervals of time. The conductor is moving at a uniform rate from a to a_6 around its circular path, but its movement in a vertical direction as indicated by a_1, c_1, a_2, c_2, etc., increases less rapidly as the conductor approaches the position a_6 where it will be moving parallel to the horizontal line $a\,b$.

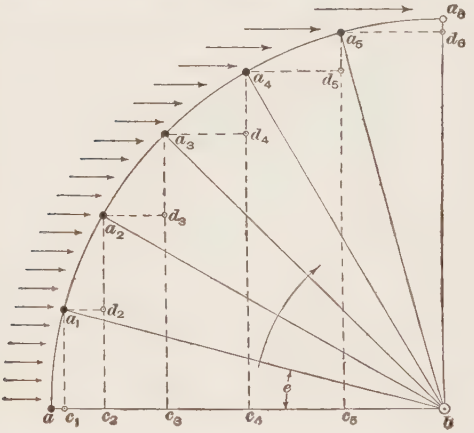


Fig. 4

The voltage generated in the conductor depends on the rate of cutting lines of force. The number of lines of force actually cut at right angles by the conductor in moving from a to a_1 may be represented proportionally by the length of the vertical line $a_1\,c_1$; from a_1 to a_2, by the line $a_2\,d_2$; and from a_5 to a_6, by the line $a_6\,d_6$. It should be noted that these lines are at right angles to the direction of the flux and also that the lines are decreasing in length as the arm approaches a_6. The voltage generated, therefore, decreases as the conductor approaches a horizontal movement.

Line $a_1\ c_1$ is proportional to the sine of angle e because $\sin c = \frac{a_1\ c_1}{a_1\ b}$. The angle c in Fig. 4 is 15°, but any other angle may be taken. The line $a_1\ b$ is of constant length since it is the lever arm, and if its length is unity, that is, 1 inch, or 1 foot, etc., the vertical line dropped from the end of the lever to the horizontal base line would be numerically equal to the sine of the angle. For example, if line $a_1\ b$ is 1 inch, the line $a_1\ c_1$ is equal to the sine of 15° multiplied by 1 inch, which is .26 inch, approximately. The other vertical lines such as $a_2\ c_2$, $a_3\ c_3$, etc., are also proportional to the sines of the corresponding angles formed between the positions of the revolving arm and the horizontal line. The lines $a_2\ d_2$, $a_3\ d_3$, etc., representing proportionally the rate at which the lines of force are cut and, therefore, the voltage generated in the conductor in its movement through equal angles at different points in its circular path, are the differences in the lengths of the corresponding vertical lines $a_2\ c_2$ and $a_1\ c_1$, and $a_3\ c_3$ and $a_2\ c_2$, etc. The maximum voltage is generated when the conductor is moving near position a when it is cutting lines of force at a maximum rate, and the minimum voltage, when near position a_6 when it is not cutting across any lines.

In the second quarter of a revolution, the conductor is cutting down across the lines of force. The voltage is reversed and builds up to maximum in the reversed direction. In the third quarter, the voltage decreases to zero; and in the fourth quarter, the voltage is reversed and again is in its original direction and increases from zero to its maximum at position a. A voltage curve may be started at the zero point, the maximum point, or any intermediate point, but in order to indicate a cycle, a full set of changes in voltage must be shown.

With constant angular rotation of conductor a, the voltage at any instant depends on the position of the conductor in its circular path, and a curve, or wave, that indicates the voltage generated under the conditions just considered follows a sine wave, the construction of which is based on the sines of angles formed between a fixed line and a rotating line of constant length.

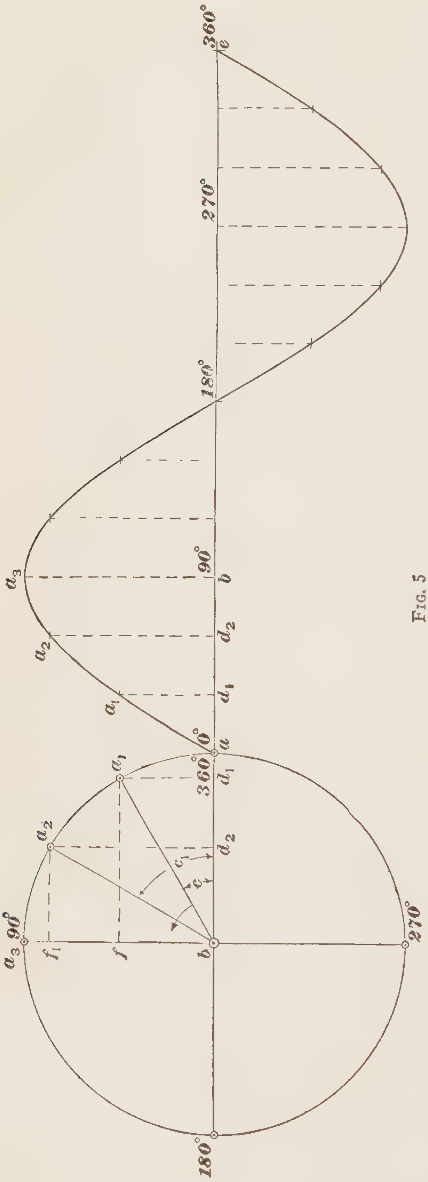


Fig. 5

14. Construction of a Sine Curve.—A sine curve may be developed as indicated in Fig. 5. A point *a* at the end of a rotating arm *a b* is moved at a uniform rate around the circumference, or 360°, of a circle of which *a b* is the radius. Consider positions of the arm for the first 90° of movement. When the point in moving through angle *c*, reaches position a_1, it has risen above the horizontal line a vertical distance represented by the line $a_1\ d_1$ which is proportional to the sine of angle *c*; $\sin\ c = \frac{a_1 d_1}{a_1 b}$. For the same reason, vertical line $a_2\ d_2$ is proportional to the sine of angle c_1; $\sin\ c_1 = \frac{a_2 d_2}{a_2 b}$. Vertical line $a_3\ b$ is proportional to the sine of 90°, which is 1, therefore the vertical distance above the horizontal line of point a_3 is equal to the length of the rotating arm. The point *a* in passing through the first 180° rises to a maximum distance above the horizontal line and then falls to minimum distance, or zero. The ordinates of this 180° movement are above the base line and can be considered positive. The point in moving from 180° to 360° repeats these variations in the values of the vertical distances from the base line, but as the point is now moving below the base line, the ordinates may be considered as negative.

The horizontal line $a\ b$ is extended to the right and laid off in equal parts such as $a\ d_1$, $d_1\ d_2$, etc. These lines on the extension are proportional to the equal angles passed through by the point a of the arm $a\ b$ in its movements to positions a_1, a_2, a_3, etc., that is, line $a\ d_1$ is proportional to angle c and line $d_1\ d_2$ is proportional to angle $a_2\ b\ a_1$. The total length $a\ e$ of the line will be proportional to the circumference of the circle, but not necessarily equal in length to the circumference. At the points on line $a\ e$ corresponding to the angles c, c_1, etc., vertical lines are erected that are equal in length to the corresponding vertical lines $a_1\ d_1$, $a_2\ d_2$, etc., of the circle diagram. Five of these lines will be above line $a\ e$ and five below it. A curve drawn through the ends of these lines will form a sine curve since the height of the curve at any point is proportional to the sine of the corresponding angle in the circle diagram. The curve is a record of the *simple harmonic motion* of the projection of the moving point a on a vertical plane as it passes through 360° of circular motion. For example, in the left-hand part of Fig. 5, the projection of point a_1 is f; and the projection of point a_2 is f_1, on the vertical line $a_3\ b$.

For equal angular movements of the point, the increases or decreases in vertical distances from the fixed line vary in value as indicated by points f, f_1, a_3, etc., or by the differences in distances from the base line of points a_1, a_2, a_3, etc. on the sine curve.

15. Average Value.—The average value of the ordinates of a sine curve is the average of all the ordinates of the curve for either the positive or the negative half cycle. For example, in the sine curve $a\ b\ c\ d$, Fig. 6, the average ordinate of the half wave $a\ b\ c$ would be that ordinate $a\ e$ which, multiplied by the base *a c*, would give a rectangle *a e f c* of the same area as the surface *a b c* of the half wave. If the whole sine curve is considered, the average value would be zero, because the average ordinate for the negative wave would be the equal and opposite of the average positive ordinate. In the case of a voltage curve based on a sine curve, the average value of the voltage during a cycle is always .636 times the maximum value of the voltage. The same relation obtains for the maximum and average values of current.

The exact proof of the relation between the maximum and average value of an alternating current or voltage involves the use of higher mathematics. The relation may, however, be verified approximately by taking a considerable number of the values of the sines of the angles from 0° to 90° from a trigonometric table, adding these together, and dividing by the number of values taken. The average so found will be nearly .636. The greater the number of angles considered, the nearer the result will be to .636. The value can be obtained with a fair degree of accuracy by considering the sines of every fifth degree, such as 0°, 5°, 10°, 15°, etc. If each of the 15° angles in Fig. 4 were divided into three equal angles, each small angle would be 5°, and the average value of the sines of the angles formed between the radial lines and the fixed line *a b* would be nearly .636. If the maximum value of the electromotive force is 1 volt, the average value of the electromotive force is the numerical value of the average of the values of the sines of a very large number of angles between 0° and 90°, or .636 volt. For any other maximum

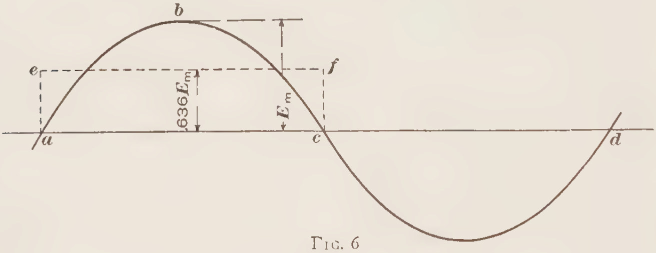


Fig. 6

value E_m, the average value of the electromotive force is .636 E_m volts.

16. Effective Value.—The *effective value* of an alternating current may be defined as that value *which will produce the same heating effect in a circuit as a continuous current of the same amount.* Thus, an alternating current has an effective value of 10 amperes when it produces the same heating effect as a direct current of 10 amperes. The effective value of alternating voltage is the value corresponding to the effective current, and bears the same relation to the maximum voltage as does effective current to maximum current. Voltmeters, ammeters, and wattmeters usually *indicate effective values.* The effective value of a sine-wave current or voltage is .707 times the maximum value.

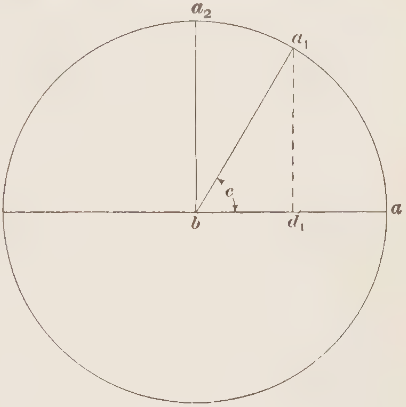


FIG. 7

17. On account of the importance of the effective value, the following proof is given of the relation between effective and maximum values. If a continuous current I be sent through a wire of resistance R, the wire becomes heated, and the power expended in heating the wire is $p = I^2 R$ watts, or is proportional to the square of the current. If an alternating current be sent through the same wire, the heating effect is at each instant proportional to the square of the current at that instant. The average heating effect will therefore be proportional to the average of the squares of all the different instantaneous values of the current, and the effective value of the current will be the square root of the average of the squares of the instantaneous values. The same proportional relation holds when comparing the effective and maximum values of an alternating voltage.

In Fig. 7 the arm $a\,b$, pivoted at b, is assumed to be moving in a circular path. Consider that the length of the arm is 1 inch; therefore, the maximum value of the sine of the angle between the rotating line and the fixed horizontal line is one divided by one, or 1. When the arm is in position $a_1\,b$, making an angle c with the horizontal line $a\,b$, the length of the vertical side $a_1\,d_1$, expressed in inches, of the right-angled triangle $d_1\,b\,a_1$ is numerically equal to the sine of angle c, since $\frac{a_1\,d_1}{a_1\,b=1}=\sin\,c$. The length of the side $b\,d_1$ is numerically equal to the cosine of angle c, since $\frac{b\,d_1}{a_1\,b=1}=\cos\,c$.

The square of the length of the hypotenuse, or $(a_1\,b)^2=\sin^2 c+\cos^2 c$, since the square of the hypotenuse of a right-angled triangle equals the sum of the squares of the other two sides. Therefore,

$$\sin^2 c+\cos^2 c=\frac{(a_1\,d_1)^2}{(a_1\,b)^2}+\frac{(b\,d_1)^2}{(a_1\,b)^2}=\frac{(a_1\,d_1)^2+(b\,d_1)^2}{(a_1\,b)^2};$$

but, since $(a_1\,d_1)^2+(b\,d_1)^2=(a_1\,b)^2$,

then, by substitution, $\sin^2 c+\cos^2 c=\frac{(a_1\,b)^2}{(a_1\,b)^2}=1$

As the line $a\,b$ revolves, the sine of the angles varies in value from 0 to $a_2\,b(=a_1\,b)$, or 1, which is the value represented by the total length of the arm, and the cosine varies from 1, or $a\,b\;(=a_1\,b)$, to 0. The sine and cosine pass through the same range of values; consequently, the average of the squares of the sine and of the cosine must be the same. Since as just shown

$$\sin^2 c+\cos^2 c=1$$

$$\text{average } \sin^2 c+\text{average } \cos^2 c=1$$

From the above statement of the relation of the average of the squares of the sine and cosine,

$$2 \text{ average } \sin^2 c=1$$

$$\text{average } \sin^2 c=\tfrac{1}{2}$$

$$\sqrt{\text{average } \sin^2 c}=\frac{1}{\sqrt{2}}$$

If an alternating voltage follows the sine law, the instantaneous value of the voltage e at any instant corresponding to the angle c, is $e=E_m \sin c$; hence,

$$e^2 = E^2{}_m \sin^2 c$$
$$\text{average } e^2 = E^2{}_m \text{ average } \sin^2 c$$
$$\text{average } e^2 = E^2{}_m \times \tfrac{1}{2} = \frac{E^2{}_m}{2}$$

Since average $\sin^2 c = \frac{1}{2}$,

$$\sqrt{\text{average } e^2} = \frac{E_m}{\sqrt{2}} = \frac{E_m}{1.414} = .707\, E_m$$

Then from the relation between E_e and the square root of the average of the squares of the instantaneous values of the voltage as stated in the first paragraph of this article,

$$E_e = .707\, E_m$$

18. In Fig. 8 the value of the square-root-of-mean-square ordinate is obtained graphically. The dotted curve is obtained by squaring the ordinates of the sine curve, and the area of the dotted rectangle is equal to the area enclosed by the curve of squares. The height of the ordinate of the rectangle is $\frac{E^2{}_m}{2}$ and it represents the average of all the values of e^2. The square root of $\frac{E^2{}_m}{2}$ is the effective value E_e of the voltage represented by the sine curve, and is $\frac{E_m}{\sqrt{2}} = .707\, E_m$.

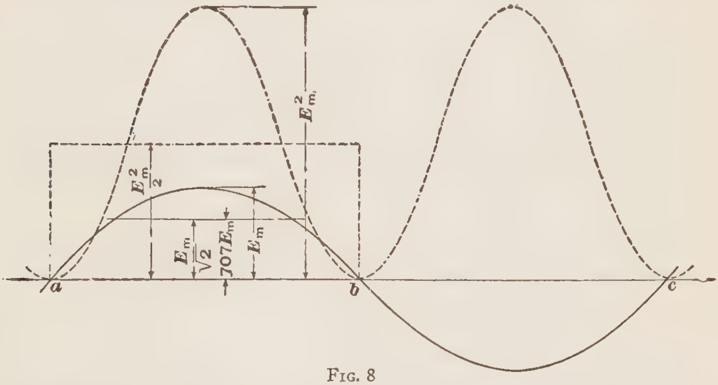


FIG. 8

PHASE RELATIONS

19. In Phase, Out of Phase.—When two alternating currents vary in exact unison, passing through corresponding instantaneous values, as zero, maximum, etc., together, the currents are said to be *in phase,* or *in step.* If they have the same frequency, but do not pass through corresponding instantaneous values together, they are said to be *out of phase.* The same statements apply to two alternating voltages or to a voltage and a current.

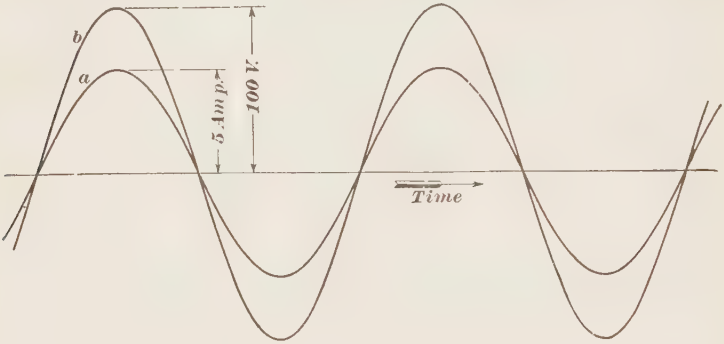


Fig. 9

In Fig. 9, the curve *a* represents a current of the maximum value of 5 amperes, and the curve *b,* a voltage of the maximum value of 100 volts; the current and voltage are in phase, or in step, with each other. Fig. 10 represents the same voltage and

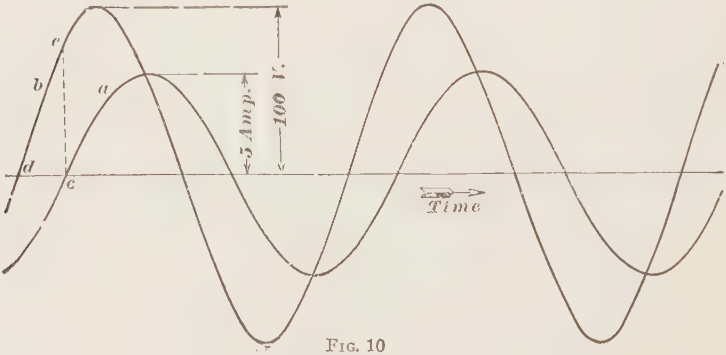


Fig. 10

current out of phase with each other; that is, corresponding instantaneous values, such as zero and maximum values, do not occur simultaneously. As time is measured along the horizontal axis in the direction shown by the arrow, point *c* represents a later instant than point *d*, or curve *a* does not start until curve *b* has reached the value represented by the vertical line *c e*. The current represented by curve *a* is therefore said to *lag* behind the voltage curve *b*, though it is just as correct to say that the voltage is *leading* the current.

20. Phase Difference.—The **phase difference** is usually expressed by the number of time-degrees between two regularly varying quantities, or variables, such as an alternating voltage and its resulting current. The phase difference is called an *angle of lag* when speaking of the lagging variable and an *angle of lead* when speaking of the leading variable.

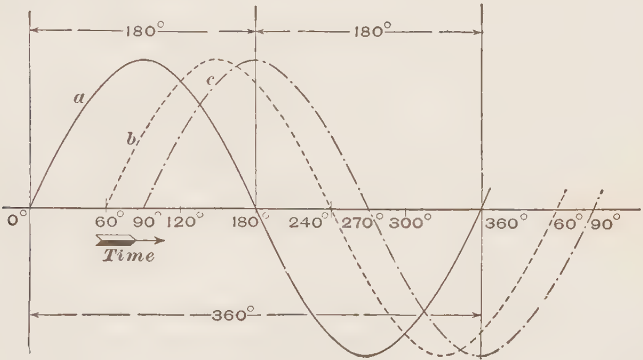


FIG. 11

For example, Fig. 11 shows a phase difference of 60 time-degrees between the voltage represented by the curve *a* and the current represented by the curve *b*; that is, the current lags behind the voltage by an angle of 60 time-degrees, and, similarly, the voltage leads the current by the same angle. The current represented by curve *c* lags behind the voltage *a* by an angle of 90 time-degrees, and behind the current *b* by an angle of 30 time-degrees. When the voltage *a* is maximum, current *c* is minimum. In practice it is usually understood that the current is lagging when the phase difference is referred to as the angle of lag, or that the current is leading when the phase difference is referred to as the angle of lead.

When the phase difference between two currents or voltages is 180 time-degrees, they are said to be in *direct opposition* to each other. The condition for *equal values* is shown in Fig. 12. When a voltage (or a current), represented by curve *a*, is exactly at its maximum *positive* point, the other voltage (or a current), represented by curve *b*, is at its maximum *negative* point; therefore, they counterbalance, or neutralize, each other and their combined effect on the circuit is zero.

21. Synchronism.—When the currents from two alternators have the same frequency and are in phase with each other, the two machines are said to be *in synchronism*. The process of adjusting the speeds of alternators for synchronism is called *synchronizing*. Alternators must always be synchronized and their voltages equalized before connecting them in parallel; the methods employed are explained in another Section.

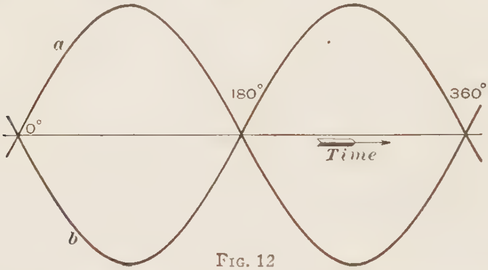


FIG. 12

EXAMPLES FOR PRACTICE

1. A voltmeter connected to an alternating-current circuit indicates 10,000 volts. What is the maximum voltage stress to which the insulation of the circuit is subjected? Ans. 14,140 volts, approx.

2. If the maximum value of an alternating electromotive force is 163 volts, what is the effective value? Ans. 115 volts

CALCULATIONS WITH WAVE FORMS

22. Addition of Sine Waves in Phase.—The method of adding two alternating currents or voltages that are in phase, and represented by sine curves, is shown in Fig. 13. One current has the maximum value I, and is represented by the dotted curve; the other current has the maximum value I_1, and is represented by the dot-and-dash curve. The curve representing the sum is shown in full line and is found by adding the instantaneous values of the separate currents, which may be called *components of the sum.*

For example, take any instant, say a on the zero axis. The ordinate, or vertical, $a\ b$ represents the instantaneous value of the current I, and the ordinate $a\ c$, the instantaneous value of the current I_1. The value of their sum at this particular instant is found by adding the ordinates $a\ b$ and $a\ c$, giving $a\ d$, and thus locating the point d. Similarly, other points at different instants are determined, and the resultant curve, shown by a full line, is drawn through these points, giving a curve with maximum value $I_2 = I + I_1$. This resultant curve is also a sine curve, and its position indicates that the resultant is in phase with the two components.

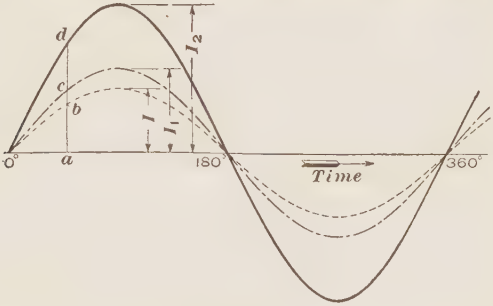


FIG. 13

23. Addition of Sine Waves Out of Phase.—The method of adding sine waves out of phase is shown in Fig. 14. Two alternating currents are represented by the curves I and I_1, current I lagging behind current I_1 by an angle represented by the distance $b\,a$ on the zero axis. In this case, at some instants, the ordinates of the two component curves have opposite signs, one positive and the other negative. Hence, when adding the ordinates, their positive or negative values must be considered; that is, when both component ordinates are positive or both negative, their sum is an ordinate of the resultant curve, as $c\,d + c\,e = c\,f$; again, when one component ordinate is positive and the other negative, their difference is the ordinate of the resultant curve. The resultant curve crosses the zero axis where the ordinates of curves I and I_1 are equal, but one is positive and the other is negative, hence the resultant ordinate is zero. The *algebraic sum* is the result obtained by addition in which the positive and negative signs of the values are considered. The variations of the resultant current I_2 follow a sine curve of the same frequency as the component curve, but the resultant current is not in phase with either of the other currents; its maximum value is less than the sum of the maximum values of I and I_1.

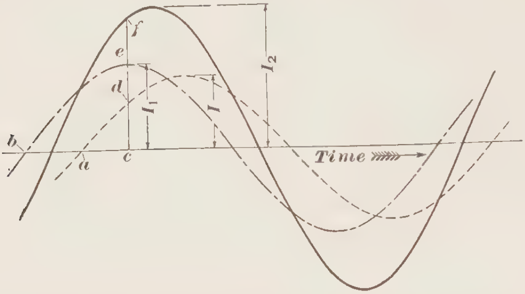


FIG. 14

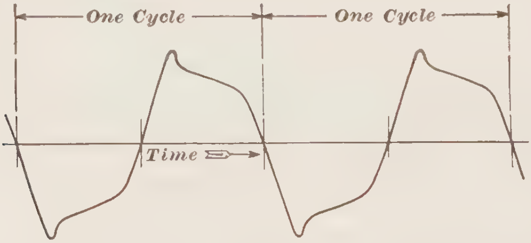


FIG. 15

24. Irregular Shapes of Waves.—The assumption that variations of an alternating current or voltage follow a sine curve is accurate enough for practical problems; in many cases, however, the actual shape of curves differs from a sine curve. For example, the shape of curves may, under certain conditions, be similar to those shown in Figs. 15 and 16.

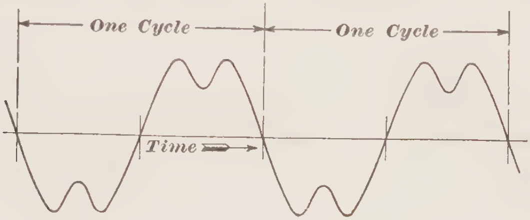


FIG. 16

Whatever may be the shape of the curve, it is in most cases symmetrical with regard to the zero axis; that is, the positive half wave has the same shape as the negative half wave. The shape of the wave depends on the shape of the poles and some other features of the design.

25. Harmonics.—Any distorted curve may be considered as a resultant of a sine curve of the *same frequency* as the distorted curve, combined with a number of other sine curves of *higher frequencies.* The sine wave of the same frequency as the distorted wave is called the *fundamental wave,* and the higher frequency components are called *harmonics.* Harmonics are distinguished by the ratio of their frequency to that of the fundamental. If double the fundamental frequency, they are

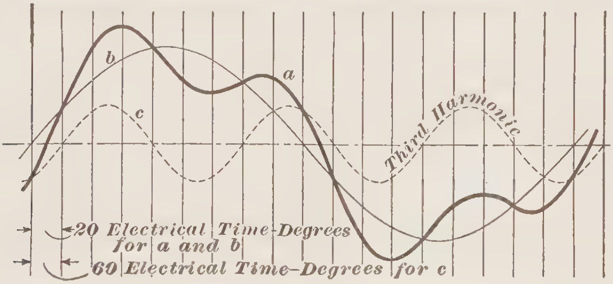

b
a
c
Third Harmonic
20 Electrical Time-Degrees for a and b
60 Electrical Time-Degrees for c

FIG. 17

called *second harmonics;* if three times, they are third harmonics; and so on. In electrical practice, harmonics are usually of the odd order, such as the third, fifth, etc.

Fig. 17 shows the resultant curve *a* consisting of the fundamental wave *b* and the third harmonic *c*. It should be noted that the third harmonic *c* passes through *three* complete cycles in the same time as required for either curve *b* or *a* to pass through one complete cycle. In Fig. 17, none of the curves are in phase. The ordinate of each point of curve *a* equals the algebraic sum of the ordinates of the corresponding instantaneous points of curves *b* and *c*. The relative magnitudes of the fundamental and the harmonics and their phase relation affect the shape of the resultant wave.

SELF-INDUCTION

EFFECTS OF SELF-INDUCTION

26. The fundamental principles of self-induction were explained in an earlier Section. The effects of self-induction on the phase relation of the voltage and current of an alternating-current circuit will here be considered.

The value of the magnetic flux around any conductor depends on the strength of the current. Any change in the current causes a corresponding change of flux, and the change of flux induces in the conductor an electromotive force in such a direction as to oppose and retard the change of current. Because of its opposing character, this induced electromotive force is called the *counter electromotive force of self-induction.*

When an alternating electromotive force increases from zero to maximum, this counter electromotive force prevents the simultaneous increase of the current. When the voltage decreases from maximum to zero, the counter electromotive force prevents the simultaneous decrease of the current. The increase or decrease of the current is thus retarded relative to the change of voltage, and the current reaches the maximum or zero values later than the voltage, or the current lags behind the voltage.

HYDRAULIC ANALOGUE OF SELF-INDUCTION

27. The hydraulic analogue shown in Fig. 18 may serve to explain the important phenomenon of phase difference between voltage and current in an inductive circuit. A wheel *a*, view (*a*), is rotated at uniform speed. A pin *b*, mounted on the wheel, engages with a link *c* on an extension of the piston rod *d* of a pump. As the wheel rotates, the link, the rod, and the piston *e* are moved toward the right or left in a straight line. Below the pump and connected to it by short pipes is a cylinder containing a heavy unattached piston *f*. Both the pump and cylinder are filled with water, and as the piston *e* moves, the rush of water into and out of the lower cylinder causes the unattached piston *f* to move in exact unison with the movements of piston *e*; that is, at any instant both pistons move at the same velocity and they reverse their direction of movement at the same instant. This simultaneous action occurs because water is practically incompressible; therefore, if piston *e* moves to the right, piston *f* will move to the left.

The motion imparted to both pistons by the pin and link is known as a rectilinear simple harmonic motion. In view (*b*), the path of the pin *b* is indicated for a movement of 180°, divided into divisions of 10°. It should be noted that when pin *b* is at or near the position shown in views (*a*) and (*b*), an angular movement of 10° causes the pin, link, and piston to move a greater distance in a horizontal direction than when the pin is at or near positions *b′* or *b″*, view (*b*). The linear, or straight-line, movements for the 10° angular movements are indicated by the horizontal distances between the vertical lines drawn from the ends of the 10° arcs of the half circle.

The velocity of movement of the piston is greatest when the piston is at the middle of the cylinder; its retardation or acceleration near or at this point is, however, very small or zero, since the speed is changing very slowly, as shown by the fact that the distances *0–1*, or *1–2* to the left of the central point *0*, or *0–3*, or *3–4* to the right, are nearly equal. These distances, however, indicate a high velocity as they are nearly equal to the 10° arcs.

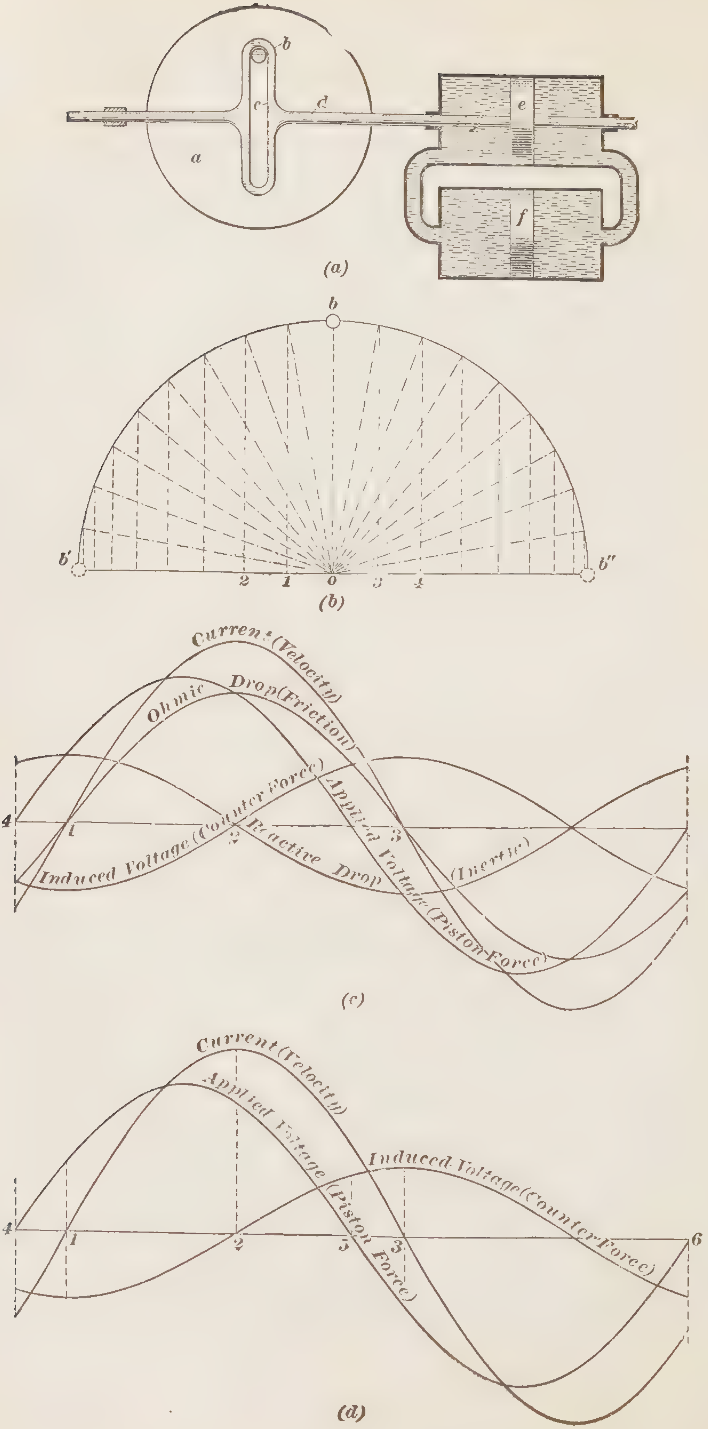

a
b
c
d
e
f
(a)
b
b′
b″
2
1
0
3
4
(b)
Current (Velocity)
Ohmic Drop (Friction)
Induced Voltage (Counter Force)
Reactive Drop (Inertia)
Applied Voltage (Piston Force)
4
1
2
3
(c)
Current (Velocity)
Applied Voltage (Piston Force)
Induced Voltage (Counter Force)
4
1
2
5
3
6
(d)

FIG. 18

When the piston is nearing or just leaving the end of the cylinder, the horizontal movements for 10° angular movements are small and the velocity of the piston is very low, but the change in velocity, either retardation or acceleration, is maximum as indicated by the considerable differences in the widths of the spaces between any two of the adjacent vertical lines near *b′* or *b″*.

28. A certain amount of friction between the pipes and the water is assumed, but the water is here considered as devoid of inertia and the inertia of the whole circuit concentrated in the heavy piston *f*, Fig. 18 (*a*). The velocity of the water is analogous to the velocity or rate of flow of electricity (current) in the electric circuit; the friction between the flowing water and the pipes, to the ohmic resistance; and the inertia of the piston *f*, to the inductance of the circuit.

The pump corresponds to the alternating-current generator in an electric circuit, and the lower cylinder with its piston *f* corresponds to a circuit containing an electromagnet or other highly inductive device. The force exerted on the water and piston *f* by the piston *e* rises and falls in value and reverses in direction. This is analogous to the rise and fall and reversal of the electromotive force of an alternator.

It requires the application of force to accelerate the movement of any body, and the piston *e* must exert the force that, acting through the body of water, is required to accelerate the piston *f*. Energy is stored in the moving piston *f* and this piston will exert a counter force against any force that tends to retard its motion or to reverse its direction of movement. The piston *f* will, therefore, at certain parts of a cycle really act as a prime mover or generator and it will supply energy to the pump. At times, therefore, in a cycle, the movement of piston *e* is caused by the force due to the momentum of piston *f* which overpowers the outside, or applied, piston force. In the electric circuit, **there are times during the cycle** when the electromotive force of the generator is less than the counter-electromotive force of self-induction, and the action of the changing magnetic flux of the inductive device in the external circuit is then similar to that of a generator, since the electromotive force of self-induction thus set up really forces current through the circuit.

The force exerted by piston *e* on the water and on piston *f* is the resultant of two components, one to overcome the friction between the water and the pipes and the other to change the velocity of piston *f*. In a similar manner, the electromotive force applied to an inductive circuit is the resultant of two components, one to force current against the ohmic resistance and the other to overcome the electromotive force of self-induction.

29. The change in the velocity of the water or the instantaneous variation in the value of an alternating current is represented by the curve marked *Current* in Fig. 18 (*c*). The rise and fall of the magnetic flux set up by the current also follows quite closely the general form of the *Current curve*. When the piston *e*, view (*a*), is at the end of the pump cylinder, the velocity of the water is zero, since for an instant the piston does not move. The frictional reaction is also zero because the water is not moving. The curve marked *Ohmic Drop*, view (*c*), represents the component of the applied piston force that is required to equalize and balance the counter force due to friction. Care should be taken in distinguishing between a counter force and the force 180 time-degrees out of phase with it which is the component of the applied force necessary to neutralize the counter force. In the electric circuit, the ohmic drop is the component voltage necessary to send current through the circuit against the ohmic resistance of the circuit. The amount of frictional reaction increases and decreases with the corresponding changes in the velocity of the water and, therefore, the *Current curve* and *Ohmic-Drop curve* are in phase.

If the piston *f*, view (*a*), were removed and only friction were considered, the pump would act as a generator throughout the entire cycle and the *Applied-Voltage curve*, or *Piston-Force curve*, view (*c*), would then be identical in value and phase relation with the *Ohmic-Drop curve*.

If a constant turning force were applied to the wheel *a*, view (*a*), the force of the piston *c* when the piston is at the end of the pump would be zero because the driving pin at positions *b′* or *b″*, view (*b*), is moving at right angles to the horizontal movement which the piston rod must make. When the piston is at the middle of its stroke, the driving pin is in position *b*, view (*b*), and the piston *e*, view (*a*), has maximum force applied to it because the circular movement of the pin is nearly identical with the straight horizontal movement which the piston rod must make.

30. At the end of the stroke of piston *e*, Fig. 18 (*a*), the acceleration or retardation of piston *f* is maximum; therefore, the counter force of piston *f* is maximum, that is, the piston *f* exerts its greatest counter force against being made to suddenly change its velocity. The curve marked *Induced Voltage*, view (*c*), has its maximum negative value when the *Current curve* is passing through zero to a positive value.

At the middle of the stroke of piston *e*, view (*a*), the acceleration or retardation of piston *f* is zero; therefore, the counter force of piston *f* is zero and the *Induced-Voltage curve*, view (*c*), crosses the zero line at the point above which the *Current curve* has its positive maximum value. The two curves differ in phase by 90 time-degrees.

In an electric circuit, the counter electromotive force of self-induction is maximum when the current is passing through zero, because then the magnetic flux is changing most rapidly, and zero when the current is maximum because the flux for an instant is not changing. This induced voltage opposes change in current, either increase or decrease, as indicated by the comparison of the instantaneous positive and negative values of the *Induced-Voltage curve* and the *Current curve*, view (*c*), during a cycle.

The component force of the applied piston force that is necessary to overcome this counter force due to inertia of piston *f*, view (*a*), must be exactly equal and opposite; therefore, the *Reactive-Drop curve*, which represents this component, is 180 time-degrees out of phase with the *Induced-Voltage curve*.

31. Consider the piston force when the inertia reaction, indicated by the curve of the *Reactive Drop,* Fig. 18 (*c*), is assumed to be the active external force applied to the piston. In this case, the *Ohmic-Drop curve* is not considered. The *Applied-Voltage curve* would then be identical in value and phase relation with the *Reactive-Drop curve.* The pistons *e* and *f,* view (*a*), pass through the same range of velocity values as considered in the case of the pump having only frictional reaction and indicated by the *Current curve,* view (*c*), but the points of the cycle where maximum and minimum piston force are applied by piston *e,* view (*a*), to the water and piston *f,* in the case considering inertia reaction, are not in phase with the corresponding points in the first case. The action of the wheel *a,* pin *b,* and link *c* need not be considered in the following explanation.

In the case under consideration, maximum force is applied by piston *e* to the water when the piston *e* is at the end of the stroke, as indicated by a comparison of the instantaneous values of the curves of *Reactive Drop* and *Current,* view (*c*). As the piston *e,* view (*a*), moves to the middle of the cylinder, the force drops to zero. The force at the central point reverses in direction, and from this point to the other end of the pump cylinder, the piston force is in opposition to the actual movement of the piston. Note that from *1* to *2,* view (*c*), the two curves under consideration are both positive, while from *2* to *3* the *Reactive-Drop curve* is negative while the *Current curve* is still positive. The same amount of energy furnished by piston *e,* view (*a*), to move the piston *f,* during the first half of its passage through the lower cylinder is given up by piston *f* to keep piston *e* moving in the last half of the stroke of piston *e* in the upper cylinder. Note that from *2* to *3,* view (*c*), the *Induced-Voltage curve* and the *Current curve* are both positive. From *1* to *3* the energy furnished by the piston *e,* view (*a*), to piston *f* is exactly balanced by the energy received from piston *f* to move piston *e*; therefore, the total work done is zero. This is an impossible condition, since all moving bodies involve friction and all electric circuits, ohmic resistance. If a circuit possessing inductive reactance but no ohmic resistance were possible, energy would pass from the generator to the circuit at parts of the cycle and the equivalent energy pass from the circuit to the generator at other parts of the cycle.

32. The total force applied to the piston *e*, Fig. 18 (*a*), or the voltage of the generator in an electric circuit, is the resultant of the addition of the ordinates of the two component forces indicated by the curves *Ohmic Drop* (*Friction*), view (*c*), and *Reactive Drop* (*Inertia*). The resultant curve is marked *Applied Voltage* (*Piston Force*), and it leads the *Current curve* by the number of time-degrees represented by the space *4–1*. In other words, the current lags in phase behind the voltage. Considering the movement of the piston *e*, view (*a*), from the left end toward the right, the force applied to the water by the piston *e* is maximum at an intermediate point between the left end and the middle of the stroke. The force of piston *e* becomes zero and reverses in direction before the piston *e* itself has reached the right end of the cylinder. During parts of the cycle, the piston *e* furnishes energy to move piston *f*, and at other parts of the cycle the energy of piston *f* drives piston *e*. As the angle of lag is less than 90 time-degrees, more energy is furnished by piston *e* than is returned to the pump by piston *f*.

If the values of ohmic resistance and of inductance of a circuit are changed, the shapes of the component curves are altered and the phase difference of the applied voltage and current are altered. If the frictional reaction or the inertia reaction are changed in value, the point in the stroke at which the piston *e* exerts maximum force is also changed.

If the frequency of the alternating voltage impressed on the circuit is increased, the rate of change in the magnetic flux increases and the reactive drop will be larger. The angle of lag of the current behind the applied voltage will then be greater.

33. In Fig. 18 (*d*), the three curves shown have been redrawn from view (*c*). The instantaneous values of the *Induced-Voltage curve* and of the *Applied-Voltage curve*, view (*d*), should be compared at several points of the cycle. At point *4*, the applied voltage is zero so that the negative induced voltage forces current through the circuit in the negative direction. The value of this instantaneous current is indicated by the negative ordinate to the *Current curve* at point *4*. From point *4* to *1*, the negative induced voltage predominates and produces the negative current. At point *1*, these two opposing forces are equal, therefore they neutralize each other and the current is zero. The positive applied voltage now predominates and the current changes to positive direction and rises to maximum at point *2*. At this point, the induced voltage becomes positive and assists the decreasing applied voltage in producing a positive current. When the applied voltage becomes zero and then negative between points *5* and *3*, the current still remains positive because the positive instantaneous values of the induced voltage exceed the negative instantaneous values of the applied voltage. At point *3*, the two opposing forces are equal and the current passes through zero to a negative value. The current is then negative to the end of the cycle at point *6* because of the predominating values of the negative applied voltage or because both the induced voltage and the applied voltage are negative, as indicated near the right end of view (*d*).

This discussion indicates that both the induced voltage and the applied voltage are active in forcing current through the circuit. These voltages also determine the point at which the current reverses, and this point in comparison with the point at which the applied voltage reverses determines the angle of lag of the current.

In any circuit in which the voltage and current are out of phase with each other, the generator supplies electrical energy to the circuit during part of a cycle, and in other parts of the cycle the inductive circuit is supplying energy to the generator. Further discussion of the power in alternating-current circuits is taken up later.

INDUCTANCE OF A CIRCUIT

34. Relation Between Inductance and Reluctance. The **inductance** of a circuit is the ability with which magnetic flux can be established around it by an electric current. Inductance therefore depends on the reluctance of the path for the magnetic flux due to the current, and the induced counter-electromotive force depends on the rapidity with which that flux varies.

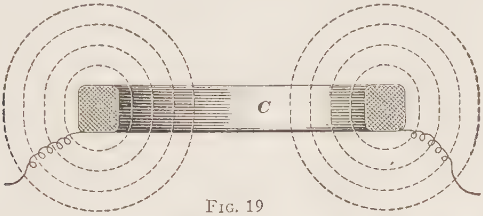


Fig. 19

For example, in a coil of wire *C*, Fig. 19, surrounded by air, the magnetic circuit is of high reluctance and the value of the flux is correspondingly small; the inductance of such a coil is therefore low. If the coil is surrounded by magnetic material *M*, Fig. 20, the value of the flux set up by the same current is considerably greater, and the inductance therefore higher. If the flux in both coils is allowed to decrease to zero in 1 second, the rate of change in the number of lines in the second coil will be greater than that in the first coil because of the greater flux. Since induced voltage is proportional to the rate of change of magnetic flux, greater counter electromotive force is included in the second coil.

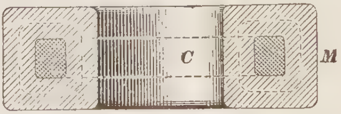


Fig. 20

The inductance of a given piece of apparatus may be considered constant when air or any material of constant permeability surrounds the conductors. If iron is present, as in case of the coil shown in Fig. 20, the inductance will be nearly constant until the magnetizing current reaches such a value that the magnetization approaches the saturation point. Circuits that have practically no inductance are known as *non-inductive circuits;* liquid resistances and incandescent lamps are practically non-inductive.

35. The Henry.—The *unit of inductance* is the **henry;** 1 henry is the inductance of a circuit in which current change at the rate of 1 ampere per second will induce 1 volt of counter electromotive force. A coil of one turn has an inductance of 1 henry when a current of 1 ampere passing through it sets up a flux of 100,000,000, or 10^8, lines of force. If the current in such a coil is uniformly changed by the increase or decrease of one ampere during 1 second, the flux through the coil will be changed by 10^8 lines of force, and when the flux through a circuit changes at the rate of 10^8 lines of force per second, 1 volt will be generated.

The henry is too large for measuring inductances found in practice, for which reason the **millihenry,** $\frac{1}{1000}$ henry, is much used. In problems, however, the value of inductance must always be expressed in henrys on account of the relation of the henry to the ampere and the volt. Thus, in calculations, 20 millihenrys should be expressed as .02 henry. The number expressing the inductance of a circuit is called the *coefficient of self-induction.*

36. Inductive Reactance.—The **inductive reactance** of an alternating-current circuit is that part of the apparent resistance caused by inductance, and is always expressed in ohms. Inductive reactance not only limits the current in the circuit, but also causes it to lag behind the voltage. The value of inductive reactance is 2 π times the product of the frequency and the inductance, π being equal to 3.1416.

Let X_s = inductive reactance, in ohms;
f = frequency, in cycles per second;
L = inductance, in henrys.

Then,
$$X_s = 2\,\pi\,f\,L$$

The constant 2 π is used in the formula because of the relation between the maximum, average, and effective values of a sine curve. The maximum flux Φ (Greek letter pronounced *fi*) of a coil cuts across the turns of wire of the coil four times during each cycle. The flux increases from zero to positive maximum, then decreases to zero, increases to negative maximum, and then decreases to zero. The average value of the instantaneous voltages induced in the coil is equal to the average number of lines of force cut per second divided by 10^8. If the coil has N turns in series and the frequency is f cycles per second, the average number of lines of force cut by all of the turns per second will be $4\ \Phi\ N f$ and the value of the average volts induced will be

$$E_a = \frac{4\,\Phi N f}{10^8}$$

Since the value of the effective volts is equal to 1.11 times that of the average volts,

$$E_e = \frac{4 \times 1.11\,\Phi N f}{10^8}$$

The inductance of a coil expressed in henrys is,

$$L = \frac{\text{maximum flux} \times \text{number of turns}}{\text{maximum current} \times 10^8}$$

If I_e is the effective current, the maximum current I_m must be 1.414 I_e, or $\sqrt{2}\, I_e$, then

$$L = \frac{\Phi \times N}{\sqrt{2}\, I_e \times 10^8}$$

$$\Phi N = \sqrt{2}\, I_e \times 10^8\, L$$

From the relation $E_a = .636\, E_m$, or $E_m = \frac{E_a}{.636}$, and $E_e = .707\, E_m$, the value $E_e = .707 \times \frac{E_a}{.636} = \frac{.707}{.636} E_a = 1.11\, E_a$. The constant $.707 = \frac{1}{1.414} = \frac{1}{\sqrt{2}}$ and the constant $.636 = \frac{2}{\pi}$, and $1.11 = \frac{.707}{.636} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\pi}}$.

Note the term 1.11 ΦN as used in the formula for E_e; note also the formula for ΦN; then the term $1.11\,\Phi N = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\pi}} \times \sqrt{2}\, I_e \times 10^8 L$

$=\frac{\pi I_e \times 10^8 L}{2}$. Substituting this value for the term in the formula for E_e, gives

$$E_e=\frac{4\times\frac{\pi I_e\times 10^8 L}{2}\times f}{10^8}$$

and simplifying,

$$E_e=2\,\pi\, f\, L\, I_e$$

$$\frac{E_e}{I_e}=2\,\pi\, f\, L$$

Consider a coil of high inductive reactance and negligible ohmic resistance, then by a modified form of Ohm's law the value $\frac{E_e}{I_e}=X_s$, which is the inductive reactance, in ohms, and

$$X_s=2\,\pi\, f\, L$$

Example 1.—On a 60-cycle circuit, what is the inductive reactance of a coil having an inductance of 25 millihenrys?

Solution.— 25 millihenrys $=.025$ henry.

$X_s=2\times 3.1416\times 60\times .025=9.425$ ohms. Ans.

Example 2.—If the inductive reactance of an electrical device is 5 ohms on a 25-cycle circuit, what is its inductance?

Solution.— $5=2\,\pi\times 25\times L$, or

$L=\frac{5}{2\,\pi\times 25}=\frac{5}{157.1}=.032$ henry, or 32 millihenrys (nearly). Ans.

37. Inductive Reactance Voltage Drop.—The **inductive reactance voltage drop** is the applied voltage that is necessary to overcome the induced counter electromotive force in an inductive circuit and this voltage drop differs in phase from the counter electromotive force by 180 time-degrees. This voltage drop is found by a formula similar to that used for finding voltage drop in direct-current circuits, namely,

$$U_s=I\,X_s$$

in which U_s = inductive reactance voltage drop, in volts;
I = alternating current, in amperes;
X_s = inductive reactance, in ohms.

EXAMPLE.—A device having an inductance of .02 henry and a full-load current rating of 16 amperes is connected with a 60-cycle circuit; what is the applied voltage necessary to overcome the counter-electromotive force of the device at: (*a*) one-fourth load? (*b*) one-half load? (*c*) three-fourths load? and (*d*) full load?

SOLUTION.—According to the formula for inductive reactance of Art. **36**,

$$X_s = 2\times3.1416\times60\times.02 = 7.54 \text{ ohms, approx.}$$

According to the formula of this article,

(*a*) $U_s = 4\times7.54 = 30.16$ volts. Ans.
(*b*) $U_s = 8\times7.54 = 60.32$ volts. Ans.
(*c*) $U_s = 12\times7.54 = 90.48$ volts. Ans.
(*d*) $U_s = 16\times7.54 = 120.64$ volts. Ans.

38. Phase Relations in Inductive Circuits.—Fig. 18 (*c*) indicates the phase relations in a circuit possessing both ohmic resistance and inductive reactance. In Fig. 21 the circuit is assumed to have high inductive reactance and a negligible amount of ohmic resistance. Curve *a* represents an alternating current, and curve *b* the magnetic flux due to that current. The flux increases and decreases simultaneously with the current; hence, the current and the flux caused by it are in phase. The flux varies most rapidly when passing its zero values, or points *c* and *d* on curve *b*, as can be seen by considering two equal time intervals *e d* and *f g*; from *e* to *d*, the change of flux is considerable, while from *f* to *g* the number of lines is nearly constant. Consequently, when the current and the flux pass through zero, the counter electromotive force is maximum, making the phase difference between the current and the counter electromotive force 90 time-degrees; that is, the counter electromotive force lags 90° behind the current and is represented by the curve *h*. The applied voltage to overcome this counter voltage must be in direct opposition to it and of the

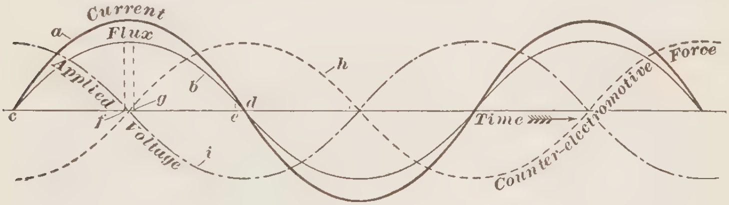


FIG. 21

same effective value; that is, the reactive voltage drop, where the purely theoretical condition of no ohmic resistance is considered, is 90 time-degrees ahead of the current and is represented by the curve *i*, 180 time-degrees from the counter-voltage curve *h*, or in direct opposition to it. In actual practice it is impossible to have a circuit without some ohmic resistance; therefore, the current will not lag just 90 time-degrees behind the applied voltage, and the applied voltage and counter electromotive force will not be exactly 180 time-degrees out of phase with each other. In a circuit there must be differences between the instantaneous corresponding values of the applied voltage and the counter electromotive force in order to establish a current.

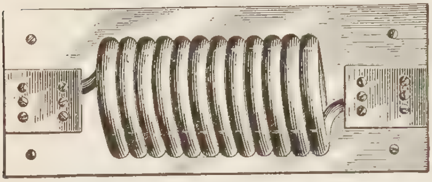


FIG. 22

In case the circuit possessed considerable ohmic resistance as well as inductive reactance, the angle of lag of the current behind the applied voltage would be less than 90 time-degrees, as, for example, in the case indicated in Fig. 18 (*d*).

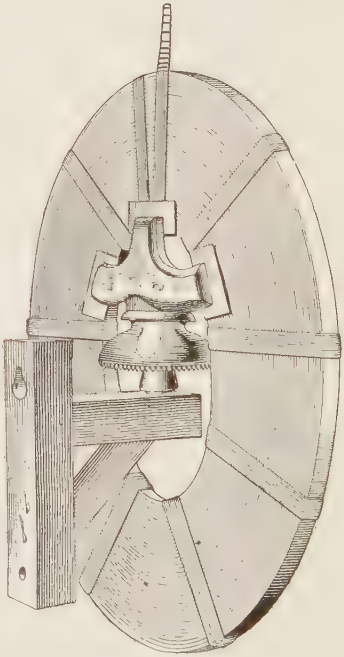


FIG. 23

39. Choke Coils.—Every circuit is at times subject to high voltage stresses of very high frequency that are caused mainly by lightning discharges or by the voltages of self-induction induced by any sudden and considerable change of current, as when opening or closing a switch. To lessen the chance of damage to apparatus by these voltage stresses, inductive devices, known as *choke,* or *reactance, coils,* are connected in series with the apparatus.

Choke coils are coils of wire, of either the solenoid or the spiral type, shown in Figs. 22 and 23, with insulation between the turns. In some choke coils the wire is bare and the turns are separated from each other. Air then acts as the insulator. The inductive reactance of these coils for ordinary frequencies is almost negligible; but with high-frequency currents, the reactance greatly increases, thus protecting the apparatus. Choke coils wound on iron cores are connected in series with some devices, for example, arc lamps, to reduce the voltage across the device; part of the applied voltage across the choke coil and the device is required to overcome the counter electromotive force induced in the reactance coil. Choke coils, called *reactors,* consisting of a few turns of heavy conductor mounted on a non-magnetic core, are sometimes used in series with large alternators to lessen the harmful effect of a short circuit.

EXAMPLES FOR PRACTICE

1. A choke coil having an inductance of 59 millihenrys is connected to a 60-cycle system. What is the inductive reactance of the coil?
Ans. 22.24 ohms

2. What is the voltage across the terminals of a choke coil having an inductive reactance of .01 ohm, when the current through the coil is 800 amperes?
Ans. 8 volts

CAPACITY

EFFECTS OF CAPACITY

40. Alternating voltage applied to a condenser alternately charges and discharges it; an ammeter connected in the circuit indicates the presence of current though the plates of the condenser are actually insulated from each other. This current is known as a charging current, because it is the current that is alternately taken up and discharged by the condenser. A long aerial transmission line or a long line of underground cable acts as a condenser of considerable capacity; if an alternating voltage is applied to either of these condensers, the ammeter will indicate a charging current, even though there is no load connected to the line and no appreciable leakage from the conductors to the ground.

When charging a circuit containing capacity effect only, the nature of the counter electromotive force is such that the charging current leads the applied voltage by 90 time-degrees. In this characteristic, capacity effect is exactly the opposite of inductive effect, because the latter causes the applied voltage to lead the current by 90 time-degrees. When both inductance and capacity are present in a circuit, their effects tend to balance each other.

HYDRAULIC ANALOGUE OF CAPACITY

41. The hydraulic analogue shown in Fig. 24 may serve to explain why the current leads the voltage in a circuit possessing large capacity as compared with inductance. The pump *a*, Fig. 24 (*a*), (*b*), (*c*), corresponds to an alternator in an electric circuit. The piston *b* is moved by the wheel and link in the manner explained in connection with Fig. 18. Below the pump, Fig. 24 (*a*), (*b*), (*c*), is placed a cylinder *c* which corresponds in its function to a condenser in an electric circuit. The *elastic partition d* of cylinder *c* corresponds to the dielectric of a condenser. The pipes and cylinders are filled with water. It is assumed that the moving parts of the machinery and the water have negligible inertia. A small amount of friction between the pipes and the moving water is assumed and this is analogous to ohmic resistance.

As the piston *b* moves to the right or left, water is forced into one end or the other of cylinder *c*, and the elastic partition *d* is distorted toward the opposite end of the cylinder. Force must be exerted by the piston to distort the partition, since the stretched partition exerts a counter force, due to its *elasticity*, that tends to move it to its straight, or undistorted, position. The part of the piston force required to distort the partition opposes the counter force.

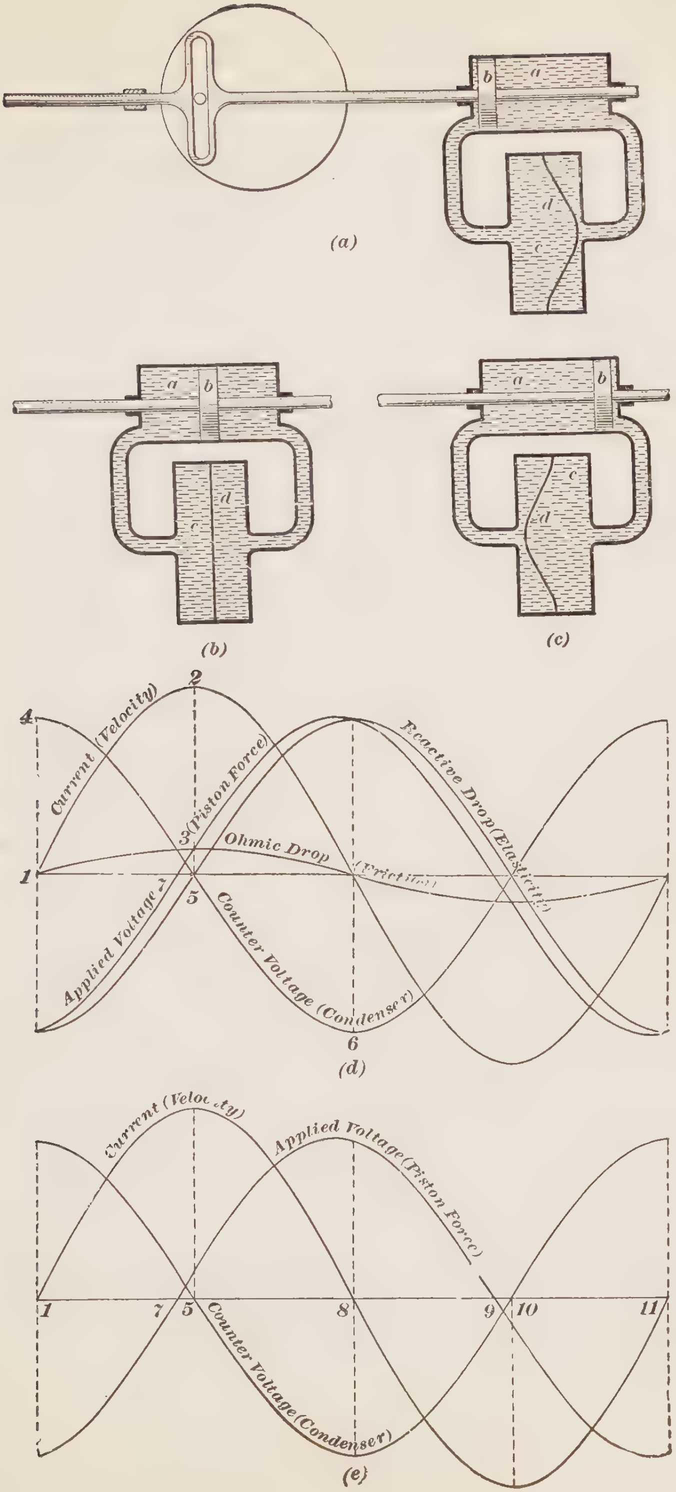

b
a
d
c
(a)
a
b
d
c
(b)
a
b
c
d
(c)
2
4
Current (Velocity)
3 (Piston Force)
Ohmic Drop
(Friction)
Reactive Drop (Elasticity)
1
Applied Voltage 7
5
Counter Voltage (Condenser)
6
(d)
Current (Velocity)
Applied Voltage (Piston Force)
1
7
5
8
9
10
11
Counter Voltage (Condenser)
(e)

FIG. 24

The amount of water shifted by the movement of the piston from its central position to either end of the pump is analogous to the electric charge, in coulombs of electricity, received by a condenser. The amount of water shifted by the partition in returning to its normal from its most distorted position is analogous to the quantity of electricity given up by the discharge of a condenser.

42. As explained in connection with Fig. 18, the velocity of the water is zero when the piston is at the ends of the pump, Fig. 24 (*a*) and (*c*), and maximum when the piston is at the center of its stroke, view (*b*). The *Current curve,* view (*d*), indicates the changes in the velocity of the water, or the variations in the current in the electric circuit. Point *1* represents zero velocity when the piston is at the end of the stroke, view (*a*).

The *Ohmic-Drop curve,* view (*d*), represents the part of the force, applied to the piston, that overcomes the effect of friction. In the electric circuit it represents the part of the applied voltage that overcomes ohmic resistance of the connecting wires from the generator to the condenser terminals. The current and ohmic drop are in phase because both the *Current curve* and the *Ohmic-Drop curve* must pass through zero at point *1* and have maximum values at points *2* and *3*, respectively, which are in a vertical line. These maximum values are obtained when the piston is moving at its greatest velocity, which occurs at the center of the stroke, view (*b*).

When the piston, view (*a*), is moving toward the left from its central position and is approaching the end of the stroke, practically all of the water that was to the left of the piston has been forced into the left end of the cylinder and the partition is greatly distorted to the right. It, therefore, requires a considerable force applied to the piston to force any more water against the partition when greatly distorted because of the nearly maximum value of the counter force due to the elasticity of the partition. At the end of the stroke, view (*a*), the *Counter-Voltage curve,* view (*d*), is maximum, as indicated at point *4*.

When external force applied to the piston starts to move it from the left end toward the center position, view (*b*), the counter force due to the tendency of the partition to straighten itself also tends to move the piston toward the center. This is indicated by the first quarter of a cycle, view (*d*), of both *Counter-Voltage curve* and the *Current curve,* which are both positive. The counter voltage decreases in value as the partition approaches its normal undistorted position at the center of the cylinder *c,* view (*b*). The velocity of the piston and of the water is maximum when the piston is at the middle of its stroke, due to the motion of the piston imparted by the pin and link, but the *Counter-Voltage curve,* view (*d*), is zero (point 5) because the partition when in normal, or undistorted, position does not exert any force on the water.

As the piston moves from the central position to the right end of the pump, view (*c*), the velocity of the water decreases, but since the partition is now being distorted toward the left, its counter force is increasing and this force acts toward the right in cylinder *c* and toward the left in pump *a* or in opposition to the movement of the piston. This condition is indicated in the second quarter of a cycle, view (*d*), by the negative *Counter-Voltage curve* and the positive *Current curve.* At the end of the piston stroke, view (*c*), the value of the counter voltage is negative maximum, view (*d*), (point 6). The current and ohmic drop are both zero at this point in the cycle.

43. The *Reactive-Drop curve,* Fig. 24 (*d*), represents the part of the piston force necessary to overcome the counter voltage and is equal to, but 180 time-degrees out of phase with, the *Counter-Voltage curve.* The addition of the corresponding instantaneous values of the *Reactive-Drop curve* and of the *Ohmic-Drop curve* is indicated by the *Applied-Voltage curve,* which represents the variations in the total force applied by the piston or the electromotive force applied to an electric circuit. The *Applied-Voltage curve* slightly leads the *Reactive-Drop curve,* but the *Current curve* leads the *Applied-Voltage curve* by nearly 90 time-degrees. The time-degrees of the angle of lead is represented by the distance along the zero axis between points *1* and *7*. The less the value of the ohmic resistance as compared to the capacity of a circuit, the nearer 90 time-degrees the current will lead the applied voltage.

44. It is of interest to note the action of the applied voltage and of the counter voltage as determining the direction of the current. The curves in Fig. 24 (*c*) are similar to those of the same titles in view (*d*). At point *1*, view (*e*), the instantaneous values of the positive counter voltage and the negative applied voltage are equal and opposite and the ohmic drop at this instant is zero, view (*d*). The current is, therefore, zero. From *1* to *7*, view (*c*), the positive counter voltage exceeds the negative applied voltage and forces the current in a positive direction. From *7* to *5*, both voltages are positive and the current passes through positive maximum when the counter voltage passes through zero at point *5*. From *5* to *8*, the positive applied voltage exceeds the negative counter voltage and the current is positive. At *8*, the two opposing forces neutralize and the current passes through zero. From *8* to *9*, the negative counter voltage exceeds the positive applied voltage and produces a negative current. From *9* to *10*, both forces are negative and the current is negative maximum below point *10*. From *10* to *11*, the negative applied voltage exceeds the positive counter voltage and the current is negative. At *11*, the end of the cycle, the forces are equal and opposite and the current is zero.

45. Self-induction in a circuit tends to retard any change, either of increase or decrease, in the existing current in a circuit. For example, in Fig. 18 (*d*), the induced voltage is always negative when the positive current is *increasing;* positive when the negative current is *increasing;* positive when the positive current is *decreasing;* and negative when the negative current is *decreasing.* Capacity in a circuit, however, has directly the opposite effect of self-induction. In Fig. 24 (*e*), the counter voltage tends to increase the rate at which the current is increasing and to increase the rate at which the current is decreasing, since the counter voltage and the current are both either positive or negative when the current is increasing and one is positive and the other negative when the current is decreasing.

46. The pump *a*, Fig. 24 (*a*), (*b*), and (*c*), may be considered as forcing water into the cylinder *c* during parts of the cycle and in other parts of the cycle the action of the partition in the cylinder is to force water into the pump. The movement of the piston is due to the action of both the force impressed on it by its driving mechanism and by the force exerted by the elastic partition. Where the applied force is nearly 90 time-degrees behind the velocity of the water, the applied force is maximum when the piston *b* is near the end of its stroke.

In case the partition is removed and only the effects of friction considered, the maximum applied force would be impressed on the piston at its center position, which would be the normal condition for the pin and link mechanism as previously explained. In an electric circuit this condition would obtain when the circuit possesses ohmic resistance only.

CAPACITY OF A CIRCUIT

47. The Farad.—The **farad** is the unit usually employed to measure the capacity of a condenser or the capacity of an open electric circuit. If a condenser be of such dimensions, or the open electric circuit have such properties, that electricity flowing into it for one second at the rate of one ampere causes the voltage across its terminals to rise one volt, its capacity is said to be 1 *farad*. The capacity of a condenser or of an electric circuit may also be expressed in *microfarads;* one microfarad is 1 millionth of a farad, or 1,000,000 microfarads = 1 farad.

48. Condensive Reactance.—The **condensive reactance** of an alternating-current circuit is that part of the apparent resistance caused by capacity; it is expressed in ohms. Condensive reactance limits the current and causes it to lead the impressed voltage. The value of condensive reactance is the reciprocal of the product of 2 π, the frequency, and the capacity, where $\pi = 3.1416$.

Let X_c = condensive reactance, in ohms;
f = frequency, in cycles per second;
C = capacity, in farads.

Then,
$$X_c = \frac{1}{2\,\pi\,f\,C}$$

The following explains the use of $2\,\pi$ in the formula. Consider a condenser of capacity C connected across a circuit of maximum volts E_m. The condenser will take up a maximum charge $Q = C\,E_m$ coulombs. The electromotive force passes through f cycles per second. The condenser is charged up to a maximum in one direction, then discharged, and the process repeated in the opposite direction, f times per second. The average rate of charge and discharge is, therefore, $4\,f$; that is, $4\,f$ times per second.

The maximum rate $= \dfrac{\text{average rate}}{.636 = \frac{2}{\pi}} = \text{average rate} \times \dfrac{\pi}{2}$; hence, the maximum rate of charge and discharge is $\dfrac{4\,f \times \pi}{2} = 2\,\pi\,f$. The maximum charge is $C\,E_m$, and if the maximum rate of charge and discharge is $2\,\pi\,f$, the maximum current must be
$$I_m = 2\,\pi\,f\,C\,E_m$$

hence,
$$E_m = \frac{I_m}{2\,\pi\,f\,C}$$

This formula gives the relation between the maximum electromotive force E_m and the maximum current I_m. The effective electromotive force $E_e = .707\;E_m = \dfrac{E_m}{1.414} = \dfrac{E_m}{\sqrt{2}}$. Dividing each side of the equation for E_m by $\sqrt{2}$ gives, $\dfrac{E_m}{\sqrt{2}} = \dfrac{I_m}{2\,\pi\,f\,C\,\sqrt{2}}$, or $E_e = I_e \dfrac{1}{2\,\pi\,f\,C}$. This last equation shows the relation between the effective electromotive force and the effective current.

If only a condenser is in circuit and the wires between the source of the electromotive force and the condenser have negligible ohmic resistance, then $\dfrac{E_e}{I_e} = X_c$, the condensive reactance, in ohms. Then divide each side of the equation for E_e by I_e and simplify, and

$$X_c = \frac{1}{2\,\pi\,f\,C}$$

EXAMPLE.—A condenser having a capacity of .1 microfarad (.0000001 farad) is connected to a 25-cycle system. What is its condensive reactance?

SOLUTION.—According to the formula for condensive reactance,

$$X_c = \frac{1}{2\times 3.1416\times 25\times .0000001} = 63{,}662 \text{ ohms, approx.}\quad \text{Ans.}$$

49. Condensive Reactance Voltage Drop.—The **condensive reactance voltage drop** is the applied voltage that is necessary to overcome the counter electromotive force caused by capacity effect and this reactive voltage drop, Fig. 24 (*d*), differs in phase from the counter electromotive force by 180 time-degrees.

Let U_c = condensive reactance voltage drop;
I = current;
X_c = condensive reactance.

Then,

$$U_c = I\,X_c$$

EXAMPLE.—A condenser having a capacity of .3 microfarad (.0000003 farad) and connected with a 60-cycle circuit receives a charging current of 2 amperes. What is the condensive reactance voltage drop?

SOLUTION.—According to the formula for condensive reactance of Art. 48,

$$X_c = \frac{1}{2\times 3.1416\times 60\times .0000003} = 8{,}842 \text{ ohms;}$$

therefore, according to the formula of Art. 49,

$$U_c = 2\times 8{,}842 = 17{,}684 \text{ volts.}\quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. What is the value of the condensive reactance in a 60-cycle circuit if the capacity of the condenser is 15 microfarads?
Ans. 176.8 ohms, approx.

2. What is the capacity, in microfarads, of a 22,000-volt, 25-cycle line, if the charging current is .11 ampere, the effect of the line resistance being neglected?
Ans. .03183 microfarad

NOTE.—Consider the line as a condenser, the condensive reactance voltage drop of which is 22,000; find the condensive reactance X_c by applying the formula of Art. 49, and substitute it in the formula for condensive reactance of Art. 48.

ALTERNATING-CURRENT CONDUCTORS AND CIRCUITS

50. Skin Effect.—**Skin effect** is the retardation of a current due to the property that alternating currents have of apparently passing along the outer surface of the conductor, thus not utilizing the full cross-sectional area. A conductor may be considered as made up of a group of small parallel conductors. If an alternating voltage is applied to the circuit of which the group of conductors forms a part, the alternating current thus established will cause a varying magnetic flux around each conductor. The action of the flux that is within the main conductor is here considered. Every conductor is not, however, affected by the same number of lines of force, because of their different positions in the group. The conductors near the center of the group are cut by more of the moving lines of force than those near the surface and, therefore, the counter-electromotive force of self-induction in the center conductors is greater than in those near the surface.

The *same voltage* is applied to the ends of all of the conductors and if the counter-voltage component is higher in the central conductors than in the others, the ohmic-drop component in these center conductors must be less than the ohmic-drop component in the outer conductors. This condition is brought about by the main current being so distributed that the center conductors carry less current than the exterior conductors. The current density throughout the cross-section of the main conductor, therefore, is not uniform, being higher near the surface than near the center. This action results in a greater resistance being offered by a conductor to the passage of an alternating current through it than to a direct current.

Skin effect depends on the frequency, on the cross-sectional area of the conductor, and on the permeability. At low frequency, in small copper conductors, the skin effect is negligible; but at high frequency or with large conductors it is sometimes important. In steel rails, used as conductors, for example, the resistance is considerably greater with alternating current than with direct current because the reluctance of the path for the magnetic lines is low and a considerable flux is established. Large copper conductors are usually stranded so as to decrease the skin effect.

51. Fig. 25 shows factors, or *skin-effect coefficients,* by which to multiply the resistance of solid conductors as given in tables, or as measured with direct current, in order to obtain the resistance to alternating current. To find the coefficient

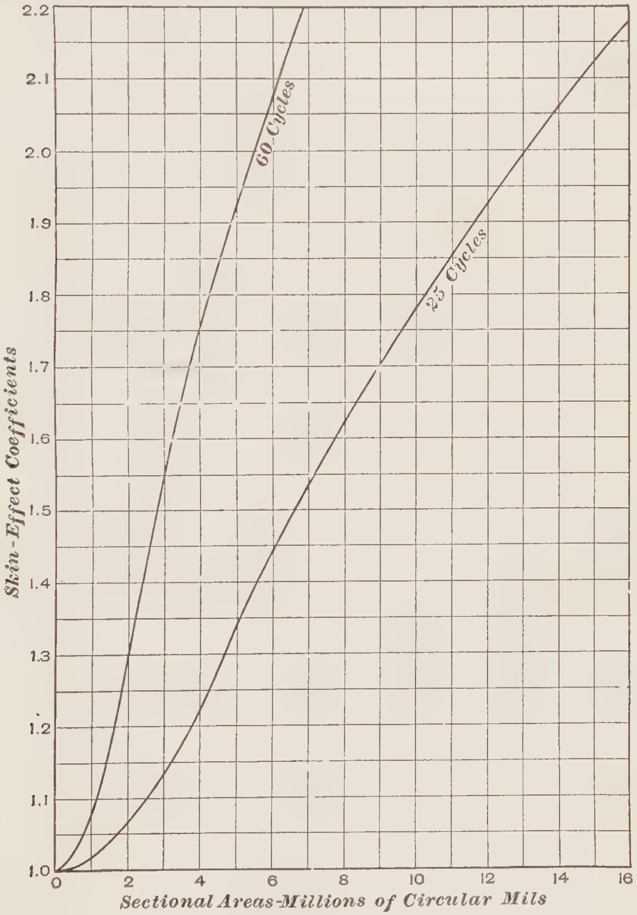


Fig. 25

for any conductor, find on the proper frequency curve the point vertically above the sectional area of the conductor, and follow the horizontal line from this point to the left-hand margin. For example, to find the coefficient for a six-million circular-mil conductor, follow the vertical line from the number *6* at the bottom to the 25- or the 60-cycle curve, as the case may be; then follow the horizontal line to the left-hand margin, where the value 1.44 is given for 25 cycles and 2.08 for 60 cycles.

Let R_a = resistance of a conductor to alternating current;
R_c = resistance to direct current;
K = skin-effect coefficient as given by the curves.

Then,

$$R_a = K\,R_c$$

Example.—If the resistance of a 5,000,000-circular-mil conductor to direct current is .001 ohm, what is its resistance to 60-cycle alternating current?

Solution.—The skin-effect coefficient, as read from the curve, Fig. 25, is 1.92; by applying the formula,

$R_a = 1.92 \times .001 = .00192$ ohm. Ans.

52. Impedance.—The **impedance** of a circuit is its total opposition to alternating current, including resistance, inductive reactance, condensive reactance, and skin effect. Each of the first three components has a distinct influence on nearly all alternating-current circuits; the last named is of importance only with high frequencies and large solid conductors. The law of alternating-current circuits is similar in general form to Ohm's law for continuous-current circuits, as is shown in the following formulas:

$$I = \frac{E}{Z} \qquad (1)$$

$$E = IZ \qquad (2)$$

$$Z = \frac{E}{I} \qquad (3)$$

in which Z = impedance, in ohms;
E = number of volts;
I = current, in amperes.

53. Circuits With Resistance Only.—The effect of resistance alone in an alternating-current circuit is the same as in a continuous-current circuit. The current is limited by the resistance according to Ohm's law, $I = E \div R$, but no phase displacement between voltage and current results. For example, incandescent lamps are practically non-inductive, and the current in a lighting load is therefore approximately according to Ohm's law, whether direct or alternating.

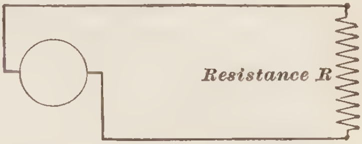


FIG. 26

In diagrams, resistance is usually represented by a zigzag line, as R in Fig. 26. An alternator is indicated conventionally at the left end of the figure.

54. Circuits With Inductance Only.—The current I established by an alternating voltage E in a circuit consisting of inductive reactance only (resistance and condensive reactance negligible) is in accordance with the formula

$$I = \frac{E}{X_s}$$

in which X_s is the inductive reactance. As the circuit is purely inductive, the current lags 90 time-degrees behind the voltage, or the angle of lag is 90. For example, in a coil having an inductive reactance of 10 ohms (negligible resistance and capacity), 300 volts of alternating current would establish a current of $300 \div 10 = 30$ amperes. Inductive reactance is usually indicated as shown at X_s, Fig. 27.

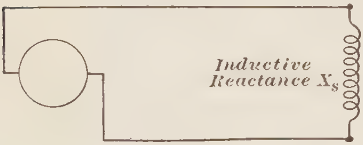


FIG. 27

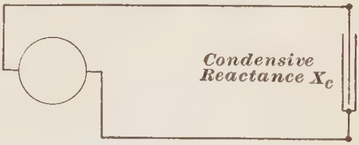


FIG. 28

55. Circuits With Capacity Only.—The current I established by an alternating voltage E through a condenser (resistance and inductive reactance negligible) is in accordance with the formula

$$I=\frac{E}{X_c}$$

in which X_c is the condensive reactance, which is usually indicated as shown at X_c in Fig. 28. The current leads the voltage by 90 time-degrees, or the angle of lead is 90. For example, if the voltage is 300 and the condensive reactance 1,500 ohms, with no resistance or inductive reactance, the current is $300 \div 1,500 = .2$ ampere.

EXAMPLES FOR PRACTICE

1. A round copper bar having an area of 4,000,000 circular mils is used as a conductor on a 60-cycle alternating-current system. How much does the skin effect increase the resistance of this bar?

Ans. 1.75 times

2. The voltmeter and ammeter connected to an alternating-current circuit give readings of 440 volts and 80 amperes, respectively. What is the impedance of the circuit? Ans. 5.5 ohms

3. A device, the inductance of which is .1273 henry, the resistance being negligible, is connected across a 220-volt circuit of a 25-cycle system. What is the current value through the device? Ans. 11 amp.

VECTORIAL REPRESENTATIONS

56. Vectors.—Any quantity that requires for its complete specification both a magnitude and a direction is called a *vector quantity;* for example, an electromotive force and an electric current are vector quantities. To describe either quantity fully, its value in volts or amperes, as well as the direction in which the electromotive force acts or in which the flow of electricity occurs, must be stated. Vector quantities may be represented graphically by a line, drawn to scale, having a length that represents the magnitude of the quantity and a direction that is parallel to the direction in which the quantity acts. The direction is usually specified by the angle which the line makes with some other line or with arbitrarily chosen axes of reference. A line representing both magnitude and direction is called a *vector*.

Vectors are used in some electrical problems to determine graphically the resultant of two or more component electromotive forces or currents that differ in phase. The maximum, effective, or average values of the quantities may be represented by vectors, but in any one problem the same classification must be used for all the vectors. In certain cases, the vectorial method will produce a result with much less work than that required if a purely mathematical process is employed.

57. Addition of Vectors.—Fig. 29 (*a*) shows in a conventional manner two alternators *a* and *b*, the rotors of which are connected rigidly on the same shaft. The electromotive force generated in the stator windings of *a* is 6 volts; in the windings of *b*, 8 volts; and the rotors are so placed on the shaft that the two electromotive forces are in phase. The line *a*, view (*b*), which is drawn to a scale of 1 division for 1 volt, represents 6 volts; and line *b*, to the same scale, represents 8 volts. The two forces have different values, but both are at all times either positive or negative since they are exactly in phase. They act, therefore, in the same direction, as indicated

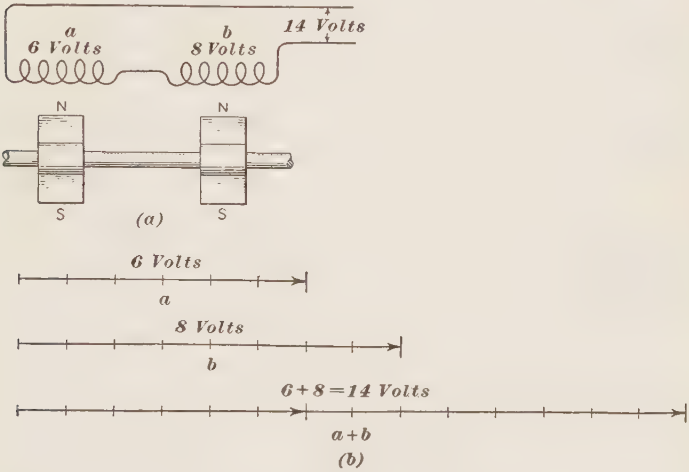


Fig. 29

by the arrowheads, and the resultant electromotive force is determined by drawing a straight line equal in length to the total length of lines *a* and *b*. The lines are joined so that the arrowheads agree in direction. The resultant line is 14 divisions long, and the resultant electromotive force, which is that applied to the circuit, view (*a*), is 6+8=14 volts.

Any number of electromotive forces, all of which are in phase, may be added directly together to obtain the resultant

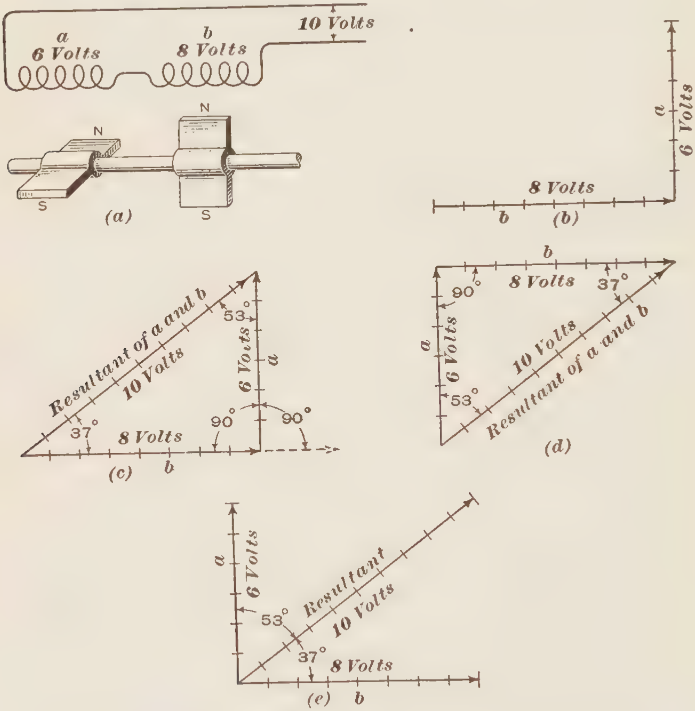


Fig. 30

voltage, and as this is a simple mathematical process, there is no advantage in employing vectors.

58. Fig. 30 (*a*) shows the same alternators, but the rotors are now mounted on the shaft so that similar poles of the two rotors differ in position by 90 space-degrees and the two voltages differ in phase by 90 time-degrees. The arithmetical sum of the voltages does not now give the resultant voltage, since both of the components are not always either positive or negative at the same time, and their maximum values do not occur at the same instant.

The components *a* and *b*, view (*b*), are drawn at right angles to each other, view (*c*). When lines are to be drawn so as to form an angle with each other, a simple protractor such as shown in Fig. 2 may be used. First draw one vector to the scale desired on a thin piece of paper. Lay the paper on Fig. 2 so that the vector coincides with the line *o d*, with one end resting on *o*; then trace the other vector in its proper angular relation to the first vector. For example, place vector *b*, Fig. 30 (*b*), along the line *o d*, Fig. 2, with its right end at *o*. Then trace vector *a*, Fig. 30 (*b*), on the paper so as to coincide with the line *o e*, Fig. 2, which forms an angle of 90° with *o d*. The length of vector *a*, Fig. 30 (*b*) and (*c*), is determined by the scale chosen. For problems containing any other angles, Fig. 2 may be used in a similar manner. For example, if an angle of 45° between lines is desired, trace the second vector so as to coincide with the line *o g*.

To determine the value and the direction of the resultant voltage, the components are drawn end to end, Fig. 30 (*c*), and the length of the line joining the beginning and end of the compound line thus formed represents the magnitude and the arrowhead on it, which is placed as explained later, shows the direction of the resultant voltage.

When drawing a vector in its proper phase relation to another vector, the phase angle should be considered as being formed between **the directions** in which the forces act, that is, the departure from the line of direction of one force made by the other force. The inside angle formed by the lines representing the two vectors does not always represent the phase angle. In Figs. 30 and 31, where the components differ in phase by 90 time-degrees, the inside angle has the same value as the angle between the directions of the forces, both being 90°, as indicated in Fig. 30 (*c*). In Fig. 32, however, the component forces differ by 60 time-degrees, and the angle of 60° is formed between vector *a* and the extension of vector *b*. The 60°

angle represents the true phase relation between the directions of the forces; the inside angle of 120°, therefore, in this case does not represent the true phase relation of the component forces.

It is essential that the component lines be so connected that, when passing over the two lines with a pencil, the arrowheads on the component lines will agree in direction with the direction of motion of the pencil. Each line may be drawn from either end of the other line, but the rule just stated regarding the arrowheads and the motion of the pencil must be observed. For example, the diagram may be drawn like Fig. 30 (*c*) or (*d*).

As each division represents 1 volt and upon measurement the length of the resultant line is found to be 10 divisions, views (*c*) and (*d*), the resultant electromotive force is 10 volts; that is, the electromotive force applied to the circuit, view (*a*), is 10 volts as compared with 14 volts, Fig. 29 (*a*), when the voltages are in phase.

The direction in which the resultant voltage acts is indicated by the arrowhead on the resultant line, Fig. 30 (*c*) and (*d*). The resultant will always have a direction opposite in sense to that of the component forces; that is, if a pencil is passed around the triangle in the direction in which the arrowheads on the lines *a* and *b* point, the arrowhead representing the direction of action of the resultant must point in a direction opposite to that in which the pencil moves. The resultant is the geometric sum of the component forces and represents the combined effect of these component forces.

59. The direction of the resultant, as compared to that of the components, is indicated in Fig. 30 (*c*), (*d*), and (*e*). In view (*c*) the angle of 37° between the larger component *b* and the resultant is smaller than the 53° angle between the smaller component *a* and the resultant. When one component is larger than the other, the resultant is more nearly parallel to the larger component.

To determine the relative phase relations of the components and the resultant, the lines representing the vectors should be drawn extending from a common point, or origin, and the direction of each force indicated as acting away from the point. The condition may be indicated by drawing the vectors, shown in views (*c*) or (*d*), in the position shown in view (*e*). In order to indicate angles of lead and lag, the general practice is to consider vectors as **rotating counter-clockwise.** If one vector is in a counter-clockwise direction from a second vector, the first vector leads the second vector; if a third vector is in a clockwise direction from the second vector, the third vector lags behind the second vector. In view (*e*) the angle of lead of voltage *a* ahead of the resultant voltage is 53 time-degrees; the resultant leads the voltage *b* by 37 time-degrees; the voltage *b* lags behind the voltage *a* by 53+37 =90 time-degrees, and lags behind the resultant by 37 time-degrees.

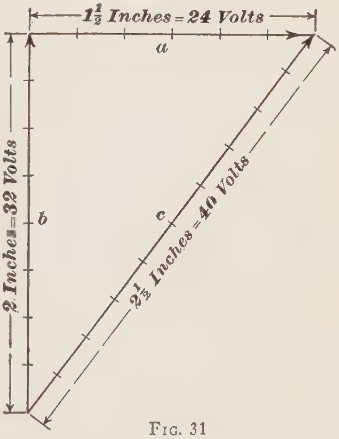


FIG. 31

In diagrams employing sine curves to represent voltages or currents, the phase difference between quantities is represented proportionally by the distance along the horizontal axis between the points where the quantities pass through their zero values when *both are becoming positive or both negative* as indicated in Fig. 11. In vectorial representations, the angles of phase differences are indicated directly by the mechanical space-degrees between the vectors when they are drawn with a common junction point as in Fig. 30 (*e*).

EXAMPLE 1.—Two electromotive forces of 24 and 32 volts, respectively, that differ in phase by 90 time-degrees are in series and are impressed on an electric circuit. What is the voltage of the circuit?

SOLUTION.—First select a scale for drawing the vectors *a* and *b*, Fig. 31; for example, allow $\frac{1}{4}$ of an inch to represent 4 volts. Then, the smaller vector *a* will be $\frac{24}{4}\times\frac{1}{4}=1\frac{1}{2}$ in. long; and the other vector *b*, $\frac{32}{4}\times\frac{1}{4}=2$ in. long. Draw vector *b* in a vertical position and assume that the force is acting upwards; draw vector *a* at the top of vector *b* and at an angle of 90° with the direction of force *b*. Assume that the force of vector *a* acts toward the right. Place the arrowheads indicating direction of action on both component vectors. The resultant vector *c* is a line drawn between the free ends of *a* and *b* and its direction of action is indicated by the arrowhead on it. The vector measures $2\frac{1}{2}$ in.; therefore, since $\frac{1}{4}$ of an inch represents 4 volts, the electromotive force impressed on the circuit is $\frac{2\frac{1}{2}}{\frac{1}{4}}\times 4 = 2\frac{1}{2}\times 4\times 4 = 40$ volts. Ans.

EXAMPLE 2.—If the component electromotive forces stated in example 1 differ in phase by 60 time-degrees instead of 90 time-degrees, what is the voltage impressed on the circuit?

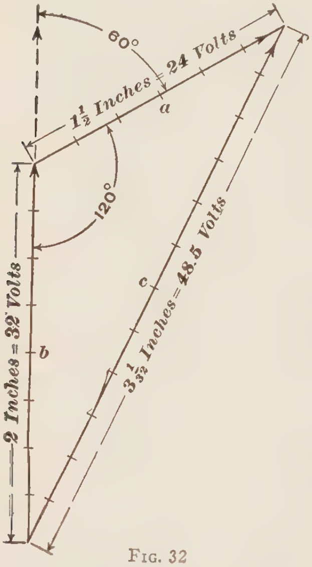

60°
$1\frac{1}{2}$ Inches = 24 Volts
a
120°
c
$3\frac{1}{32}$ Inches = 48.5 Volts
2 Inches = 32 Volts
b

FIG. 32

SOLUTION.—Draw the vectors to the same scale as in the previous example. In order to show the true relation between the directions of the two component forces, draw vector *a*, Fig. 32, at an angle of 60° with the extension of vector *b*. The inside angle 120° does not in this case represent the phase angle 60 time-degrees. The protractor, Fig. 2, may conveniently be used in drawing the 60° angle. The resultant *c*, Fig. 32, is $3\frac{1}{32}$ in., approximately, and this represents $\frac{3\frac{1}{32}}{\frac{1}{4}}\times 4 = 3\frac{1}{32}\times 4\times 4 = 48.5$ volts. Ans.

60. Resolution of Vectors. Any vector may be considered as made up of any number of component vectors which when added vectorially will produce a resultant vector equal to the given vector. The operation of finding component vectors that when added together will make up a given resultant vector is called the *resolution of vectors.*

For example, resolve the resultant vector *a*, Fig. 33, into two components, vector *b* making an angle of lead of 60° with *a*, and vector *c* making an angle of lag of 30° with *a*. From one end *d* of the resultant vector *a*, Fig. 33, draw the line *b*, inclined 60° to the resultant, with vector *b* leading vector *a*; from the other end *e* of the resultant vector *a*, draw the line *c* inclined 30° to the resultant, with vector *c* lagging behind vector *a*. The extension of line *c* also forms an angle of 30° with the extension of vector *a* and shows the true phase relation of vectors *a* and *c*. The lines *b* and *c* will intersect at the point *f*, thereby determining the lengths of the components. The arrowheads on the component vectors *b* and *c* must point in the same general direction around the triangle and in opposition to that on their resultant vector *a*, for the reasons explained before. To determine phase relations, however, all vectors must be indicated as acting from one point. Thus, transferring vector *c* to its true position c_1 shows the correct phase relations of vectors *a*, *b*, and c_1.

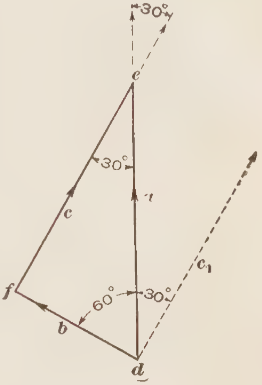


FIG. 33

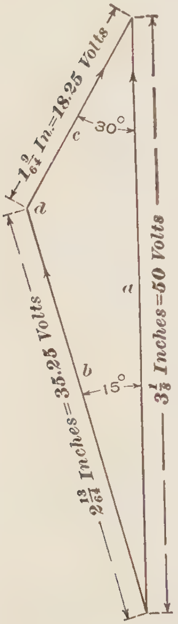

$1\frac{9}{64}$ In.=18.25 Volts
30°
c
d
a
$3\frac{1}{8}$ Inches=50 Volts
b
15°
$2\frac{13}{64}$ Inches=35.25 Volts

FIG. 34

61. Suppose an electromotive force of 50 volts is to be resolved into two component voltages, one leading the resultant by 15 time-degrees and the other lagging behind the resultant by 30 time-degrees. Any convenient length, such as $\frac{1}{16}$ inch, may be assumed to represent 1 volt. Line *a*, Fig. 34, representing the resultant electromotive force of 50 volts will be $50 \times \frac{1}{16} = 3\frac{1}{8}$ inches long. From the ends of line *a*, lines *b* and *c*, representing the component voltages, are drawn to make angles of 15° leading and 30° lagging, respectively, with line *a*. These angles also represent the true phase relations of the component voltages with the resultant voltage. The component lines meet at point *d*, and their lengths are $2\frac{13}{64}$ and $1\frac{9}{64}$ inches, respectively. At $\frac{1}{16}$ inch per volt, these lines represent 35.25 volts and 18.25 volts, which are the component voltages that with the stated phase relations will form the resultant electromotive force of 50 volts.

In order to have one definite solution in regard to magnitudes and true phase relations, the phase relations of the components to the resultant must be known. When the directions of the component vectors are not given, an unlimited number of pairs of vectors may be drawn in such manner that the vectorial sum of each pair equals the given resultant. In Fig. 34, if the angles had been either less or more than 15° or 30°, the values of the magnitudes of the component vectors would be different from those indicated in the figure.

62. Subtraction of Vectors.—To subtract a vector is to add a vector with the opposite sign of direction. A case that is to a limited extent analogous is algebraic subtraction in which the sign of the subtrahend, either positive or negative, is changed and the subtrahend then added to the minuend. It should be understood, however, that the values of the magnitudes of vector quantities cannot either be directly added or subtracted unless the forces are in the same direction or in exactly opposite directions. If the forces differ by an angle other than 0° or 180°, the problem may be solved by a vectorial diagram or by the laws of trigonometry.

Fig. 35 (*a*) shows two vectors *a* and *b*, drawn to such a scale that each represents 100 volts. The electromotive forces differ in phase by 90 time-degrees, as indicated by the relative positions of the vectors. If these two voltages in series act on a circuit, the electromotive force of the circuit is equal to the vectorial sum of the two voltages.

View (*b*) shows the addition of the vectors. The resultant electromotive force $+a+b$ is 141 volts and lags 45 time-degrees behind voltage *a* and leads voltage *b* by 45 time-degrees, as indicated when the forces are drawn with a common junction point.

View (*c*) indicates the resultant voltage produced by subtracting vector *b* from vector *a*. In the vectorial diagram the direction of $-b$ is reversed from its original direction as shown in view (*a*) and the vector in its reversed position, view (*c*), added to vector *a*. Vector $-b$ is drawn at the arrowhead end of *a*, and the resultant $+a-b$, produced by the addition of the

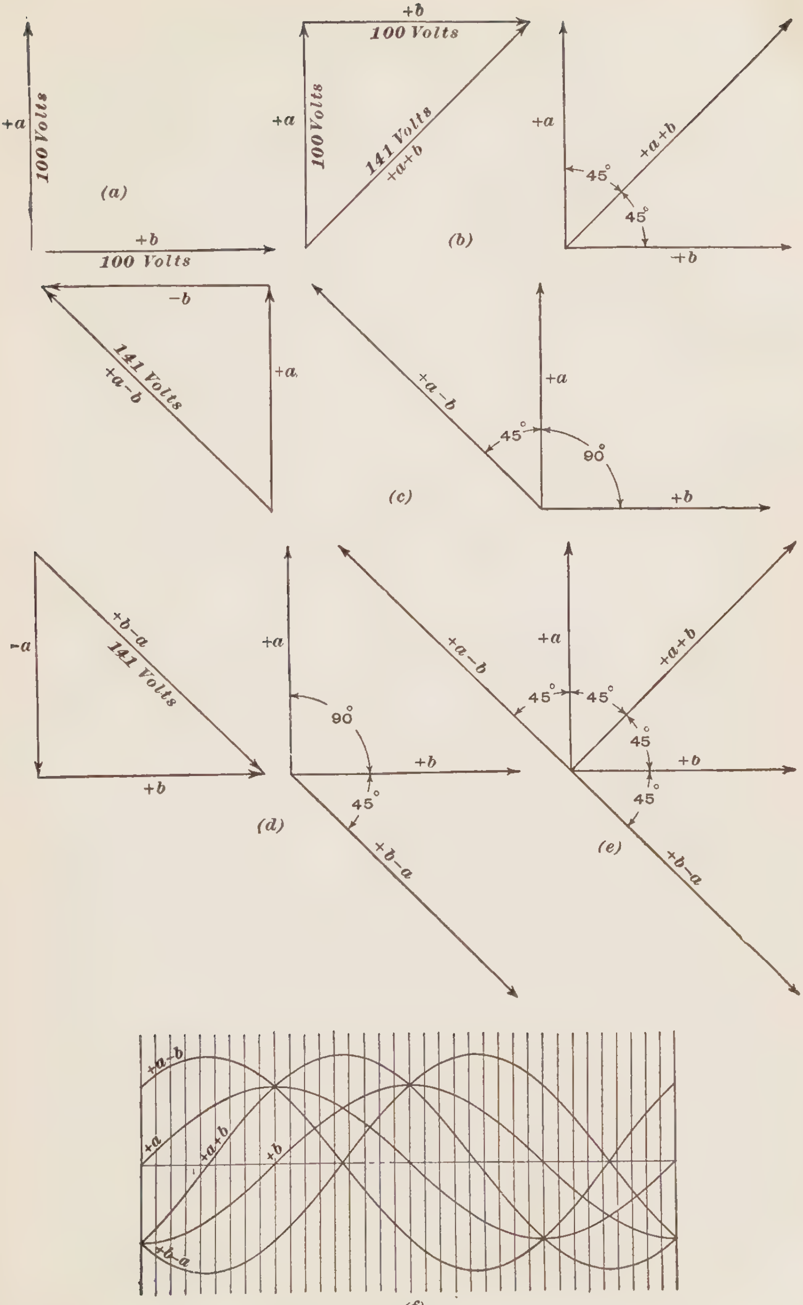

+a
100 Volts
(a)
+b
100 Volts
+b
100 Volts
+a
100 Volts
141 Volts
+a+b
(b)
+a
45°
+a+b
45°
+b
−b
141 Volts
+a−b
+a
+a−b
+a
45°
90°
(c)
+b
−a
+b−a
141 Volts
+b
+a
90°
+b
45°
(d)
+b−a
+a−b
+a
45°
45°
+a+b
45
+b
45
(e)
+b−a
+a−b
+a
+a+b
+b
+b−a
(f)

FIG. 35

two vectors in their present positions, has a value of 141 volts, as before, but this resultant leads voltage *a* by 45 time-degrees and leads voltage *b* (original direction) by 90+45=135 time-degrees. It should be noted that while the resultant vector $+a-b$, view (*c*), represents the *addition* of component vector $+a$ and the reversed component vector $-b$, it also represents the result of the subtraction of vector *b* from vector *a*, both considered in their original positions.

In view (*d*), vector *a* is subtracted from vector *b*. The direction of vector $-a$ is reversed from its original direction and the vector in its new position added to vector *b*. The resultant $+b-a$ has a value of 141 volts, as in the other cases, but the resultant now lags behind voltage *b* by 45 time-degrees and lags behind voltage *a* (original direction) by 135 time-degrees. The phase relation of the vectors representing the two components and the three resultants just obtained are indicated in view (*e*).

In view (*f*), the component voltages and the resultant voltages are represented by sine curves. In making curve $+a+b$, the ordinates are either added or subtracted according as the actual instantaneous values are either both positive or both negative or one positive and the other negative. When tracing the resultant curve $+a-b$, the ordinates of $+b$ are *considered* positive when actually negative and negative when actually positive. The ordinates are then added to or subtracted from the ordinates of $+a$ depending on whether curves $+a$ and $+b$ are then both *considered* positive or both negative or one positive and the other negative. In a similar manner the ordinates of curve $+a$ are *considered* as reversed in direction when they are combined with the ordinates of $+b$ in tracing the resultant curve $+b-a$.

The space between any two parallel vertical lines represents 10 time-degrees. By considering the horizontal spaces between the zero positions of the various curves when those being compared are all passing to a positive or all to a negative value, the phase relations indicated are found to be the same in view (*e*). Views (*e*) and (*f*) show two different methods of indicating phase relations between quantities.

It should be noted that when the components are equal and at an angle of 90 time-degrees, the resultants due to addition and to subtraction all have the same form and height of curve, but that all differ in their phase relations with themselves and with the components. If the components, with either the same or different magnitudes, differ in phase by other than 90 time-degrees, the resultant due to addition is not of the same magnitude as the resultant due to subtraction, but for any angle of phase displacement and pair of component vectors of the same or different magnitudes, the two resultants due to subtraction will agree in magnitudes and will differ in phase by 180 time-degrees. If the magnitudes of the pair of vectors are equal and they differ in phase displacement by any angle except 180°, the resultant due to addition will differ in phase relation by 90 time-degrees from either of the resultants obtained by subtraction. This relation is indicated in Fig. 35 (*e*).

63. Fig. 36 relates to vectors $+a$ and $+b$ of equal magnitudes, that differ in phase by 120 time-degrees. View (*a*) shows the vectors, each of which represent 100 volts. The resultant $+a+b$ due to the addition of these vectors is shown in view (*b*); the resultant $+a-b$ due to the subtraction of vector b from a, in view (*c*); the resultant $+b-a$ due to the subtraction of vector a from b, in view (*d*); and the phase relations of the components and the resultants in view (*e*). The resultant $+a+b$, due to addition, lags behind $+a$ by 60 time-degrees and leads $+b$ by 60 time-degrees; the resultant $+a-b$ leads $+a$ by 30 time-degrees and leads $+b$ by 150 time-degrees; the resultant $+b-a$ lags behind $+a$ by 150 time-degrees and behind $+b$ by 30 time-degrees. In this problem, the resultant due to addition is less in magnitude than either of the resultants due to subtraction. The two resultants due to subtraction differ in phase by 180 time-degrees.

It should also be noted in both Figs. 35 and 36 that since the two vectors are of equal magnitudes and differ in phase relation, the resultant obtained by addition differs in phase relation by 90 time-degrees from either of the resultants obtained by subtraction.

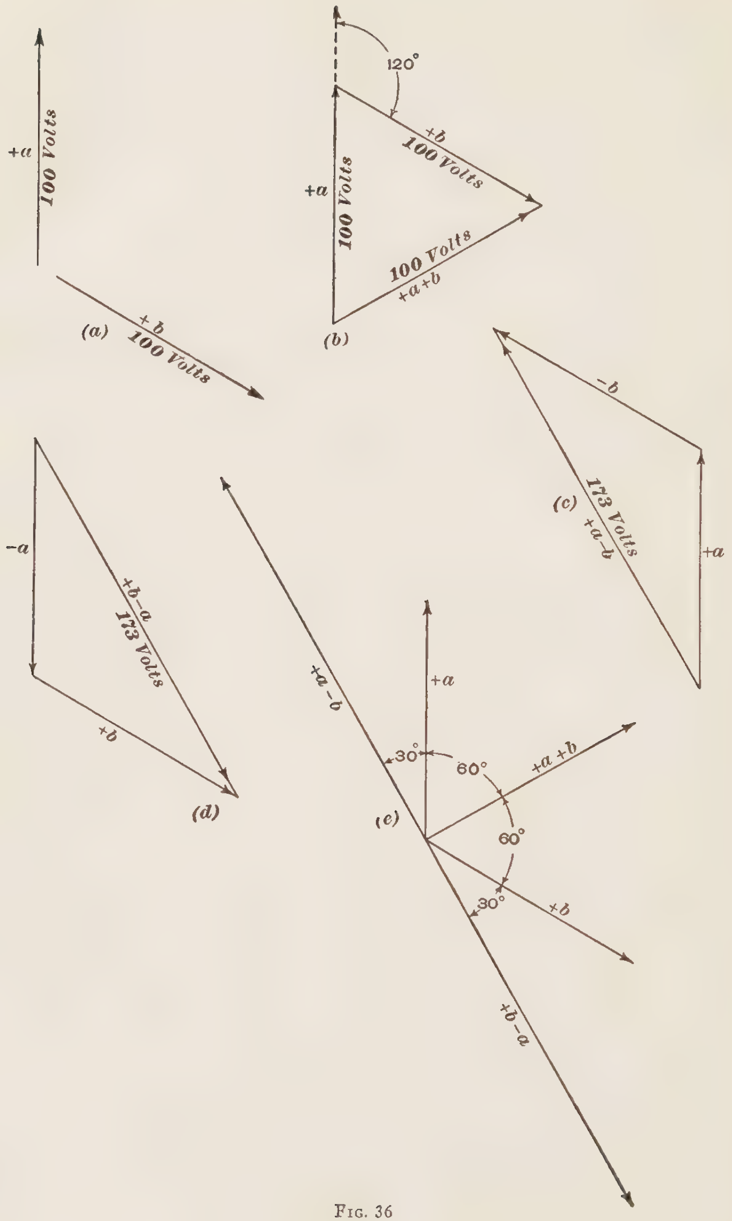

+a
100 Volts
+b
100 Volts
(a)
120°
+a
100 Volts
+b
100 Volts
100 Volts
+a+b
(b)
−b
173 Volts
+a−b
+a
(c)
−a
+b−a
173 Volts
+b
(d)
+a−b
+a
30°
60°
+a +b
60°
30°
+b
+b−a
(e)

FIG. 36

64. In alternating-current operation there are many cases in which the voltage impressed on the line wires of a circuit or on a machine is the resultant of two or more component voltages. In order to understand how these resultant voltages are produced, it is well to fix firmly in mind the principles of vectorial combinations of forces.

EXAMPLES FOR PRACTICE

1. Find, by vectorial addition, the sum of two currents of 48 and 20 amperes, respectively, and differing in phase by 90 time-degrees. Ans. 52 amp.

2. Find, by the addition of vectors, the sum of two electromotive forces of 63.5 volts each and differing in phase by 60 time-degrees. Ans. 110 volts, approx.

3. Find, by vectorial subtraction, the difference between two electromotive forces of 63.5 volts each and differing in phase by 60 time-degrees. Ans. 63.5 volts

4. With reference to the data in examples 2 and 3, what is: (*a*) the arithmetical difference in magnitudes? and (*b*) the phase relation between the resultant obtained by vectorial addition and the two resultants that may be obtained by vectorial subtraction?

Ans. (*a*) 46.5 volts
(*b*) The resultant obtained by addition leads one of the resultants obtained by subtraction by 90 time-degrees and lags behind the other resultant due to subtraction by 90 time-degrees.

5. Two component voltages of equal magnitude differ in phase by 90 time-degrees, and each component differs in phase from the resultant by 45 time-degrees. Find the value of the equal component voltages by the resolution of vectors, if the resultant vector is 440 volts. Ans. 311 volts, approx.

ALTERNATING CURRENTS

(PART 2)

SINGLE-PHASE CIRCUITS

SERIES CIRCUITS

1. Preliminary Considerations.—All electric circuits possess to a greater or less extent the properties of resistance, inductance, and capacity, but the proportional values of these for different circuits vary through a wide range. It is important that the effects of these properties on the voltage and current of an alternating-current circuit be well understood. In case two only are considered, the third property is assumed to have such a small value that its effect may be regarded as negligible. The effects of each one of these properties alone were discussed in *Alternating Currents,* Part 1.

It is of importance in the following discussions to distinguish carefully between the *counter electromotive force due to the inductance or to the capacity of a circuit and the component of the applied electromotive force required to balance the counter electromotive force. The component of the applied electromotive force differs in phase by 180 time-degrees from the counter electromotive force which it balances and is exactly the same in magnitude.* In a vector diagram, if a vector pointing in a given direction from its origin represents a counter force, the component of the applied force necessary to balance the counter force is represented by a vector of similar length to the first line, but extending from the common origin of both vectors in exactly the *opposite direction* to the direction of the first vector.

COPYRIGHTED BY INTERNATIONAL TEXTBOOK COMPANY. ALL RIGHTS RESERVED

This phase relation as applied to sine curves is indicated in the figures accompanying the analogies of the effects of inductance and capacity in *Alternating Currents,* Part 1.

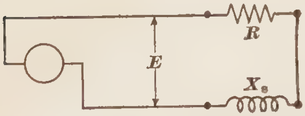


FIG. 1

2. Resistance and Inductance in Series.—In a series circuit possessing both resistance R and inductive reactance X_s, as shown in Fig. 1, alternating voltage follows the general law $E = I Z$, in which I is the current and Z the impedance. The current in all parts of a series circuit is the same, and a component voltage $U = I R$ is required to balance the voltage drop due to ohmic resistance, and a component voltage $U_s = I X_s$ is required to overcome the counter electromotive force of self-induction.

In the case of the ohmic-resistance drop, there is no active counter electromotive force generated, and the resistance drop is in phase with the current. The effect of inductance is to establish an active counter electromotive force and the component of the applied voltage that overcomes the counter force must be out of phase with it by 180 time-degrees; that is, this component of the applied force at all points in a cycle opposes and balances the counter force. The counter electromotive force of self-induction lags behind the current 90 time-degrees; and since the inductive reactance voltage drop differs in phase by 180 time-

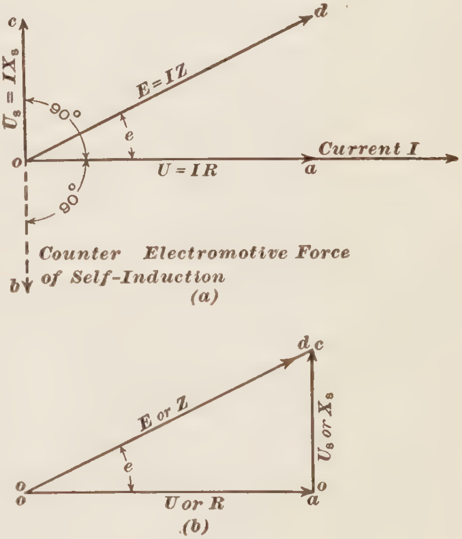


FIG. 2

degrees from the counter force, the inductive reactance drop is 90 time-degrees ahead of the current. The voltage E impressed on the circuit is equal to the vectorial sum of the ohmic-resistance drop and the inductive reactance voltage drop.

3. In Fig. 2 (a) the direction of the current vector I is indicated by the arrowhead at its right end. The vector for the component voltage $U = I R$ is in phase with the current and its magnitude is represented by the length of line between the common origin o and point a. The counter electromotive force of self-induction is represented by the vector $o\,b$ and it lags behind the current by 90 time-degrees. The vector $o\,c$ for the inductive reactance voltage drop $U_s = I X_s$ is opposite in direction to $o\,b$ and leads the current by 90 time-degrees.

The resultant vector $o\,d$, view (b), of the two component vectors $o\,a$ and $o\,c$ represents the voltage applied to the circuit that is necessary to force the current against the opposition offered by the resistance and by the inductance of the circuit. The applied voltage $o\,d$, views (a) and (b), leads the current by the angle e in electrical time-degrees, that is, the current in a circuit possessing resistance and inductance lags behind the applied electromotive force.

From the values of I and E, the value of the impedance Z may be determined by mathematics from the general formula,

$$Z = \frac{E}{I}$$

4. The triangle, Fig. 2 (b), is usually referred to as the *voltage and impedance triangle*. The impedance of a circuit is the resultant of the resistance component and the inductive reactance component, all of which are expressed in ohms. In a series circuit, the value of the current is the same in all parts; therefore, the resistance, the inductive reactance, and the impedance are proportional to the voltages necessary to overcome them. The vector $o\,a$, view (b), may also represent the resistance R; the vector $o\,c$ may represent the inductive reactance X_s; and the resultant vector $o\,d$ may represent the impedance Z.

The value of the voltage applied to the circuit or of the impedance of the circuit can be determined with approximate accuracy by measuring the resultant vector to the same scale as used for the component vectors, but greater accuracy is obtained by calculation, by use of the formulas,

$$E=\sqrt{U^2+U_s^2} \qquad (\mathbf{1})$$

$$Z=\sqrt{R^2+X_s^2} \qquad (\mathbf{2})$$

in which E = applied voltage;
U = voltage drop in the resistance;
U_s = voltage to overcome counter electromotive force of self-induction;
Z = impedance;
R = resistance;
X_s = inductive reactance.

Example 1.—The voltage necessary to overcome the resistance of a circuit is 64, and that to overcome the inductive reactance is 48. Find the value of the impressed electromotive force.

Solution.—According to formula **1**,

$$E=\sqrt{64^2+48^2}=\sqrt{4{,}096+2{,}304}=80 \text{ volts. Ans.}$$

Example 2.—The resistance of a circuit is 4 ohms, and the inductive reactance 3 ohms. Find the impedance of the circuit.

Solution.—According to formula **2**,

$$Z=\sqrt{4^2+3^2}=\sqrt{16+9}=5 \text{ ohms. Ans.}$$

Example 3.—A choke coil is required in series with an 80-volt arc lamp in order to connect the latter across a 120-volt, 60-cycle circuit; the current through the lamp is 4 amperes. What is the inductive reactance of the coil?

Solution.—The voltage drop across the lamp is considered as used in overcoming ohmic resistance, while the voltage across the choke coil is considered as used in overcoming inductive reactance. The vectors of the two are therefore at right angles to each other, and their vectorial sum, or the hypotenuse of the triangle, is equal to 120 volts.

According to formula **1**, $120=\sqrt{80^2+U_s^2}$; therefore, $120^2=80^2+U_s^2$, and $U_s^2=120^2-80^2$; from which $U_s=\sqrt{120^2-80^2}=89.44$ volts. The choke coil and the lamp are connected in series; therefore, the current through both is the same, namely, 4 amp., and the inductive reactance is

$$X_s=U_s \div I=89.44 \div 4=22.36 \text{ ohms. Ans.}$$

5. Resistance and Capacity in Series.—Alternating voltage E applied to a circuit containing both a resistance R and a condensive reactance X_c, as shown in Fig. 3, establishes a current I in the circuit. The resultant applied voltage E is equal to the vectorial sum of the component voltage $U=I\,R$ necessary to balance the resistance voltage drop, and the component condensive reactance voltage drop $U_c=I\,X_c$ necessary to overcome the counter electromotive force due to capacity.

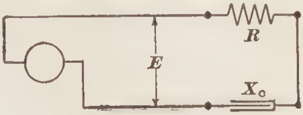


FIG. 3

Fig. 4 (a) shows the phase relations of the resistance voltage drop, vector $o\,a$, in phase with the current, vector I; the counter electromotive force, vector $o\,b$, due to capacity, which is 90 time-degrees ahead of the current; the condensive reactance voltage drop, vector $o\,c$, which is 90 time-degrees behind the current; and the applied voltage, resultant vector $o\,d$, views (a) and (b), which lags behind the current an amount represented in time-degrees by the angle e. The current in a circuit possessing resistance and capacity leads the applied voltage.

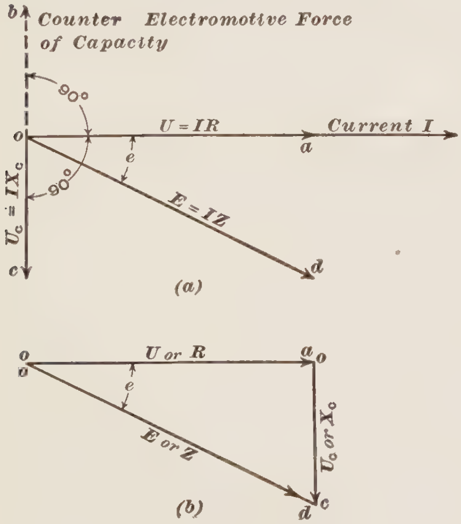


FIG. 4

For more accurate results than can be obtained by the vectorial method, the following formulas may be employed:

$$E=\sqrt{U^2+U_c^2} \qquad (1)$$

$$Z=\sqrt{R^2+X_c^2} \qquad (2)$$

EXAMPLE.—A noninductive resistance of 1,000 ohms and a condensive reactance of 1,600 ohms are connected in series across a 440-volt circuit. What is: (a) the impedance? (b) the value of the current?

SOLUTION.—(*a*) According to formula **2**,

$$Z=\sqrt{1{,}000^2+1{,}600^2}=1{,}887 \text{ ohms, approx. Ans.}$$

(*b*) Transposing the general law $E=I\,Z$,

$$I=\frac{440}{1{,}887}=.233 \text{ amp., or 233 milliamp. Ans.}$$

6. Inductance and Capacity in Series.—In a circuit containing inductance and capacity *only*, with *negligible resistance*, as indicated in Fig. 5, the effect of capacity tends to neutralize the effect of inductance. Component voltages of such a circuit are shown in Fig. 6, in which the vector *o a* represents the current; the vector *o b*, the voltage $U_c=I\,X_c$ necessary to *overcome* the counter electromotive force due to capacity; and the vector *o c*, the voltage $U_s=I\,X_s$ necessary to *overcome* the counter electromotive force due to inductance. The voltage U_c lags 90 time-degrees behind the current and the voltage U_s leads the current by 90 time-degrees.

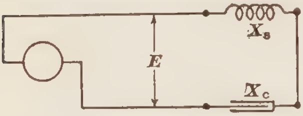


FIG. 5

The two effects neutralize completely when the power $I\,U_c$ is equal to the power $I\,U_s$. In a series circuit, in which the current through both reactances is the same, the neutralization takes place when the voltages U_s and U_c are equal. When these voltages are not equal, a part of the greater voltage neutralizes the smaller voltage, and the resulting condition is that of a simple circuit possessing either inductive or condensive reactance only, according to whether U_s or U_c is the greater. The resultant voltage differs by 90 time-degrees from the current and is, in case U_s is the greater, U_s-U_c, and in case U_c is the greater, U_c-U_s.

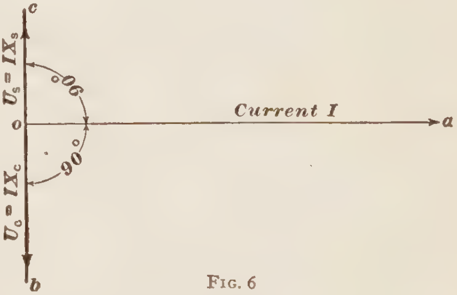


FIG. 6

Such conditions are purely theoretical; they never occur in practice, because a circuit entirely devoid of resistance is impossible. Nevertheless, a knowledge of such conditions is essential to a clear understanding of practical problems.

7. Resistance, Capacity, and Inductance in Series. A voltage E applied to a circuit containing resistance R, condensive reactance X_c, and inductive reactance X_s, as shown in Fig. 7, is equal to the vectorial sum of the voltages necessary to overcome the drop of voltage due to the resistance and the two counter electromotive forces.

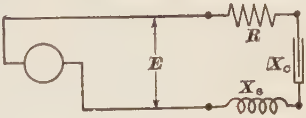


FIG. 7

Two methods of determining this sum are shown in Fig. 8, in which the vector $o\,a$ represents the component voltage $U = I\,R$; the vector $o\,b$, the component voltage $U_c = I\,X_c$; and the vector $o\,c$, the component voltage $U_s = I\,X_s$. In the case considered, the condensive reactance X_c has a greater value than the inductive reactance X_s; therefore, vector $o\,b$ is longer than vector $o\,c$. One method of determining the resultant vector $o\,e$, representing the applied voltage E, consists in adding vectorially three electromotive forces displaced in phase by the angles indicated. Vectors $o\,a$ and $o\,c$ are added first, forming the resultant $o\,d$, the vector $a\,d$ at the right end of $o\,a$ being similar in direction and magnitude to vector $o\,c$; then $o\,b$ and $o\,d$ are added, forming the final resultant vector $o\,e$, which represents the impressed, or applied, voltage $E = I\,Z$ lagging behind the current I. Vectors $b\,e$ and $o\,d$ are similar in direction and magnitude. When vector $b\,e$ is drawn at the lower end of $o\,b$, the line connecting o with e is the resultant vector for the components $o\,b$ and $o\,d$. If U_c and U_s were equal, they would neutralize and the vectors $o\,e$ and $o\,a$ would coincide;

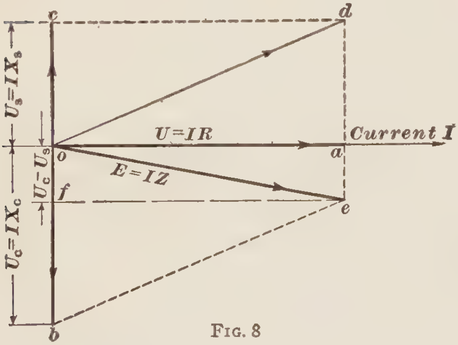


FIG. 8

that is, the voltage E would be in phase with the current I and proportional to the resistance. If the value of X_s is greater than the value of X_c, the current I will lag behind the voltage E. The other method of finding the vectorial sum consists in drawing the vector $o\,f$, representing the difference between the voltages U_c and U_s, in the direction of the greater of the two voltages, in this case U_c, and then adding vectors $o\,a$ and $o\,f$. Vector $a\,e$ is similar in direction and magnitude to vector $o\,f$. This gives the resultant $o\,e$, as before.

The following formulas may be used for calculating values of the applied voltage and of the impedance:

$$E=\sqrt{U^2+(U_c-U_s)^2}, \qquad (1)$$

or

$$E=\sqrt{U^2+(U_s-U_c)^2}, \qquad (2)$$

depending on whether U_c or U_s is the greater.

Likewise, impedances may be determined from formulas:

$$Z=\sqrt{R^2+(X_c-X_s)^2}, \qquad (3)$$

$$Z=\sqrt{R^2+(X_s-X_c)^2}, \qquad (4)$$

depending on whether X_c or X_s is the greater.

EXAMPLE.—An alternator is connected to a circuit having a resistance of 96 ohms, an inductance of 249 millihenrys, and a capacity of 19.8 microfarads. The value of the current through the circuit is 5 amperes and the frequency is 60. (*a*) Find the drop of potential across the resistance, and the voltages necessary to overcome the effects of inductance and of capacity. (*b*) Determine whether the current leads the main voltage or lags behind it. (*c*) Find the value of the main voltage necessary to maintain the current at 5 amperes. (*d*) Find the impedance of the circuit.

SOLUTION.—(*a*) The inductance is .249 henry and the capacity .0000198 farad; therefore, by substituting values in the formulas for inductive reactance and condensive reactance, given in *Alternating Currents*, Part 1, the inductive reactance is $X_s=2\times 3.1416\times 60\times .249$ $=94$ ohms, and the condensive reactance is $X_c=\frac{1}{2\times 3.1416\times 60\times .0000198}$ $=134$ ohms. The voltages necessary are:

$$\left.\begin{array}{ll}\text{To overcome resistance,} & U\ =I\,R\ =5\times\ 96=480\\ \text{To overcome ind. reac.,} & U_s=I\,X_s=5\times\ 94=470\\ \text{To overcome cond. reac.,} & U_c=I\,X_c=5\times 134=670\end{array}\right\}\text{Ans.}$$

(*b*) The current must lead the voltage because the condensive reactance exceeds the inductive reactance. These relations are made clear by a vectorial diagram, Fig. 9.

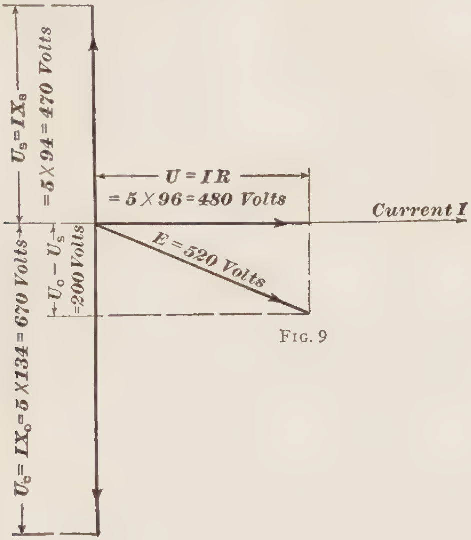

$U_s = IX_s$
$= 5 \times 94 = 470$ Volts
$U = IR$
$= 5 \times 96 = 480$ Volts
Current I
$E = 520$ Volts
$U_c - U_s$
$= 200$ Volts
$U_c = IX_c = 5 \times 134 = 670$ Volts

FIG. 9

(*c*) The value of the line voltage is determined by formula **1.** Thus, $E = \sqrt{480^2 + (670 - 470)^2} = 520$ volts. Ans.

(*d*) The value of the impedance may be determined either by formula **3**, as $Z = \sqrt{96^2 + (134 - 94)^2} = 104$ ohms or by the general formula of Art. **3**,

$$Z = \frac{E}{I} = \frac{520}{5} = 104 \text{ ohms. Ans.}$$

EXAMPLES FOR PRACTICE

1. An inductive reactance of 8 ohms in series with a non-inductive resistance of 15 ohms is connected with a 119-volt circuit. Find the value of the current. Ans. 7 amp.

2. A coil connected to a 130-volt, continuous-current circuit carries 26 amperes. The same coil connected to a 130-volt, 60-cycle, alternating-current circuit, carries only 10 amperes. What is the inductance of the coil? Ans. 31.8 millihenrys

NOTE.—From the first pair of voltage and current values, determine the resistance; from the second pair, determine the impedance. Thus, one side and the hypotenuse of an impedance triangle are determined. Find the third side, which is the inductive reactance of the coil, and from this value, knowing the frequency, determine the inductance by the formula for inductive reactance given in *Alternating Currents,* Part 1.

PARALLEL CIRCUITS

8. Resistance and Inductance in Parallel.—A voltage E applied to the terminals of a resistance R and an inductive reactance X_s connected in parallel, Fig. 10, establishes in them the currents I_r and I_s. These currents are determined by the formulas $I_r = \frac{E}{R}$ and $I_s = \frac{E}{X_s}$, and the resultant, or the line current I, is equal to their vectorial sum.

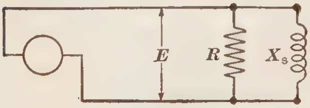


FIG. 10

The method of determining this sum is shown in Fig. 11, in which the vector $o\,a$ represents the current I_r in phase with the voltage E, and the vector $o\,b$, the current I_s lagging 90 time-degrees behind the voltage. The resultant, or the line current I, is represented by the vector $o\,c$, which indicates that the line current I lags behind the line voltage E by the angle d in electrical time-degrees.

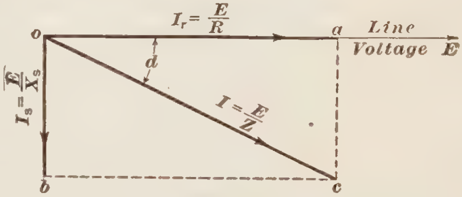


FIG. 11

The resultant current may be determined also by means of the triangle of forces, as shown in Fig. 12. The sides $a\,b$ and $b\,c$ represent the component currents, and the hypotenuse $a\,c$ represents their vectorial sum, or the line current, lagging behind the line voltage by the angle d.

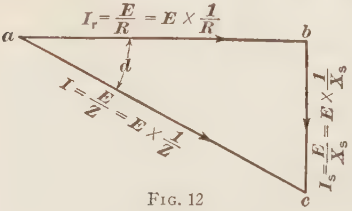

$I_r = \frac{E}{R} = E \times \frac{1}{R}$
a
b
d
$I = \frac{E}{Z} = E \times \frac{1}{Z}$
$I_s = \frac{E}{X_s} = E \times \frac{1}{X_s}$
c

FIG. 12

9. When the values of the currents in the parallel paths, one possessing only resistance and the other only inductive reactance, are known, the current in the line wires on the generator side of the points where the branch circuits are connected, Fig. 10, may be found by the formula,

$$I = \sqrt{I_r^2 + I_s^2}$$

EXAMPLE.—An alternating voltage of 220 is applied to a circuit having connected in parallel a non-inductive resistance of 18.3 ohms and an inductive reactance of 44 ohms. Determine the currents through the branches and the line current.

SOLUTION.—The component currents are

$$I_r=\frac{220}{18.3}=12 \text{ amp., approx.} \quad \text{Ans.}$$

$$I_s=\frac{220}{44}=5 \text{ amp.} \quad \text{Ans.}$$

According to the preceding formula,

$$I=\sqrt{12^2+5^2}=13 \text{ amp.} \quad \text{Ans.}$$

10. Conductance, Admittance, and Susceptance. Just as the reciprocal of resistance $\frac{1}{R}$ is called *conductance*, so the reciprocal of impedance $\frac{1}{Z}$ is called *admittance* and the reciprocal of reactance $\frac{1}{X_s}$ or $\frac{1}{X_c}$, is called *susceptance*. The **conductance** measures the degree of ease in forcing either an alternating or a direct current through a circuit possessing only ohmic resistance; the **susceptance** measures the degree of ease in forcing an alternating current through a circuit possessing only inductive reactance or condensive reactance; and the **admittance** measures the degree of ease in forcing an alternating current through a circuit possessing two or more of the properties of resistance, inductance, and capacity.

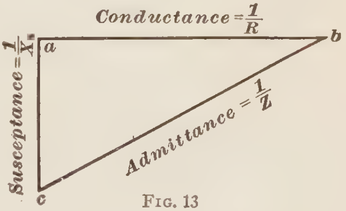

Conductance = $\frac{1}{R}$
a
b
Admittance = $\frac{1}{Z}$
Susceptance = $\frac{1}{X_s}$
c

FIG. 13

Admittance and susceptance are expressed in the same unit as conductance, which is the *mho*, or the word ohm spelled backwards. The admittance of a parallel circuit, which is analogous to the joint conductance of a direct-current circuit, is found by adding vectorially the component conductances and susceptances. The impedance can then be determined by taking the reciprocal of the admittance. The reciprocal of any quantity is 1 divided by that quantity.

The method of determining an admittance is shown in Fig. 13. If the side *a b* represents the conductance and the side *a c* the susceptance, the hypotenuse *c b* represents the admittance; or, by formula,

$$\frac{1}{Z}=\sqrt{\left(\frac{1}{R}\right)^2+\left(\frac{1}{X_s}\right)^2}$$

EXAMPLE.—A resistance of 24 ohms and an inductive reactance of 10 ohms are connected in parallel. What is the impedance of the circuit?

SOLUTION.—The conductance is $\frac{1}{24}$ mho and the susceptance $\frac{1}{10}$ mho. According to the formula just given,

$$\frac{1}{Z}=\sqrt{\left(\frac{1}{24}\right)^2+\left(\frac{1}{10}\right)^2}=\sqrt{\frac{1}{576}+\frac{1}{100}}=\sqrt{\frac{1}{576}+\frac{5.76}{576}}=\sqrt{\frac{6.76}{576}}=\frac{2.6}{24}\text{ mho}$$

If $\frac{1}{Z}=\frac{2.6}{24}$, $Z=\frac{24}{2.6}=9.23$ ohms. Ans.

11. Resistance and Capacity in Parallel.—If a voltage E is applied to the terminals of a resistance and a condensive reactance that are connected in parallel, as shown in Fig. 14, the currents I_r and I_c will be established in them. The value of these currents is determined by the formulas $I_r=\frac{E}{R}$ and $I_c=\frac{E}{X_c}$; the resultant, or the line current I, is equal to the vectorial sum of these currents, as illustrated in Fig. 15, in which the vector *o a* represents the current I_r, in phase with the line voltage, and the vector *o b* represents the current I_c through the condensive reactance, this current leading the line voltage by 90 time-degrees. The vectorial sum of these currents, or the line current I, is represented by the vector *o c*, the position of which indicates that the line current I leads the line voltage E by the angle d in time-degrees.

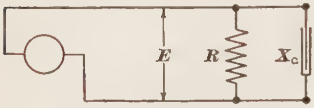


FIG. 14

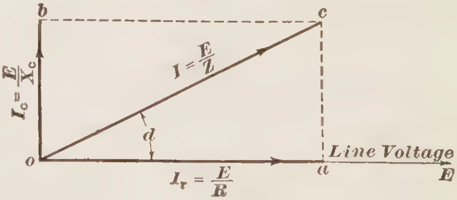


FIG. 15

If this problem is solved by means of the triangle, the component vector I_c and the resultant vector I now being above the vector I_r, Fig. 12, the following relations will be established:

$$I=\sqrt{I_r^2+I_c^2} \qquad (1)$$

$$\frac{1}{Z}=\sqrt{\left(\frac{1}{R}\right)^2+\left(\frac{1}{X_c}\right)^2} \qquad (2)$$

12. Inductance and Capacity in Parallel.—Alternating voltages applied to a circuit containing no resistance (an impossible condition in practice), but condensive and inductive reactances only, connected in parallel, as shown in Fig. 16, would establish the currents $I_s=\frac{E}{X_s}$ and $I_c=\frac{E}{X_c}$. Inasmuch as I_s would lag 90 time-degrees behind the line voltage and I_c would lead the line voltage by 90 time-degrees, the two currents would be in opposition, as indicated in Fig. 17. The resultant current would be the arithmetical difference between the two components, and it would lead or lag, with reference to the line voltage E, according to which of the components were the larger. If I_s were the larger, the line current would lag by 90 time-degrees, and if I_c were the larger, the line current would lead by 90 time-degrees.

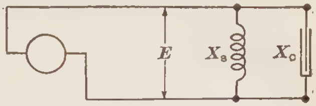


FIG. 16

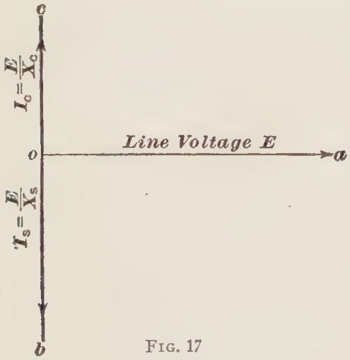


FIG. 17

13. All circuits have resistance, and this property may be combined with either, or both, inductance and capacity. The applied voltage for any such circuit may be considered as composed of a component voltage drop necessary to force current through the ohmic resistance, and a component voltage drop required to overcome the counter electromotive force established by inductance or capacity.

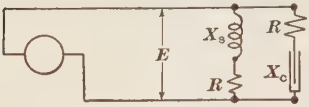


FIG. 18

In an actual case, each of the branch circuits, Fig. 16, also has the property of ohmic resistance, therefore the conditions may be indicated with greater accuracy by Fig. 18. The effects of resistance and inductance in series were explained in Art. **2**, and of resistance and capacity in series, in Art. **5**. If the condensive reactance X_c, Fig. 18, equals the inductive reactance X_s and if both X_c and X_s are large compared with the two resistances R, also assumed equal, the condition in these two branches, considered separately, will be as represented in the vector diagrams, Figs. 19 and 20.

In the inductive branch circuit, Fig. 19, the resistance drop U, vector $o\,a$, is in phase with the current in branch $X_s\,R$, Fig. 18. The inductive reactance voltage drop U_s, vector $o\,b$, Fig. 19, is 90 time-degrees ahead of the current in this branch circuit; that is, the current must be 90° behind the inductive reactance voltage drop U_s. The vector sum of these two voltage drops is represented by $o\,c$, and is the voltage required to force current through branch circuit $X_s\,R$, Fig. 18. The branch current I_s, Fig. 19, lags behind the applied voltage E for that branch by angle ϕ, called *fi*, expressed in time-degrees. The value of this angle depends on the resistance and the inductance. Likewise, Fig 20 shows the relation of the vectors for the condensive branch circuit $R\,X_c$, Fig. 18. The branch current leads the applied voltage for that branch by angle ϕ_1 expressed in time-degrees. The value of this angle depends on the relative values of the resistance and the capacity.

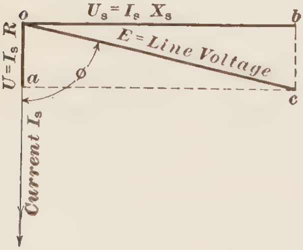


FIG. 19

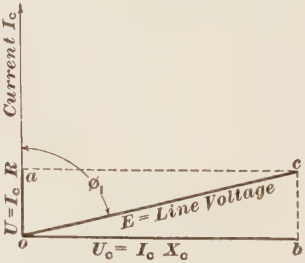


FIG. 20

The current vector diagram for the complete circuit, Fig. 18, is as indicated in Fig. 21. As the two circuits $X_s R$ and $R X_c$, Fig. 18, are in parallel and adjacent to each other, the branch voltages may be considered as being the same as the line voltage. Vector I_s, Fig. 21, representing the current in the inductive circuit, lags behind the voltage vector E by the angle ϕ; vector I_c, representing current in the condensive circuit, leads the voltage vector by the angle ϕ_1; and their vectorial sum, vector I (from the origin to the junction of the dotted lines), representing line current in the wires between the generator and the branch circuits, is in phase with the voltage, a condition that prevails in this case because it is considered that $X_c = X_s$ and the ohmic resistances of both branch circuits are equal.

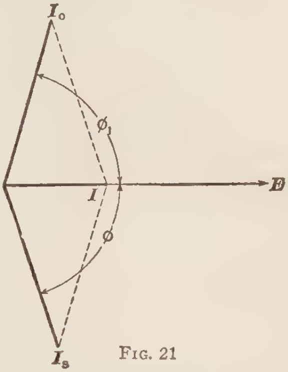


FIG. 21

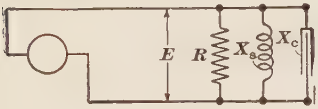


FIG. 22

It should be noted that the line current in this case is less in value than either of the components I_s or I_c. This condition is established when the phase difference between the two branch currents is 120 time-degrees or more. In such cases during a considerable portion of a cycle the branch currents are in opposite directions; therefore, the combined, or line, current will be less than either branch current.

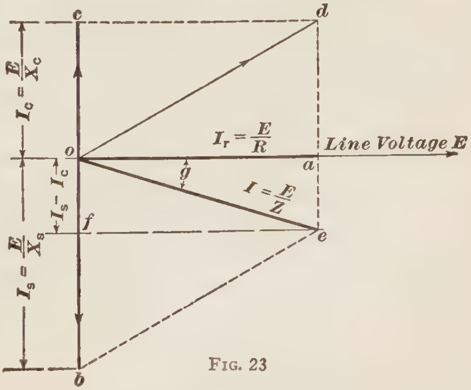


FIG. 23

14. Resistance, Inductance, and Capacity in Parallel.—An alternating voltage applied to a circuit containing resistance and inductive and condensive reactances connected in parallel, as shown in Fig. 22, establishes the currents I_r, I_s, and I_c through the corresponding branches. The line current I is equal to the vectorial sum of these currents, determined as shown in Fig. 23, in which the vector $o\,a$ represents the current I_r, in phase with the line voltage E; vectors $o\,b$ and $o\,c$ represent the currents I_s and I_c; and the resultant vector $o\,e$ represents the line current I. This resultant can be determined graphically as shown by either of the two methods, similar to those described in Art. **7**, or it can be calculated by the following formulas:

$$I=\sqrt{I_r^2+(I_s-I_c)^2} \qquad (1)$$

$$I=\sqrt{I_r^2+(I_c-I_s)^2}, \qquad (2)$$

depending on whether I_s or I_c is the greater.

Likewise, admittance may be determined from formulas.

$$\frac{1}{Z}=\sqrt{\left(\frac{1}{R}\right)^2+\left(\frac{1}{X_s}-\frac{1}{X_c}\right)^2} \qquad (3)$$

$$\frac{1}{Z}=\sqrt{\left(\frac{1}{R}\right)^2+\left(\frac{1}{X_c}-\frac{1}{X_s}\right)^2}, \qquad (4)$$

depending on whether $\frac{1}{X_s}$ or $\frac{1}{X_c}$ is the greater. The impedance can be determined by taking the reciprocal of the admittance or by dividing the line voltage by the line current.

EXAMPLE.—A resistance of 30 ohms, an inductive reactance of 37.5 ohms, and a condensive reactance of 600 ohms are connected in parallel to a 480-volt circuit. Determine: (*a*) the currents through each branch; (*b*) the line current; and (*c*) the impedance of the circuit.

SOLUTION.—(*a*) The current through the resistance

$$I_r=\frac{480}{30}=16 \text{ amp.} \quad \text{Ans.}$$

The current through the inductive path

$$I_s=\frac{480}{37.5}=12.8 \text{ amp.} \quad \text{Ans.}$$

The current through the condenser

$$I_c=\frac{480}{600}=.8 \text{ amp.} \quad \text{Ans.}$$

(*b*) The line current, according to formula **1**, is

$$I=\sqrt{16^2+(12.8-.8)^2}=20 \text{ amp.} \quad \text{Ans.}$$

(*c*) The impedance can be determined by the general formula

$$Z=\frac{E}{I}=\frac{480}{20}=24 \text{ ohms.} \quad \text{Ans.}$$

Also, it can be determined, without first determining the line current, by formula **3**. Thus,

$$\frac{1}{Z}=\sqrt{\left(\frac{1}{30}\right)^2+\left(\frac{1}{37.5}-\frac{1}{600}\right)^2}$$

Consider the quantity $\left(\frac{1}{37.5}-\frac{1}{600}\right)^2$. Multiply the first fraction by $\frac{16}{16}$; then,

$$\frac{1}{37.5}\times\frac{16}{16}=\frac{16}{600}$$

$$\left(\frac{16}{600}-\frac{1}{600}\right)^2=\left(\frac{15}{600}\right)^2=\left(\frac{1}{40}\right)^2. \quad \frac{1}{Z}=\sqrt{\left(\frac{1}{30}\right)^2+\left(\frac{1}{40}\right)^2}=\sqrt{\frac{1}{900}+\frac{1}{1{,}600}}.$$

Multiply the first fraction by $\frac{16}{16}$ and the second fraction by $\frac{9}{9}$; then,

$$\frac{1}{Z}=\sqrt{\frac{16}{14{,}400}+\frac{9}{14{,}400}}=\sqrt{\frac{25}{14{,}400}}=\sqrt{\frac{1}{576}}=\frac{1}{24} \text{ mho.}$$

If $\frac{1}{Z}=\frac{1}{24}$, $Z=24$ ohms. Ans.

EXAMPLES FOR PRACTICE

1. A choke coil, a condenser, and a non-inductive resistance are connected in series. The inductive reactance is 32 ohms, the condensive reactance 160 ohms, and the resistance 240 ohms. Find the impedance of the circuit. Ans. 272 ohms

2. If the choke coil, condenser, and resistance described in example 1 were connected in parallel, what would be the impedance of the circuit? Ans. 39.5 ohms

3. A non-inductive resistance of 15 ohms and an inductive reactance of 8 ohms are connected in parallel across a 120-volt circuit. What is the value of the main current? Ans. 17 amp.

POWER IN TWO-WIRE CIRCUITS

THEORETICAL CONSIDERATIONS

15. Power in Non-Inductive Circuits.—In an alternating-current circuit, the power at any instant is equal to the product of the voltage and the current values at that instant. For example, let Fig. 24 represent the condition of an alternating-current, non-inductive circuit. The current represented by the curve *a* has the maximum value *I*, assumed to be 10 amperes, and is in phase with the voltage, curve *b*, of the maximum value *E*, assumed to be 110 volts. Since maximum amperes and maximum volts occur simultaneously, the maximum power is 10×110=1,100 watts, as shown by the curve *g* and the scale *W*. The power at any other instant, such as *c*, is likewise equal to the product of the instantaneous ordinates *c d* and *c e*. The values of these ordinates can be determined by any convenient scales, as *I* and *E*, and the value of their product by the power scale *W*, giving an ordinate *c f* of the power curve. Other points on this curve are established in the same way, and the complete curve indicates the rate at which energy is supplied to the circuit. In order to indicate all of the curves in one figure, different scales are used to lay out the values of current, voltage, and power. For example, if the watts were laid out by the same scale as used to indicate amperes, the ordinates of curve *g* would be inconveniently long.

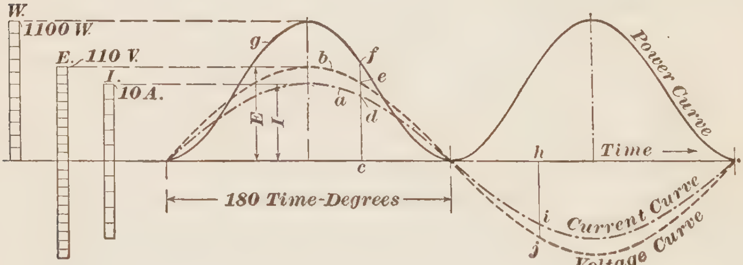


FIG. 24

16. Positive and Negative Work and Power. Under the conditions assumed for Fig. 24, the simultaneous ordinates of the voltage and the current curves, for example, *c e* and *c d* or *h j* and *h i*, are always of the same sign, both positive or both negative. Their product must therefore be positive, as the product of quantities having like signs, as two positive or two negative quantities, is always positive. The power curve, therefore, is wholly above the zero axis. In reality, energy is neither positive nor negative, nor does it have direction. It may be considered as positive or negative, however, according to whether it is supplied from a source to a system or returned from the system to the source. For example, the conditions represented in Fig. 24 show continuous energy flow, at the rate represented by the curve *g*, from the alternator to its load. No energy is reacted from the system to the alternator, and all the energy may be considered as positive.

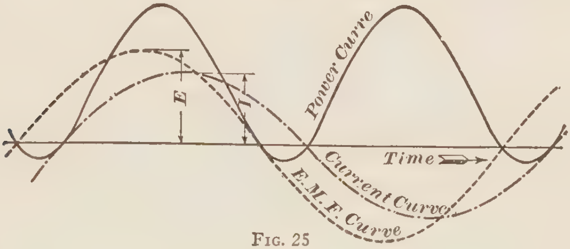


FIG. 25

17. Power in Reactive Circuits.—Alternating voltage in a circuit having inductance or capacity overcomes both reactance and resistance and establishes a current in consuming devices, such as lamps and motors. Some of the energy from the alternator is required to overcome the reactance, some is converted into heat in the resistance, and the remainder is converted into heat, light, work, or other forms of energy in the consuming devices. The energy that overcomes the reactance is returned to the source as electric energy; therefore, this part of the total energy may be considered as negative.

Fig. 25 serves to illustrate the conditions in an alternating-current circuit in which the current lags less than 90 time-degrees behind the voltage. Points on the power curve are found as explained in Art. **15,** by multiplying together simultaneous ordinates of the current and voltage curves.

When one of the simultaneous ordinates is positive and the other negative, their product is negative, as represented by the loops of the power curve below the horizontal axis. The ordinates of the negative loops represent the instantaneous rates at which the energy is returned from the circuit to the source and the positions of the negative loops, the parts of the cycle when the system returns energy to the source.

18. Wattless Current.—Current in a circuit representing no corresponding work is called *wattless current.* In Fig. 26, this condition in an inductive alternating-current circuit is represented when the angle of lag of the current behind the voltage is 90 time-degrees. In this case, the power curve lies as much above as below the axis, indicating that the circuit receives and returns energy at the same rate. The energy actually expended is zero, although a current exists in the circuit. Such a condition would exist only if an alternator were supplying current to a device having no resistance, but inductance only; that is, all the current would be wattless, or idle, because no work would be done by it. The condition is not a practical one because all conductors have some ohmic resistance. The term wattless current is somewhat misleading because if the circuit has resistance, the current in the circuit will establish heat in the conductors. The wattless component of the current may be more accurately designated as the reactive component of the current.

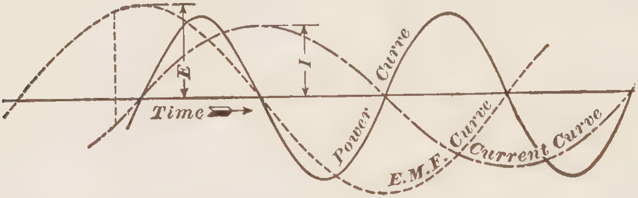


FIG. 26

If alternating voltage acted in condensive circuits without resistance, the current would lead the voltage by 90 time-degrees and would be wattless; such wattless current is called *charging current.* In every circuit containing either inductance or capacity, or both, unless the two balance each other, some of the current is wattless.

CALCULATION OF POWER

19. Current and Voltage in Phase.—When the current and the voltage are in phase, as is the case in continuous-current circuits and in non-reactive, single-phase, alternating-current circuits, the power is equal to the product of the current and the voltage, as indicated by the ammeter and voltmeter.

This may be expressed in the form of a formula; thus,

$$P = E I$$

in which P = power, in watts;
E = voltage;
I = current, in amperes.

EXAMPLE.—A non-inductive load of 550 incandescent lamps, each lamp taking energy at the rate of 40 watts, is connected to a 110-volt circuit. What is the value of the current for the entire load?

SOLUTION.—When all the lamps are burning, energy will be taken at the rate of $550 \times 40 = 22{,}000$ watts. According to the formula just given, $22{,}000 = 110 \times I$; therefore,

$$I = \frac{22{,}000}{110} = 200 \text{ amp.} \quad \text{Ans.}$$

20. Current and Voltage Out of Phase.—When voltage and current are out of phase, the indication of either the voltmeter or the ammeter may be considered as a resultant value. In a series circuit, the indicated voltage is the resultant of two components, one in phase with the current and the other differing 90 time-degrees from it, or in quadrature with it. In a parallel circuit, the indicated current is the resultant of two components, one in phase with the voltage and the other differing 90 time-degrees from it. As it makes no difference in the final result whether the voltage or the current is resolved, the common practice in all cases is to consider the current as resolved into two components, one in phase with the voltage and the other in quadrature with it. The power of the circuit is the product of the voltage and the component of the current in phase with it.

For example, let vector $o\,a$, Fig. 27, represent the indicated current I and $o\,b$ the indicated voltage E of a circuit, the phase difference being an angle ϕ. Vector $o\,a$ can be resolved into two components, namely, $o\,d$ in phase with the voltage and $d\,a$ perpendicular to it. The power is equal to the product of vector $o\,b$ and component $o\,d$. But $\frac{o\,d}{o\,a} = \cos\phi$, and $o\,d = o\,a \cos\phi$; therefore, $o\,b \times o\,d = o\,b \times o\,a \cos\phi$, or the watts,

$$P = E\,I \cos\phi$$

The product $E\,I$ is sometimes called the *volt-amperes*, or *apparent watts*, while $E\,I \cos\phi$, or P, is the *true power*, or the *real watts*, usually expressed simply as *watts*. This formula is applicable to all single-phase circuits.

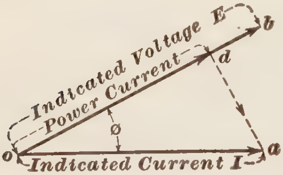


FIG. 27

21. Power Factors.—The factor, $\cos\phi$, by which the product of the volts and amperes must be multiplied to give the real watts, is called the *power factor* of the circuit. This factor must be known before the power can be determined from the indications of a voltmeter and an ammeter. An instrument called the *power-factor meter*, explained in another Section, indicates the power factor, or it can be calculated from the readings of a wattmeter, a voltmeter, and an ammeter, using the formula of Art. **20.** The wattmeter gives the true power $E\,I \cos\phi$, and from the formula referred to

$$\cos\phi = \frac{P}{E\,I} = \frac{\text{real watts}}{\text{apparent watts}}$$

If either the power factor or the angle of phase difference is known, the other can be found by referring to Table I. These power factors are the natural cosines of the angles, and are practically the same values of cosines as given in a preceding Section.

TABLE I

POWER FACTORS

Phase Difference φ Time-Degrees	Power Factor cos φ	Phase Difference φ Time-Degrees	Power Factor cos φ	Phase Difference φ Time-Degrees	Power Factor cos φ
0	1.0000	31	.857	61	.485
1	.9998	32	.848	62	.469
2	.9994	33	.839	63	.454
3	.9986	34	.829	64	.438
4	.9976	35	.819	65	.423
5	.996	36	.809	66	.407
6	.995	37	.799	67	.391
7	.993	38	.788	68	.375
8	.990	39	.777	69	.358
9	.988	40	.766	70	.342
10	.985	41	.755	71	.326
11	.982	42	.743	72	.309
12	.978	43	.731	73	.292
13	.974	44	.719	74	.276
14	.970	45	.707	75	.259
15	.966	46	.695	76	.242
16	.961	47	.682	77	.225
17	.956	48	.669	78	.208
18	.951	49	.656	79	.191
19	.946	50	.643	80	.174
20	.940	51	.629	81	.156
21	.934	52	.616	82	.139
22	.927	53	.602	83	.122
23	.921	54	.588	84	.105
24	.914	55	.574	85	.087
25	.906	56	.559	86	.070
26	.899	57	.545	87	.052
27	.891	58	.530	88	.035
28	.883	59	.515	89	.017
29	.875	60	.500	90	.000
30	.866				

EXAMPLE 1.—A motor on test develops an output of 10 horsepower with an input of 50 amperes from a 220-volt circuit. If the motor output, in watts, is 80 per cent. of the input: (*a*) what is the power factor? (*b*) what is the angle of phase difference?

SOLUTION.—(*a*) As 1 H. P.=746 watts, the output of 10 H. P. is 746×10=7,460 watts. Since this is 80 per cent., or .8, of the input, the input must be 7,460÷.8=9,325 real watts. The volt-amperes, or apparent watts, are, however, 220×50=11,000. According to the formula, the power factor

$$\cos\phi=\frac{9,325}{11,000}=.848. \quad \text{Ans.}$$

(*b*) The table of power factors shows that the phase difference corresponding to a power factor of .848 is 32 time-degrees, or the angle of lag is 32°. Ans.

EXAMPLE 2.—Electrical energy is to be supplied at the rate of 165 kilowatts to a load having a power factor of .8. The voltage of the circuit is 2,200. What is the current?

SOLUTION.—According to the formula, $.8=\frac{165,000}{\text{apparent watts}}$; therefore, apparent watts, $IE=\frac{165,000}{.8}=206,250$. But $E=2,200$; therefore,

$$I=\frac{206,250}{2,200}=93.8 \text{ amp.} \quad \text{Ans.}$$

22. Power factors ranging from .75 to 1 are common in practice; lower power factors, down to .5, are sometimes encountered, but anything lower than .5 is very rare. Low power factors are undesirable, because the lower the power factor the greater is the wattless component of the current. This wattless current adds to the heat of the circuit and thereby reduces its ability to carry power current; in fact, the useful capacity of alternators and lines may be often increased by connecting with the circuits devices that correct the power factor or bring it nearer unity.

Synchronous motors can be made to perform this function, and when so used they are known as *synchronous condensers*. Where many induction motors are operating, synchronous motors are frequently employed to perform the double duty of driving some continuously operating machines as pumps or blowers, and simultaneously improving the power factor of the circuit. This subject will be treated in another Section.

EXAMPLES FOR PRACTICE

1. A non-inductive heating device having a resistance of 27.5 ohms is connected to a 110-volt alternating-current circuit. Find the value of the power. Ans. 440 watts

2. If the cost of 1 kilowatt-hour of electrical energy is 10 cents, what will be the cost of operation for 5 hours of the heating device described in example 1? Ans. 22c.

3. A carbon-filament incandescent lamp connected to a 110-volt alternating-current circuit has a current of .5 ampere passing through it. Find the energy consumed in 1,000 hours. Ans. 55 K.-W.-hr.

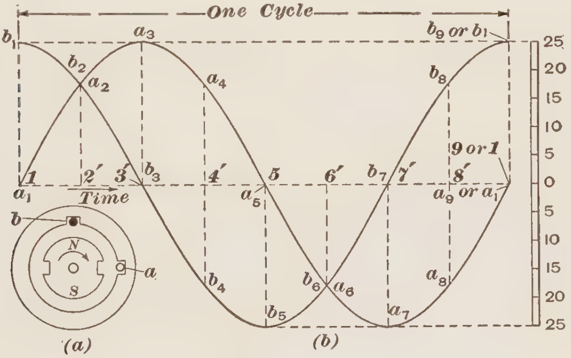
4. A line voltage is 220 and the line current is 50 amperes. If there is a phase displacement of 30 electrical time-degrees between the voltage and the current, what is the power? Ans. 9,526 watts

---

POLYPHASE CIRCUITS

FUNDAMENTAL PRINCIPLES

23. Generation of Polyphase Voltages.—Fig. 28 (*a*) shows an elementary type of alternator, the stator of which has two separate sets of windings, only one conductor of each set, however, being shown at *a* and *b*. The methods of connecting the windings of alternator stators are explained in



(*a*) (*b*)

FIG. 28

another Section. The rotor has two poles, therefore one cycle of voltage changes is established in one revolution of the rotor.

The alternating voltage generated in conductor *a* is similar to that of conductor *a*, Fig. 1, of *Alternating Currents,* Part 1. The conductor *b*, Fig. 28 (*a*), is mounted on the stator 90 space-degrees ahead of conductor *a*, taking ahead to mean in a direction opposite to that of the rotation of the rotor. At the instant when the voltage a_1 of conductor *a* is zero, views (*a*) and (*b*), but about to have a positive value, the voltage b_1 of conductor *b*

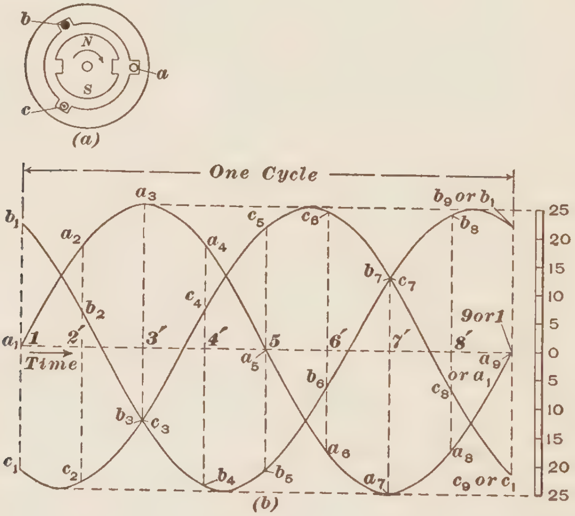


FIG. 29

is positive maximum since the flux from the north pole is cutting directly across conductor *b*. When the rotor has turned 45°, both conductors are equally affected by the flux from the north pole and the voltages b_2 and a_2 of these conductors are equal. The b_2 voltage has decreased from b_1 to b_2 and the a_2 voltage increased from a_1 to a_2. When the rotor has turned through 90°, or one-fourth of a revolution, b_3 voltage is zero and a_3 voltage is positive maximum. The variations in the voltages for both conductors during one cycle, or one revolution of the two-pole rotor, is indicated in view (*b*). In this case, the voltage of conductor *b* leads the voltage of conductor *a* by 90 time-degrees. In this connection it should be noted that voltage b_3 is passing through zero to a negative direction 90 time-degrees ahead of voltage a_5, which is passing through zero to a negative direction.

If an external circuit is connected to the ends of conductor *a* and another circuit to the ends of conductor *b*, view (*a*), the voltages in these two circuits will differ in phase by 90 time-degrees, thus forming a *two-phase,* or *quarter-phase, system.* All circuits that are a combination of two or more phases are also known as **polyphase circuits,** and the currents in such circuits are called *polyphase currents.* Fig. 28 (*a*) shows the elements of a *two-phase alternator.*

24. If conductors *a, b,* and *c,* Fig. 29 (*a*), are mounted 120 space-degrees from each other on the stator of an alternator that has a two-pole rotor, electromotive forces, view (*b*), are established in these conductors that differ in phase by 120 time-degrees. Each single conductor forms a part of one of three distinct windings mounted on the stator. The voltage of conductor *a,* view (*a*), is similar to that of conductor *a,* Fig. 28 (*a*).

The electromotive force of conductor *b,* Fig. 29 (*a*) and (*b*), leads the voltage of conductor *a* by 120 time-degrees because of the relative positions of these conductors on the stator frame. The voltage of conductor *c* leads the voltage of conductor *b* by 120 time-degrees and the voltage of conductor *a* by 240 time-degrees. The voltage of conductor *c* may also be said to lag 120 time-degrees behind the voltage of conductor *a* since there are 360 time-degrees in a cycle and $360-240=120$. The phase relations should be noted at the points where the curves cut the zero line all passing to a positive direction or all passing to a negative direction. It should be also noted that when the voltage of one conductor is maximum, the voltages of the other conductors are one-half maximum, but in the opposite direction. For example, see points a_3, b_3, and c_3.

If an external circuit is connected to the ends of each of the conductors *a, b,* and *c,* the voltages in these three circuits will differ in phase 120 time-degrees, thus forming a *three-phase system.* Fig. 29 (*a*) shows the elements of a *three-phase alternator.*

25. System of Notation for Vectors.—In the succeeding discussion, the system of subscripts used with the symbols representing current and voltage relations in polyphase circuits is chosen to indicate direction of the current or of the voltage. Junction points in circuits, as *a* and *b*, Fig. 30, are lettered, and these letters, used as subscripts, are written in such order as to denote that the direction of the current or voltage is from the junction marked with a first subscript toward the junction marked with a second subscript. For example, the current I_{ab} is the current from *a* to *b*, and E_{ab} is the voltage acting in the direction from *a* to *b* that establishes this current. The current I_{ba} is from *b* to *a*, and the voltage E_{ba} is the voltage acting from *b* to *a* that establishes current I_{ba}.

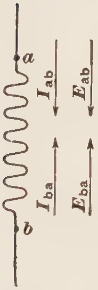


FIG. 30

Since the current I_{ab} is exactly opposite in direction to the current I_{ba}, the currents differ in phase by 180 time-degrees. If current I_{ab} is represented by a vertical vector with its arrowhead pointing downwards, current I_{ba} is represented by a vertical vector with its arrowhead pointing upwards; the two vectors differ in phase by 180 time-degrees. To indicate the direction of a current that is the *reverse* of current I_{ab}, the positions of the subscripts may be reversed, as I_{ba}, or the subscripts may remain unchanged in position and the minus sign prefixed to the symbol, as $-I_{ab}$. The *reversal* of I_{ba} may be indicated by I_{ab} or $-I_{ba}$; hence, $-I_{ba}$ has the same direction as I_{ab}, likewise $-I_{ab}$ has the same direction as I_{ba}.

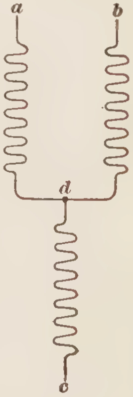


FIG. 31

The order in which the subscripts are taken from the current or voltage diagram must be observed carefully, as a change in their order will change the position of the vector representing that current or voltage in the vector diagram by 180 time-degrees. If the circuit is non-reactive, I_{ab} and E_{ab} or I_{ba} and E_{ba} are in phase; otherwise, there is an angle of lag or lead, the value of which depends on the relative proportion of the resistance and the reactance and on whether the reactance is inductive or condensive.

26. Kirchhoff's First Law.—*As much current is directed toward any junction as away from it.* If during a certain interval of time a given quantity of electricity flows toward a junction point through one or more paths, the same quantity of electricity must flow in the same interval of time away from that junction point through one or more paths. Assume that the current toward the junction point *d*, Fig. 31, is I_{cd} and that the currents away from the junction point are I_{da} and I_{db}, then the law may be expressed by a vectorial equation as follows:

$$I_{cd}=I_{da}+I_{db}$$

When the currents are in phase, the arithmetical sum of the terms in the second member will equal the first member. If, however, the currents are out of phase, the equation will not balance by the arithmetical process if the values of the currents are substituted in the equation, because this addition does not take into consideration the out-of-phase condition of the currents. This vectorial equation, however, does indicate the operation that must be performed with the vectors of the currents represented by the symbols in the equation when solving a vector diagram.

Vectorial equations may be subjected to the intermediate operations possible with algebraic equations, but the final solutions must be by vector diagrams when it is necessary to consider phase differences. For example, in the preceding equation the terms I_{da} and I_{db} of the second member, may be transferred to the first member, provided their signs are changed; the equation then becomes

$$I_{cd}-I_{da}-I_{db}=0$$

For the factors $-I_{da}$ and $-I_{db}$ may be substituted their equivalents I_{ad} and I_{bd}, and the equation becomes

$$I_{cd}+I_{ad}+I_{bd}=0$$

If all of the subscripts are reversed in position, the equation becomes

$$I_{dc}+I_{da}+I_{db}=0$$

These equations serve as a basis for the modified form of Kirchhoff's first law as used in solutions of problems involving alternating currents.

Modified First Law.—*The sum of currents, all of which are directed toward any junction or all directed away from that junction, is zero.*

The subscripts in the equation $I_{cd}+I_{ad}+I_{bd}=0$ indicate that the direction of each current in the three branches is toward the junction *d,* Fig. 31, and if the vectors of the currents, placed in proper phase relation, are added, the vectorial sum is zero. In the equation $I_{dc}+I_{da}+I_{db}=0$, the direction is taken as away from the junction. It should be understood that in each branch circuit, Fig. 31, the current considered as established, for example, toward the junction *d,* would be that due to the electromotive force of *its branch acting alone* without taking into consideration the opposing action of the electromotive forces of the other branches that also tend to establish currents directed toward the junction *d.*

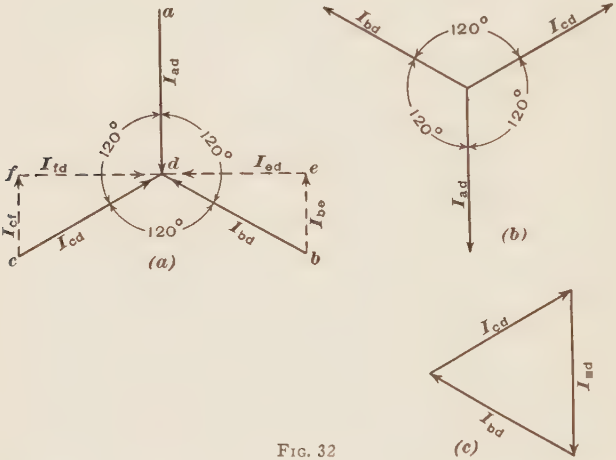


FIG. 32

In Figs. 31 and 32, the branch currents I_{cd}, I_{ad}, and I_{bd} and the corresponding vectors are assumed to be equal, but to differ in phase by 120 time-degrees as indicated when the vectors are referred to a common origin, view (*b*). In accordance with the instruction in *Alternating Currents*, Part 1, resolve vectors I_{cd} and I_{bd}, view (*a*), into vertical and horizontal components as shown. The horizontal component vectors I_{ed} and I_{fd} are of equal magnitude, but differ in phase by 180 time-degrees; therefore, these two component currents neutralize each other.

The two vertical component vectors I_{be} and I_{cf} are equal to each other and are each equal to one-half of vector I_{ad} since the sine of 30° is .5. The vertical component vectors I_{be} and I_{cf} act upward and the vertical vector I_{ad}, downward; therefore, the combined action of these two component vectors neutralizes the action of vector I_{ad}. The vectorial sum, therefore, of the currents $I_{cd}+I_{ad}+I_{bd}=0$, as stated in the equation and the modified first law.

In the vector diagram, view (*c*), the vectors of the branch currents are so placed that the arrow end of one vector is connected to the origin end of the next vector and the general direction around the triangle thus formed is clockwise. Start to trace at vector I_{cd}, and note that the arrow end of vector I_{bd} joins the origin end of vector I_{cd}; thus, the resultant is zero, because the three component vectors complete the closed vector diagram without the necessity of a resultant vector.

27. Kirchhoff's Second Law.—*In any closed electric circuit, the combined opposing voltages are equal to the combined applied voltages acting in that circuit.* A clear understanding of the term *opposing voltage* is necessary. Thus, the opposing voltage caused by inductive reactance is the *counter electromotive force of self-induction,* not the inductive reactance voltage drop necessary to overcome it. The opposing voltage is equal in value to the inductive reactance voltage drop, but differs from it by 180 time-degrees. The opposing voltage *lags* behind the current 90 time-degrees, while this inductive reactance voltage drop leads the current by the same angle.

Similarly, the opposing voltage caused by condensive reactance is the *counter electromotive force* equal in value to the condensive reactance voltage drop, but in exact opposition to it; the opposing voltage *leads* the current by 90 time-degrees, in this case, while the condensive reactance voltage drop lags behind the current by the same angle. The phase relations of the reactance voltage drops and counter voltages are indicated in Figs. 2 and 4 and explained in *Alternating Currents,* Part 1.

There is no counter electromotive force in case of resistance, the voltage drop being usually considered as consumed by the resistance. For the purpose of Kirchhoff's law, however, it is necessary to assume that the voltage drop instead of being consumed is opposed or counterbalanced by another voltage, the voltage drop being in phase with the current and the opposing voltage being 180 time-degrees out of phase with the voltage drop; that is, in exact opposition.

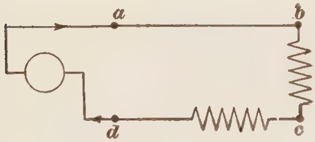


FIG. 33

28. When all of the applied voltages acting in a like direction around the circuit are denoted either by the sign of addition (+) or of subtraction (−), and all of the opposing voltages by the opposite sign, Kirchhoff's second law is usually stated thus: *The algebraic sum of all voltages in any closed circuit is zero.* In tracing the voltages around a circuit, one direction is assumed as a *positive direction.* The positive direction may correspond to the actual direction of current in the circuit or it may be assumed as opposite in direction to the flow of electricity. In any case, + or − signs should be assigned to the various applied and counter voltages of the circuit in accordance with whether these voltages *agree* or *disagree* with the assumed direction called positive.

In Fig. 33, the actual direction of current at a given instant is taken as that indicated by the arrowheads on the lines representing the circuit *a b c d.* The positive direction, in this case, is assumed to be clockwise, which corresponds to the direction of the current. The applied voltage in the alternator is, therefore, $+E_{da}$ and the opposing voltages, caused by the two resistors, are $-E_{cb}$ and $-E_{dc}$. The resistance of the connecting wires is considered negligible. According to the statement of the law and to the assignment of the + and − signs,

$$E_{da}+(-E_{cb})+(-E_{dc})=0$$

$$E_{da}-E_{cb}-E_{dc}=0;\text{ or, } E_{da}=E_{cb}+E_{dc}$$

The last equation states that the applied voltage E_{da} is equal to the sum of the opposing voltages caused by the two resistors.

In the equation $+E_{da}-E_{cb}-E_{dc}=0$, substitute for $-E_{cb}$ and $-E_{dc}$ their equivalents $+E_{bc}$ and $+E_{cd}$; then,

$$E_{da}+E_{bc}+E_{cd}=0$$

It should be noted that the subscripts indicate a clockwise tracing around the circuit.

Suppose that the positive direction is taken as counter-clockwise, which is opposite the direction of current, then the alternator voltage is $-E_{da}$ and the opposing voltages $+E_{cb}$ and $+E_{dc}$. According to the law and the assignment of the + and − signs,

$$+(-E_{da})+E_{cb}+E_{dc}=0$$

$$-E_{da}+E_{cb}+E_{dc}=0;\text{ or, } E_{da}=E_{cb}+E_{dc}$$

The last equation is the same as that previously derived when the positive direction was taken as clockwise.

In the equation $-E_{da}+E_{cb}+E_{dc}=0$, substitute for $-E_{da}$ its equivalent E_{ad}, and reverse the positions in the equation of E_{cb} and E_{dc}; then,

$$E_{ad}+E_{dc}+E_{cb}=0$$

It should be noted that in this case the subscripts indicate a counter-clockwise tracing around the circuit.

The two final equations derived for the clockwise and counter-clockwise tracings serve as a basis for the modified form of Kirchhoff's second law as used with vectorial equations in alternating-current problems

Modified Second Law.—*The vectorial sum of voltages taken completely around any circuit, with their directions assumed either all clockwise or all counter-clockwise, is equal to zero.*

29. It should be clearly understood that the equations written in accordance with Kirchhoff's laws indicate operations with vectors; that is, the factors in the equation indicate whether the vectors they represent are to be added or subtracted to obtain the resultant. When drawing the individual component vectors with relation to the others, the phase relation of each vector must be taken into consideration. The general direction of the component vectors around the diagram may be either clockwise or counter-clockwise with like results. In no case where phase differences exist should the numerical values of voltages or currents represented by these vectors be substituted in the equation and the equation solved algebraically, as the results would then be erroneous. The numerical value indicates the magnitude only; both magnitude and phase relation must be considered in combining alternating currents or voltages.

It should be remembered also, that if the connections of any coil to the line are reversed, or the direction of the current in the coil is reversed, the vector representing the electromotive force or the current of that coil is likewise reversed, or its phase relation changed 180 time-degrees.

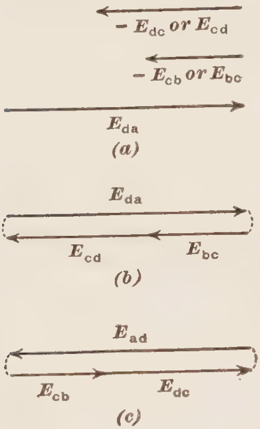


FIG. 34

30. Fig. 34 is a vector diagram indicating assumed voltage conditions in the circuit, Fig. 33. The positive direction is at first assumed to agree with the direction of the current; then,

$$E_{da}+(-E_{cb})+(-E_{dc})=0; \text{ or, } E_{da}+E_{bc}+E_{cd}=0$$

The applied, or alternator, voltage is indicated by vector E_{da}, Fig. 34 (*a*), and the opposing voltages due to the action of the resistors by vectors $-E_{cb}$ or E_{bc} and $-E_{dc}$ or E_{cd}. The latter vectors are 180 time-degrees out of phase with the vector for the alternator voltage. The directions of the vectors E_{bc} and E_{cd} are, therefore, taken as opposite the direction of vector E_{da}. When the three vectors are to be added, the arrow end of each component vector must be connected to the origin end of the next vector. In this case the vectors will really all lie along the same line because of the 180 time-degree phase relation, but in the vector diagram, view (*b*), a little space is left between the alternator vector and the opposing vectors in order to show clearly the relative positions and directions of the vectors. It should be noted that the three component vectors when added complete the vector diagram without the necessity of a resultant vector. The vectorial sum of the component voltages is, therefore, zero.

If the positive direction is assumed to be in the opposite direction to that of the current, Fig. 33, then $E_{ad}+E_{dc}+E_{cb}=0$. The direction of each of the three component vectors, Fig. 34 (*a*), are reversed in view (*c*), and this vector diagram also indicates that the vectorial sum of the three component voltages is zero.

TWO-PHASE CIRCUITS

31. Two-Phase, Four-Wire Circuits.—The simplest two-phase circuit is a four-wire system, such as is shown diagrammatically in Fig. 35. It consists of two single-phase circuits, each of which is loaded independently of the other. Here, line voltage equals phase voltage and line current equals phase current. The voltages of the two separate phase windings of the alternator differ in phase by 90 time-degrees. Care is usually taken to load the phases as equally as possible in order to have like values of current in the windings of the alternator. If the loads are equal, the system is said to be *balanced*.

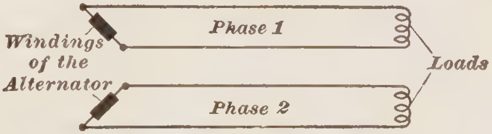


FIG. 35

The voltage and current calculations are carried out separately for each phase, in the same way as for single-phase circuits. The total power of the two-phase, four-wire system is the sum of the powers in the separate phases.

32. Two-Phase, Three-Wire Circuits.—In the case of a perfectly balanced load on a two-phase system, some economy in copper and insulators may be made by combining one conductor of phase *1* and one of phase *2*, Fig. 35, into one single conductor, usually called the *return*, or *neutral, conductor.* This system, known as a *two-phase, three-wire system,* is shown in Fig. 36. The three line wires are shown at *A*, *B*, and *C*, and *B* represents the return conductor. The two sets of phase windings *a b* and *c b* on the stator of the two-phase alternator are connected together and the return conductor is connected to the junction point *b*. The voltages of the two windings differ in phase by 90 time-degrees.

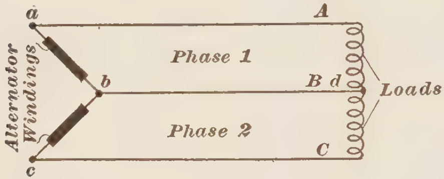


Fig. 36

The voltage and current relations in two-phase, three-wire circuits are determined as follows: Consider the currents directed toward the junction *b*, then according to Kirchhoff's first law,

$$I_{ab}+I_{cb}+I_{db}=0$$

Transfer I_{db} to the other member of the equation; then,

$$I_{ab}+I_{cb}=-I_{db}\text{; or, } I_{bd}=I_{ab}+I_{cb}$$

The last equation indicates that the vectors for the currents in the two line wires *A* and *C*, Fig. 36, are *added* to produce the resultant vector representing the current in the neutral wire *B*.

The currents I_{ab} and I_{cb} in the phase windings and in wires *A* and *C* are established by voltages E_{ab} and E_{cb} of phases *1* and *2*, respectively; these currents, therefore, differ in phase by 90 time-degrees. In a balanced system, these currents are equal and can be represented by equal vectors.

In Fig. 37 (*a*), the vector I_{cb} is assumed to lead vector I_{ab} by 90 time-degrees. In view (*b*), vector I_{ab} is added to vector I_{cb} to produce the resultant vector I_{bd} which represents the current in the neutral wire *B*, Fig. 36.

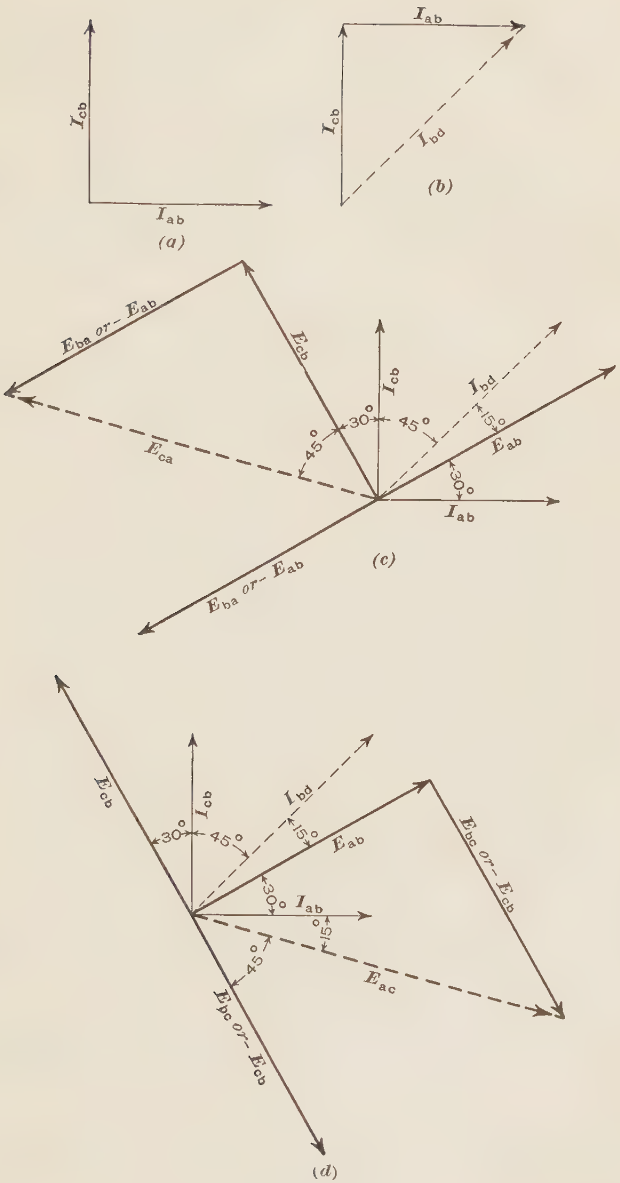

I_{cb}
I_{ab}
(a)
I_{ab}
I_{cb}
I_{bd}
(b)
E_{ba} or $-E_{ab}$
E_{cb}
I_{cb}
I_{bd}
30°
45°
45°
15°
E_{ab}
E_{ca}
30°
I_{ab}
(c)
E_{ba} or $-E_{ab}$
E_{cb}
I_{cb}
I_{bd}
30°
45°
15°
E_{ab}
E_{bc} or $-E_{cb}$
30°
I_{ab}
15°
45°
E_{ac}
E_{bc} or $-E_{cb}$
(d)

FIG. 37

33. According to Kirchhoff's second law, the equation for voltages taken in a counter-clockwise direction around the circuit *a–b–c–C–load–d–load–A–a* is

$$E_{ca}+E_{ab}+E_{bc}=0$$

In tracing E_{ca}, trace through the load in the external circuit, not through the air from terminal *c* to terminal *a*. Transfer E_{ab} and E_{bc} to the other member of the equation, then $E_{ca}=-E_{ab}-E_{bc}$, or

$$E_{ca}=E_{ba}+E_{cb}$$

The voltage E_{ca} across the two outer wires *C* and *A*, Fig. 36, is the vectorial sum of the equal voltages across the phase windings when the vectors representing the phase voltages are drawn in the direction *indicated by the subscripts*. The vectors for the currents are shown in proper phase relation in Fig. 37 (*c*). In this problem it is assumed that the voltages of the phase windings lead the currents in the phase windings by 30 time-degrees and that current I_{cb} is established by voltage E_{cb} and that current I_{ab} is established by voltage E_{ab}. In view (*c*), the component voltage vectors and component current vectors are drawn with the voltage vectors leading their respective current vectors by 30 time-degrees. In the vectorial equation $E_{ca}=E_{ba}+E_{cb}$, it should be noted that the subscripts of E_{ba} indicate a direction of voltage opposite to that of the voltage represented by E_{ab}. The vector representing E_{ba} or $-E_{ab}$, view (*c*), is, therefore, drawn at an angle representing 180 time-degrees from the vector E_{ab}. The vectors E_{ba} and E_{cb} are added to produce the resultant vector E_{ca}.

The voltage E_{ca} is that between alternator terminals *c* and *a*, Fig. 36, in the direction from line wire *C* to line wire *A*, which are the line wires connected to terminals *c* and *a*.

34. If the voltages, Fig. 36, are traced in a clockwise direction around the circuit *a–A–load–d–load–C–c–b–a*; then,

$$E_{ac}+E_{cb}+E_{ba}=0$$

Transfer E_{cb} and E_{ba} to the other member of the equation and reverse the subscripts; then,

$$E_{ac}=E_{ab}+E_{bc}$$

Fig. 37 (*d*) indicates the vector diagram for these conditions. The vector for E_{cb} is turned through an angle representing 180 time-degrees and then produces vector E_{bc}. The vectorial sum of E_{ab} and E_{bc} produces the resultant vector E_{ac}. The voltage E_{ac} is that between alternator terminals *a* and *c*, Fig. 36, in the direction from line wire *A* to line wire *C*, which are connected to terminals *a* and *c*.

Voltage E_{ca} differs in phase by 180 time-degrees from voltage E_{ac} due to the manner of subtracting the component vectors. In Fig. 37 (*c*), voltage E_{ab} is subtracted from voltage E_{cb}, and in Fig. 37 (*d*), voltage E_{cb} is subtracted from voltage E_{ab}. In connection with this problem refer to the vectorial diagrams relating to the addition and subtraction of two equal vectors differing in phase by 90 time-degrees in *Alternating Currents*, Part 1.

35. Fig. 37 shows the relative values and phase relations of the currents and voltages in a balanced two-phase, three-wire system, Fig. 36, in which the current in each phase lags 30 time-degrees behind its voltage. The vectors E_{ca} or E_{ac}, Fig. 37, are the diagonals of a square of which vectors representing the voltages in the phase windings form two sides. The vector I_{bd}, Fig. 37 (*b*), which represents the current in the neutral wire, is the diagonal of a square formed by vectors representing the currents in the phase windings or in the outer line wires, Fig. 36. Since the diagonal of a square is $\sqrt{2}$, or 1.414, times a side, the following statements are true of a balanced two-phase, three-wire system:

1. The voltage across outside wires $= 1.414 \times$ phase voltage.
2. The current in a neutral wire $= 1.414 \times$ phase current.

EXAMPLE.—In a balanced two-phase, three-wire system, the phase voltage is 440 and the phase current 100 amperes. (*a*) What is the voltage across the outside conductors? (*b*) What is the current in the neutral conductor?

SOLUTION.—(*a*) The voltage across the outside conductors is $1.414 \times 440 = 622$. Ans.

(*b*) The current in the neutral conductor is $1.414 \times 100 = 141.4$ amp. Ans.

36. Two-Phase Interconnected Circuits.—Sometimes the winding of each phase of two-phase devices is in two equal parts interconnected as in either Fig. 38 or Fig. 39. The first is known as the *star connection* and the second as the *mesh connection,* both of which terms are more commonly used with three-phase systems, as is explained later. The windings marked *I* belong to one phase and those marked *II* to the other. Considering the star connection, Fig. 38, the voltage generated in part *a o* is in phase with the voltage generated in part *o c* as these two parts form the complete winding *a o c* of phase *I*. In a similar manner the voltage generated in part *b o* is in phase with the voltage generated in part *o d* as these two parts form the complete winding *b o d* of phase *II*. The voltages of the two complete phase windings differ in phase by 90 time-degrees.

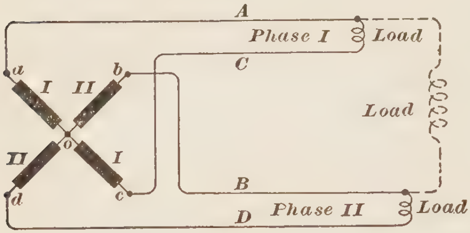


Fig. 38

37. With the star connection, Fig. 38, the voltages between the outside terminals and the common junction point *o* of the parts of the alternator windings *a o, b o, c o,* and *d o* are each equal to one-half of the voltage of the complete phase windings *a o c* or *b o d,* which total voltage may be called the *phase winding voltage*. The currents in the line wires *A, B, C,* and *D* are each equal to the current in its part of the alternator winding *a o, b o, c o,* and *d o,* since the line wires are in series with these windings. Consider the currents toward the junction point *o,* then according to Kirchhoff's first law,

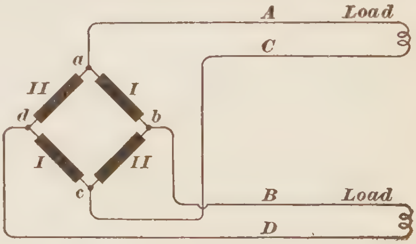


Fig. 39

$$I_{ao}+I_{bo}+I_{co}+I_{do}=0$$

If these currents are assumed to be equal, as they would be in a balanced system, their vectors, placed in proper phase

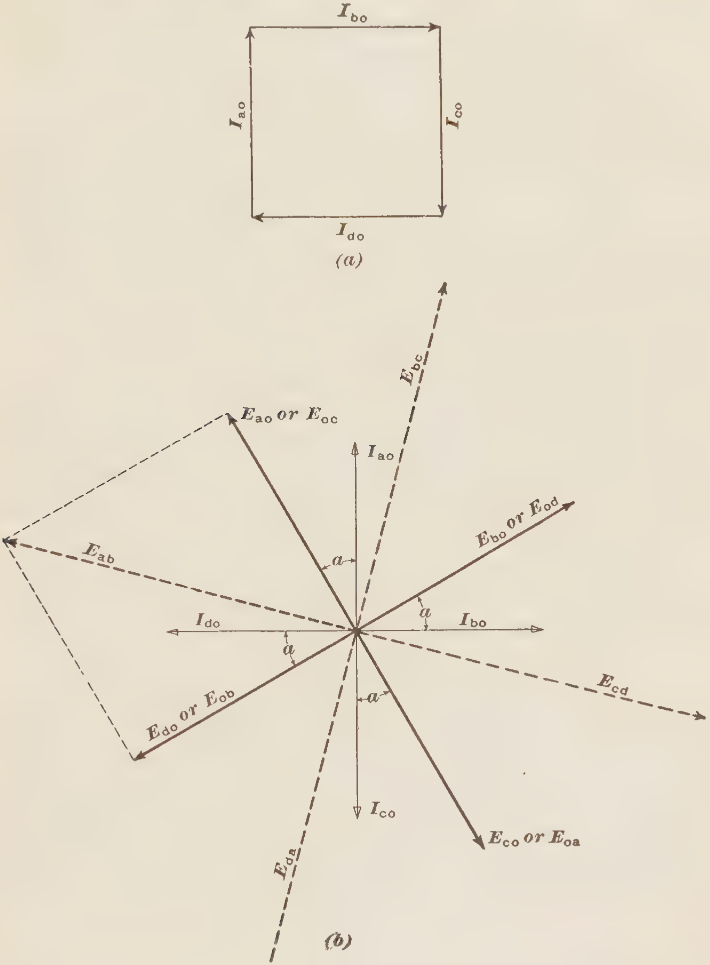


FIG. 40

relation, will be as shown in Fig. 40 (*a*). The vectorial sum of these four component vectors is zero since the diagram is closed without the necessity of a resultant vector. The current vectors can be drawn so as to have a common origin as in view (*b*). It should be noted how the subscripts are taken; if I_{co} were reversed, the vector representing I_{oc} would then coincide with vector I_{ao}, which is correct, since the current from *a* to *o*, Fig. 38, is in phase with the current from *o* to *c* because these two partial windings form the complete winding of phase *I*. The same reasoning applies to vector I_{bo} and to the reversal of vector I_{do}, Fig. 40 (*b*).

If the currents are assumed to lag behind the voltage in their respective parts of the windings by an angle of *a*, view (*b*), the corresponding voltage vectors can be drawn as shown. The arrangement of the subscripts on the voltage vectors should be noted in connection with the preceding statements regarding phase relations of the currents. For example, the vectors for E_{ao} and E_{oc} coincide in position.

The voltage across the wires *A* and *C* or *B* and *D*, Fig. 38, which may be called the *phase line voltage*, is equal to the phase winding voltage. The voltage across the wires *A* and *B*, *B* and *C*, *C* and *D*, or *D* and *A* is formed according to Kirchhoff's second law. Assume that a load is connected between line wires *A* and *B* as indicated by the dotted lines, Fig. 38, and consider the voltages in a clockwise direction around the circuit *a–A–load–B–b–o–a*. Then the voltage across *A* and *B* may be determined as follows:

$$E_{ab}+E_{bo}+E_{oa}=0;$$

therefore, $$E_{ab}=-E_{bo}-E_{oa}=E_{ob}+E_{ao}$$

At the end of vector E_{ao}, Fig. 40 (*b*), draw a vector equal and parallel to vector E_{ob}, then the resultant vector E_{ab} represents the voltage between line wires *A* and *B*. Each vector E_{ao} and E_{ob} represents one-half the phase winding voltage, therefore the line voltage between *A* and *B* is

$$E_{ab}=\frac{E}{2}\times 1.414=.707\,E$$

in which *E* is the phase winding voltage or the phase line voltage.

The voltage across the other combinations of line wires, *B* and *C*, *C* and *D*, or *D* and *A*, Fig. 38, may be found in a similar manner.

EXAMPLE.—If the phase line voltage between the conductors *A* and *C*, Fig. 38, is 220, what is the line voltage between conductors *A* and *B*?

SOLUTION.—The voltage between *A* and *C* is the phase line voltage *E*; therefore, according to the preceding formula, the line voltage between *A* and *B* is $E_{ab}=.707\times 220=155.5$. Ans.

38. Fig. 39 shows the mesh connection of the windings on the stator of the two-phase alternator. The voltages in the parts marked *I* are in phase and the voltages in the parts marked *II* are also in phase, but these latter voltages differ in phase by 90 time-degrees from the former voltages. The voltage for each part of the winding, such as between the terminals *a b*, *b c*, *c d*, and *d a*, are each equal to one-half the phase winding voltage of the complete phase winding *a o c* or *b o d*, Fig. 38, or $\frac{E}{2}$. It is at first assumed that Figs. 38 and 39 represent different connections of the windings on the same alternator. By applying Kirchhoff's second law to the voltages for the mesh connections, Fig. 39, and considering them in a clockwise direction around the closed circuit *a–b–c–d*; then,

$$E_{da}+E_{ab}+E_{bc}+E_{cd}=0$$

The vectors for these voltages can be represented as in Fig. 41. The arrangements of the subscripts should be noted in order to check the phase relations of the vectors.

To determine the voltage across line wires *A* and *C*, Fig. 39, consider the circuit *a–A–load–C–c*, and either path *c–d–a* or *c–b–a*. There are two paths in parallel from *c* to *a*. Each path consists of one part of phase winding *I* and one part of phase winding *II*; therefore, the voltages of the two paths are equal and the two voltages that are in series in each path differ in phase by 90 time-degrees. In the following equation path *c–d–a* is taken. According to Kirchhoff's second law,

$$E_{ac}+E_{cd}+E_{da}=0;\text{ or, } E_{ac}=-E_{cd}-E_{da}=E_{dc}+E_{ad}$$

From this equation the resultant vector E_{ac}, Fig. 41, is drawn. Vector E_{ac} is the diagonal of a square of which E_{dc} and E_{ad}, each equal to $\frac{E}{2}$, form two sides; therefore, the voltage across line wires A and C, Fig. 39, equals $\frac{1.414\,E}{2} = .707\,E$.

The voltage across line wires B and D may be determined in a similar manner. The voltage across line wires A and B, B and C, C and D, or D and A is $\frac{E}{2}$ since these pairs of wires are connected to the terminals of parts of phase windings $a\,b$, $b\,c$, $c\,d$, and $d\,a$.

If the voltages of the mesh connection, Fig. 39, are considered *independently* of the star connection and the voltage between line wires A and C or B and D is taken as E_1, the voltage across A and B, B and C, C and D, or D and A is $.707\,E_1$. This is shown by the following: The voltages across the pair of wires A and B and the pair B and C differ in phase

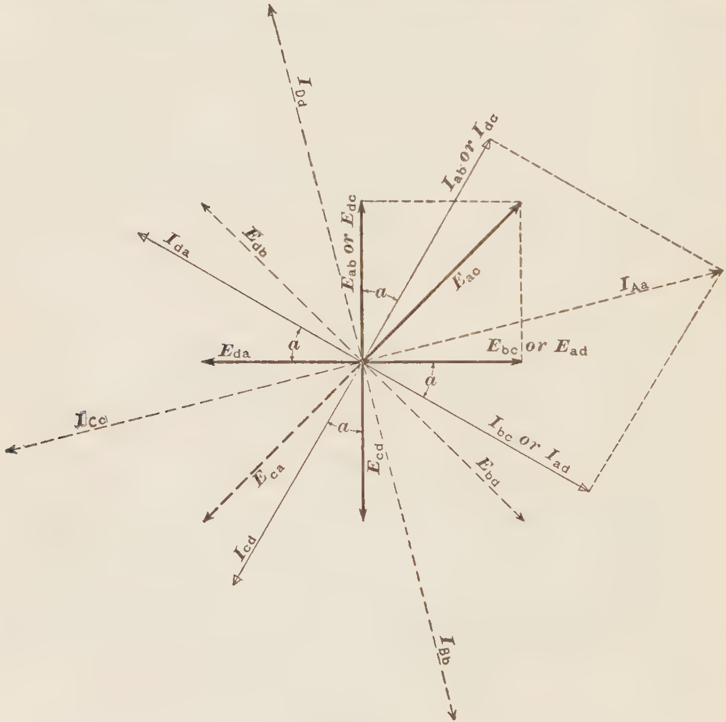


FIG. 41

by 90 time-degrees; then, the voltage between A and $C = E_1 = \sqrt{(.707\,E_1)^2+(.707\,E_1)^2}$.

39. The currents in line wires A, B, C, and D, Fig. 39, are determined by means of Kirchhoff's first law. First consider the junction point a, then the vectorial sum of the currents in the paths connected to the junction point is,

$$I_{Aa}+I_{da}+I_{ba}=0;\text{ or, } I_{Aa}=I_{ad}+I_{ab}$$

The equations for the currents in the other line wires may be formed by considering the currents directed toward the other junction points b, c, and d. Assume that the current in each partial phase winding lags behind the voltage of that winding by an angle a, Fig. 41. The vectors for the currents I_{ab}, I_{bc}, etc. are drawn at an angle a behind the corresponding vectors for voltages E_{ab}, E_{bc}, etc. In the foregoing equation, the vectorial sum of I_{ad} and I_{ab} is the resultant vector I_{Aa}.

The current in line wire A, Fig. 39, represented by vector I_{Aa}, Fig. 41, is, in the case of a balanced system, the diagonal of a square having I_{ad} and I_{ab} for two sides. The other line currents are also made up of two equal component currents that differ in phase by 90 time-degrees.

Let I = current in any one of the four parts of the winding;
I_e = current in any line wire.

Then, $$I_e=1.414\,I$$

Example.—If the current in a line wire of a balanced two-phase interconnected (mesh) system is 283 amperes, what is the current in any one of the four equal parts of the winding?

Solution.—According to the preceding formula, $283 = 1.414\,I$. Therefore, $I = 283 \div 1.414 = 200$ amp. Ans.

THREE-PHASE CIRCUITS

40. Three-Phase Star Connection.—The voltage and current conditions in the phase windings on the stator of a three-phase alternator and in the line wires to which the alternator is connected will now be considered. The voltages in the three alternator windings differ in phase by 120 time-degrees. The two usual methods of grouping the phase windings are by the **star** and **delta** methods of connection. The star connection is also called the Y connection, and the delta connection (△) is also known as the mesh connection.

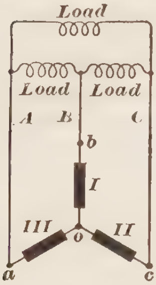


FIG. 42

Joining an end of each of the phase windings *I, II,* and *III,* Fig. 42, of a three-phase alternator to a common point, called a *neutral point,* as at *o,* forms a *three-phase star connection.* The voltage across any phase winding, as E_{oa}, E_{ob}, or E_{oc}, is the *phase winding voltage,* and the voltage between any two line wires, as E_{ab}, E_{bc}, or E_{ca}, is the *line voltage.* If the loads on the system are balanced, the three line voltages are equal.

In order to form vectorial equations for line voltages based on Kirchhoff's second law, consider the following circuits, Fig. 42.

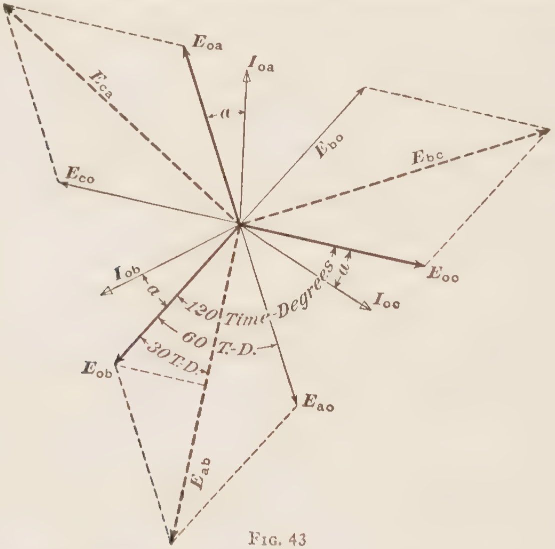


FIG. 43

Terminal *a–A-load–B–b–o–a*; terminal *b–B–load–C–c–o–b*; terminal *c–C–load–A–a–o–c*. The vectorial equations are:

$$E_{ab}+E_{bo}+E_{oa}=0; \text{ or, } E_{ab}=E_{ao}+E_{ob} \qquad (1)$$

$$E_{bc}+E_{co}+E_{ob}=0; \text{ or, } E_{bc}=E_{bo}+E_{oc} \qquad (2)$$

$$E_{ca}+E_{ao}+E_{oc}=0; \text{ or, } E_{ca}=E_{co}+E_{oa} \qquad (3)$$

Each line wire is in series with a phase winding; therefore,

line current = phase winding current

According to Kirchhoff's first law, the sum of the currents directed from the neutral point of a three-phase star-connected system equals zero, or, in Fig. 42,

$$I_{oa}+I_{ob}+I_{oc}=0$$

The vectors for the three line currents differ in phase by an angle representing 120 time-degrees and may be drawn as in Fig. 43. The three-phase winding voltages E_{oa}, E_{ob}, and E_{oc} may also be represented by vectors. If the power factor of the system is less than unity because of inductive reactance, these voltage vectors will lead the corresponding current vectors by an angle a. The value of angle a depends on the properties of the circuit.

The vectors of the three line voltages E_{ab}, E_{bc}, and E_{ca} may be drawn in accordance with equations **1**, **2**, and **3**, but it must be remembered that E_{ao} is the reverse of E_{oa}, E_{bo} the reverse of E_{ob}, and E_{co} the reverse of E_{oc}.

41. In a three-phase star connection, a phase difference of 30 time-degrees always exists between the line voltage and the phase winding voltage, as indicated by the angle between vectors E_{ab} and E_{ob}, Fig. 43. Since E_{ab} is the diagonal of a parallelogram, the sides of which are equal, a perpendicular to E_{ab} drawn from the extremity of the vector E_{ob} bisects the vector E_{ab}. Then, $\frac{\frac{1}{2}E_{ab}}{E_{ob}}=\cos 30°$, $\frac{1}{2}E_{ab}=E_{ob}\cos 30°$, and $E_{ab}=2E_{ob}\cos 30°$. In Table I, cos 30° is found to be .866; therefore,

$$E_{ab}=2\times .866\,E_{ob}=1.732\,E_{ob};$$

or, *line voltage = 1.732 × phase winding voltage*

Example 1.—If the voltage across a phase winding is 6,350, what is the line voltage with phase windings connected in star?

Solution.—According to the formula just given,

line voltage = 1.732 × 6,350 = 11,000, approx. Ans.

Example 2.—The line voltage is 6,600. What is the phase winding voltage, or the voltage between any of the line wires and the neutral point?

SOLUTION.—According to the formula, 6,600=1.732×phase-winding voltage; or,

phase-winding voltage=6,600÷1.732=3,810. Ans.

42. The **three-phase four-wire system,** shown diagrammatically in Fig. 44, is a modification of a star connection. The fourth conductor O is connected to the neutral point and is known as the *neutral conductor.* Motors are connected with the three main conductors A, B, and C, and lamps are usually connected between each of these conductors and the neutral O. The voltage between any outside conductor and the neutral is the same as the phase winding voltage; the line voltage between any two outside conductors, as A and B, B and C, or C and A, is the same as with a star connection. The line currents are the same as the phase winding currents. With equal loads on each phase winding, that is, with the system balanced, the neutral carries no current; otherwise, the current in the neutral wire depends on the extent of unbalancing.

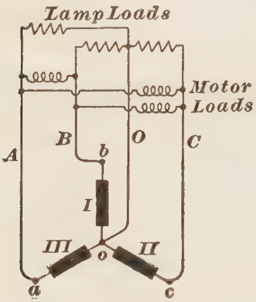


FIG. 44

43. Three-Phase Delta Connection.—Joining the three phases of a three-phase winding end to end, so as to form a closed circuit, as in Fig. 45, is called *mesh,* or *delta, connection.* According to Kirchhoff's second law applied to the closed circuit a–b–c,

$$E_{ab}+E_{bc}+E_{ca}=0$$

The vectors of these voltages and of their corresponding phase winding currents I_{ab}, I_{bc}, and I_{ca} are shown in Fig. 46, it being assumed that the current lags behind the voltage by an angle a. Fig. 45 shows that the voltage between any two of the line conductors A, B, and C is the voltage of the phase winding joining the two; that is,

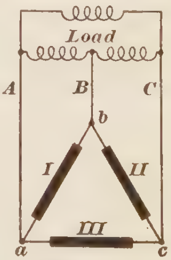


FIG. 45

line voltage = phase winding voltage

The line currents in case of a delta connection are determined according to Kirchhoff's first law. Consider the junction point *a*, Fig. 45; then,

$$I_{Aa}+I_{ba}+I_{ca}=0\text{; or, } I_{Aa}=I_{ab}+I_{ac}$$

Similar equations can be written for the line currents I_{Bb} and I_{Cc} and the line-current vectors drawn accordingly, as in Fig. 46.

44. In a three-phase delta connection, a phase difference of 30 time-degrees always exists between the line currents and phase winding currents, which are the same as the currents between any pair of line wires, or the phase currents. This phase difference is indicated between I_{Aa} and I_{ac}, Fig. 46. By the same line of reasoning as followed in Art. **41,** $\frac{1}{2} I_{Aa}=I_{ac} \cos 30°$; $I_{Aa}=2\times I_{ac}\times .866=1.732\ I_{ac}$, or

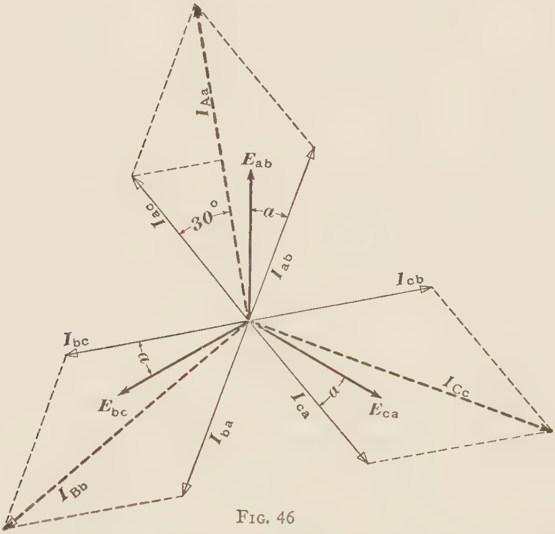


FIG. 46

line current = 1.732 × phase winding current

EXAMPLE.—The line current of a delta-connected, three-phase alternator is 3,464 amperes. What is the current per phase winding, or the current between any pair of line wires?

SOLUTION.—According to the formula just given,

$$3{,}464 \div 1.732 = 2{,}000 \text{ amp.} \quad \text{Ans.}$$

45. Fig. 47 is similar to Fig. 46 except that the three-phase delta system is assumed to have unity power factor. The vector for the phase winding voltage E_{ab}, Fig. 47, will now coincide in direction with the vector for the corresponding phase winding current I_{ab}. The same condition applies to vectors E_{bc} and I_{bc}, and E_{ca} and I_{ca}. Line-current vector I_{Cc} will then be at right angles to the vector for the phase winding voltage E_{ab}, indicating that the current and the voltage represented by these vectors differ in phase by 90 time-degrees. This feature of a three-phase system is utilized in some forms of reactive power indicators, as described in a later Section.

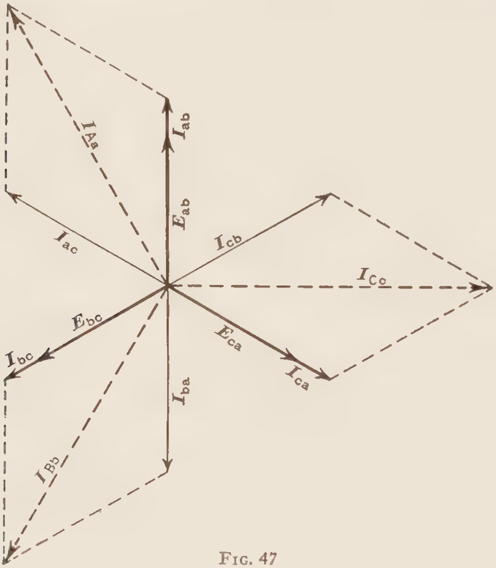


FIG. 47

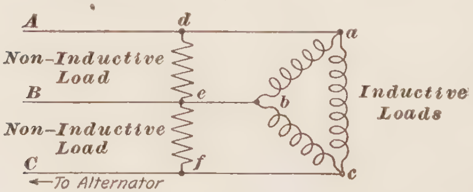


FIG. 48

46. Three-Phase Unbalanced Loads.—A vector diagram may be employed in solving a problem in which an unbalanced load on a three-phase system is involved. For example, delta-connected inductive loads, Fig. 48, are supplied from a three-phase, 440-volt circuit. If only the three equal inductive loads be considered, the assumed current per phase, which is the current passing through the inductive load connected between any two line wires, is 150 amperes, and the power factor of the load is taken as .866. In addition to the inductive loads, two equal non-inductive loads of incandescent lamps are connected to the system, one load between points *d* and *e* on line wires *A* and *B* of one phase and the other load between points *e* and *f* on line wires *B* and *C* of another phase. Each of these non-inductive loads establishes a phase current of 100 amperes at unity power factor between the line wires *A* and *B*, and *B* and *C*. There is no separate non-inductive load connected to the phase between line wires *C* and *A*, therefore the load on this phase is less than that on the others and the system as a whole is unbalanced.

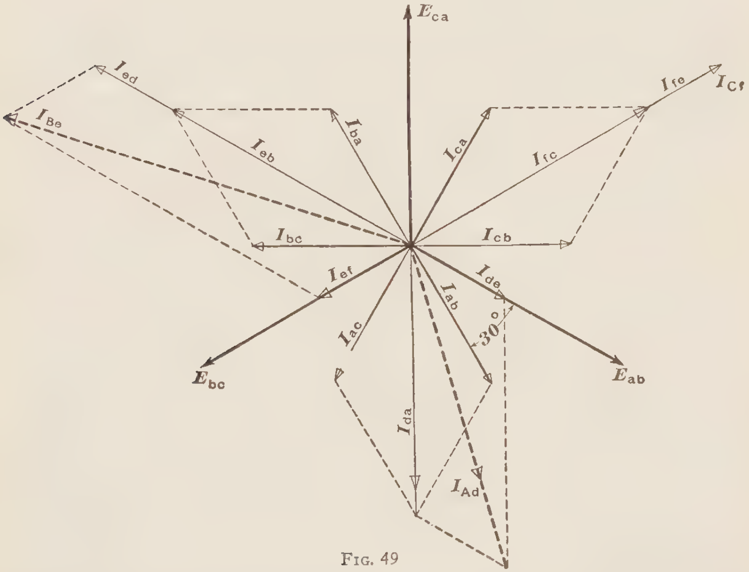


FIG. 49

First consider the vectors for the balanced *inductive loads.* The power factor of the inductive load is .866, which corresponds to an angle of the phase current of 30 time-degrees (see Table I) behind the phase voltage. The phase voltage is the voltage between a pair of line wires. The vectors for the three equal phase voltages E_{ab}, E_{bc}, and E_{ca}, Fig. 49, differ in position by angles representing 120 time-degrees. The vector for the corresponding inductive-load currents I_{ab}, I_{bc}, and I_{ca} are drawn at angles representing 30 time-degrees behind the vectors for their voltages.

To find the currents in the portions of the three line conductors between *d* and *a*, *e* and *b*, and *f* and *c*, Fig. 48, that are established by the inductive loads, equations are written for the points *a, b,* and *c* according to Kirchhoff's first law, as follows:

Point *a*,

$$I_{da}+I_{ba}+I_{ca}=0; \text{ or, } I_{da}=I_{ab}+I_{ac} \qquad (1)$$

Point *b*,

$$I_{eb}+I_{ab}+I_{cb}=0; \text{ or, } I_{eb}=I_{ba}+I_{bc} \qquad (2)$$

Point *c*,

$$I_{fc}+I_{ac}+I_{bc}=0; \text{ or, } I_{fc}=I_{ca}+I_{cb} \qquad (3)$$

In accordance with these three equations, vectors I_{da}, I_{eb}, and I_{fc}, Fig. 49, are drawn, bearing in mind that $I_{ac}=-I_{ca}$, $I_{ba}=-I_{ab}$, and $I_{cb}=-I_{bc}$. With the assumed angle of lag, vector I_{da} is 180° from vector E_{ca}, vector I_{eb} is 180° from E_{ab}, and vector I_{fc} is 180° from vector E_{bc}.

The total currents in the line conductors *A, B,* and *C,* Fig. 48, from the alternator to junction points *d, e,* and *f,* are the vectorial sum of the line currents required for the inductive and non-inductive loads. The vectors for these total line currents may be drawn in accordance with equations based on Kirchhoff's first law; thus,

Point *d*,

$$I_{Ad}+I_{ad}+I_{ed}=0; \text{ or, } I_{Ad}=I_{da}+I_{de} \qquad (4)$$

Point *e*,

$$I_{Be}+I_{de}+I_{be}+I_{fe}=0; \text{ or, } I_{Be}=I_{ed}+I_{eb}+I_{ef} \qquad (5)$$

Point *f*,

$$I_{Cf}+I_{ef}+I_{cf}=0; \text{ or, } I_{Cf}=I_{fe}+I_{fc} \qquad (6)$$

By equation **4** the vector of the current I_{Ad} in conductor A is the sum of vector I_{da}, found by equation **1,** and I_{de}, which is in phase with E_{ab}, Fig. 49. In equation **5,** vector I_{ed} is the reverse of I_{de}, and is in phase with vector I_{eb}; the combined length of vectors I_{eb} and I_{ed} therefore form one component and I_{ef}, in phase with E_{bc}, forms the other, the resultant of these two components being the vector of the current I_{Be} in conductor B. In equation **6,** I_{fe} is the reverse of I_{ef} and is in phase with I_{fc}; the combined length of I_{fc} and I_{fe} therefore gives the vector of the current I_{Cf} in conductor C.

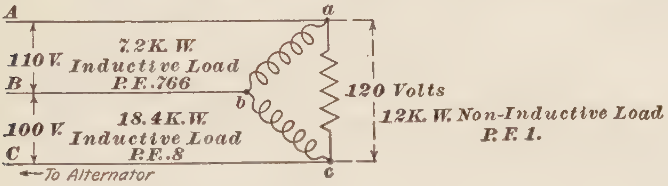


FIG. 50

The phase relations and values of the various voltages and currents in the line wires and loads of an unbalanced three-phase system, operating under the conditions stated in the problem, are indicated in Fig. 49.

47. Fig. 50 shows conditions found in a certain three-phase circuit by means of measuring instruments. The phases are badly unbalanced, yet the exact phase relations of the currents and voltages can be determined by means of vector diagrams. The phase relations of the voltages can be shown by means of a triangle, Fig. 51, with sides porportional in length to the three voltages 120, 110, and 100. The construction of this triangle is based on Kirchhoff's second law. Consider the triangle formed by the representations of the loads, Fig. 50; then,

$$E_{ab}+E_{bc}+E_{ca}=0$$

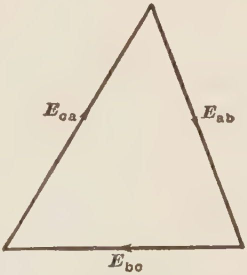


FIG. 51

The lengths of the three corresponding vectors, Fig. 51, are proportional to the voltages between line wires. The three component vectors placed end to end must form a closed figure, since their vectorial sum is 0. The phase relations of the voltages are indicated by the relative positions of the vectors. Radial vectors equal to and parallel to sides of the triangle, Fig. 51, are drawn in Fig. 52.

The current between each pair of line wires, or in each phase, Fig. 50, is the quotient of apparent watts divided by voltage, and apparent watts is watts load divided by power factor. The angle of lag of the current behind the voltage corresponding to the power factor in each phase is given in Table I.

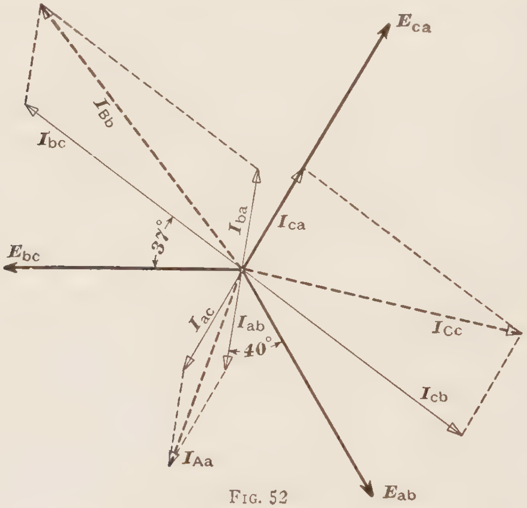


FIG. 52

$$I_{ab}=\frac{7{,}200}{.766\times110}=85 \text{ amperes. Angle of lag 40 time-degrees.}$$

$$I_{bc}=\frac{18{,}400}{.8\times100}=230 \text{ amperes. Angle of lag 37 time-degrees}$$

$$I_{ca}=\frac{12{,}000}{1\times120}=100 \text{ amperes. Angle of lag 0 time-degrees.}$$

The vectors of these phase currents are then drawn, as in Fig. 52, I_{ab} 40 time-degrees behind E_{ab}, I_{bc} 37 time-degrees behind E_{bc}, and I_{ca} in phase with E_{ca}. The line currents $A\,a$, $B\,b$, and $C\,c$, Fig. 50, can now be found according to Kirchhoff's first law.

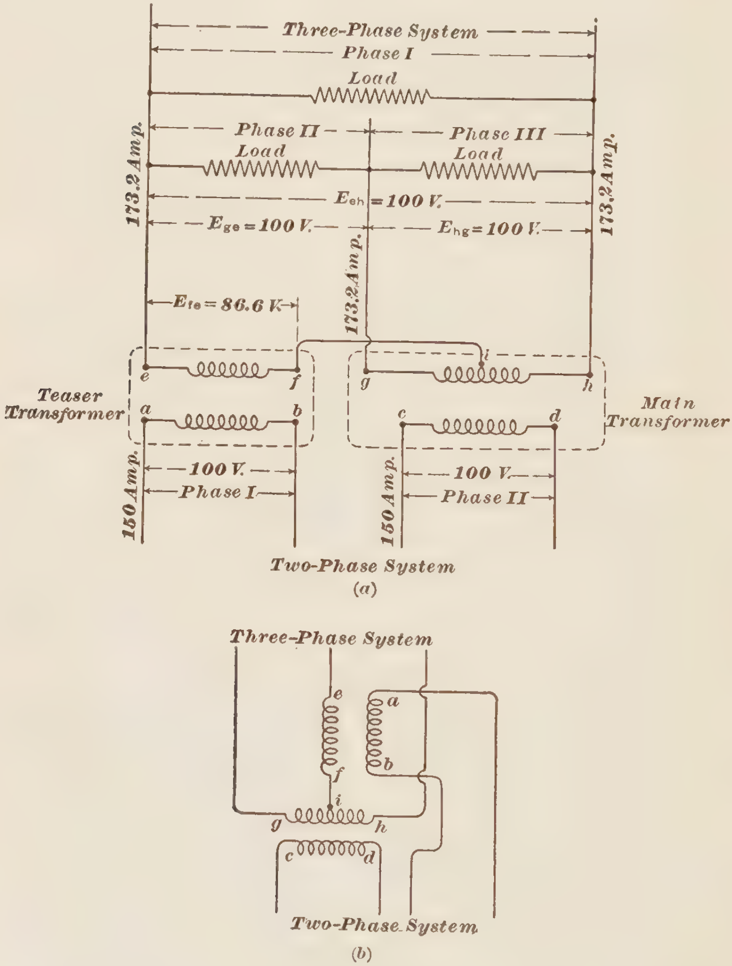
$$I_{Aa}+I_{ba}+I_{ca}=0;\text{ or, }I_{Aa}=I_{ab}+I_{ac}$$
$$I_{Bb}+I_{ab}+I_{cb}=0;\text{ or, }I_{Bb}=I_{ba}+I_{bc}$$
$$I_{Cc}+I_{ac}+I_{bc}=0;\text{ or, }I_{Cc}=I_{ca}+I_{cb}$$

In Fig. 52, vector $I_{ac}=-I_{ca}$, $I_{ba}=-I_{ab}$, and $I_{cb}=-I_{bc}$. By adding vectors in Fig. 52, in accordance with these equations, the vectors representing the currents in the line wires are determined. The actual values of the line currents can be obtained by measuring the resultant vectors by the same scale used for forming the component vectors and computing the current values from the value of each scale division, expressed in amperes.

48. Scott Two-Phase, Three-Phase Connection. In the preceding two- and three-phase problems, it was shown that a great variety of phase relations between voltages and currents may be obtained by suitably arranging the coils in which the voltages or currents are established.

By means of an arrangement of transformer coils devised by Mr. C. F. Scott, a two-phase system may be used to supply electrical energy to a three-phase system or vice versa. The fundamental principles of an induction coil, which is a form of transformer, are explained in a previous Section and a more extensive treatise on transformers is provided in a later Section. The purpose of the following instruction is to show the application of vectors to this transformation problem.

The problem to be considered is to transform (neglecting losses) 30,000 watts at 100 volts from a two-phase system to 30,000 watts at 100 volts in a three-phase system. In Fig. 53 (*a*), the primary coils, which are in this case connected to the two-phase system, are *a b* and *c d*. On the three-phase system, the corresponding secondary coils are the teaser coil *f e* and the main coil *h g*. In view (*b*) is shown one method of representing the same coil connections that resemble approximately the phase relations of the problem. One end of the teaser coil is connected to the middle point *i* of the main secondary winding. The voltage E_{fe}, view (*a*), of the teaser coil is 90 time-degrees out of phase with the voltage E_{hg} of the main coil because these voltages are established by the separate actions of the two phases of the two-phase system which set up currents through the primary coils that differ in phase by 90 time-degrees.



(a)

(b)

Fig. 53

The primary coil *a b* and *c d* are similar. The number of turns of wire on the main secondary coil *h g* is such that the voltage for a phase of the three-phase system will be established in coil *h g*. As shown in Fig. 53 (*a*), a three-phase voltage of 100 is assumed in order to show the relative voltages. The number of turns in the teaser coil *f e* is only .866 times the number of turns in coil *h g*; therefore, the voltage of coil *f e* is .866 times the voltage of coil *h g*, or the voltage of coil *f e* is 86.6. The voltages between *g* and *i* or *i* and *h* are one-half of the voltage between *h* and *g*, or 50.

By means of Kirchhoff's second law and vector diagrams, the values and phase relations of the voltages E_{hg}, E_{ge}, and E_{eh}, which are the phase voltage of the three-phase system, may be determined. Trace in a counter-clockwise direction circuit *e–f–i–g–Load Phase II–e*. Then,

$$E_{ge}+E_{ef}+E_{ig}=0;\ \text{or,}\ E_{ge}=E_{fe}+E_{gi} \qquad (1)$$

Trace in a clockwise direction circuit *e–Load Phase I–h–i–f–e*. Then,

$$E_{eh}+E_{hi}+E_{fe}=0;\ \text{or,}\ E_{eh}=E_{ih}+E_{ef} \qquad (2)$$

Trace in a counter-clockwise direction circuit *h–Load Phase III–g–i–h*. Then,

$$E_{hg}+E_{gi}+E_{ih}=0;\ \text{or,}\ E_{hg}=E_{ig}+E_{hi} \qquad (3)$$

49. In Fig. 54 (*a*), the vectors for voltages E_{fe}, E_{gi}, and E_{ih} and their general phase relations are indicated. If the order of the subscripts of a vector in one equation is reversed in another equation, the second vector should be drawn so as to point in the opposite direction to that of the first vector. Vector E_{fe} is drawn to scale to represent 86.6 volts; vector E_{gi} or E_{ih}, to represent 50 volts. Vector E_{fe} differs in phase from either E_{gi} or E_{ih} by an angle representing 90 time-degrees. Vectors E_{gi} and E_{ih} are equal and in phase.

The vector diagram, view (*b*), is based on equation **1**, and the resultant vector E_{ge} indicates the voltage of phase II, Fig. 53. The value of this voltage is $\sqrt{86.6^2+50^2}=100$. The diagram, Fig. 54 (*c*), is based on equation **2**, and the resultant vector E_{eh} represents the voltage of phase I, Fig. 53. In Fig. 54 (*d*), vectors E_{ig} and E_{hi} are added in accordance with equation **3**. The resultant vector E_{hg} represents the voltage of phase III, Fig. 53, and is equal to $50+50=100$.

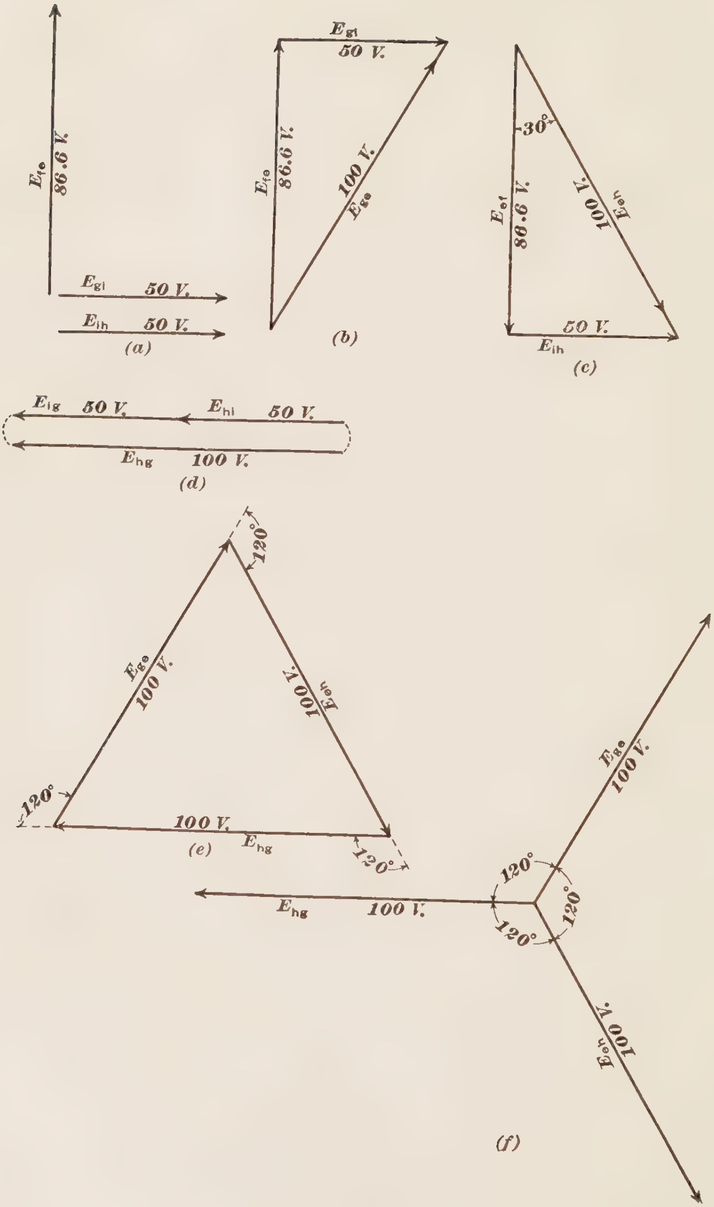

E_{fe}
86.6 V.
E_{gi}
50 V.
E_{ih}
50 V.
(a)
E_{gi}
50 V.
E_{fe}
86.6 V.
100 V.
E_{ge}
(b)
30°
E_{ei}
86.6 V.
100 V.
E_{eh}
50 V.
E_{ih}
(c)
E_{ig}
50 V.
E_{hi}
50 V.
E_{hg}
100 V.
(d)
120°
E_{ge}
100 V.
100 V.
E_{eh}
120°
100 V.
E_{hg}
120°
(e)
E_{ge}
100 V.
120°
120°
E_{hg}
100 V.
120°
E_{eh}
100 V.
(f)

FIG. 54 (*a*) to (*f*)

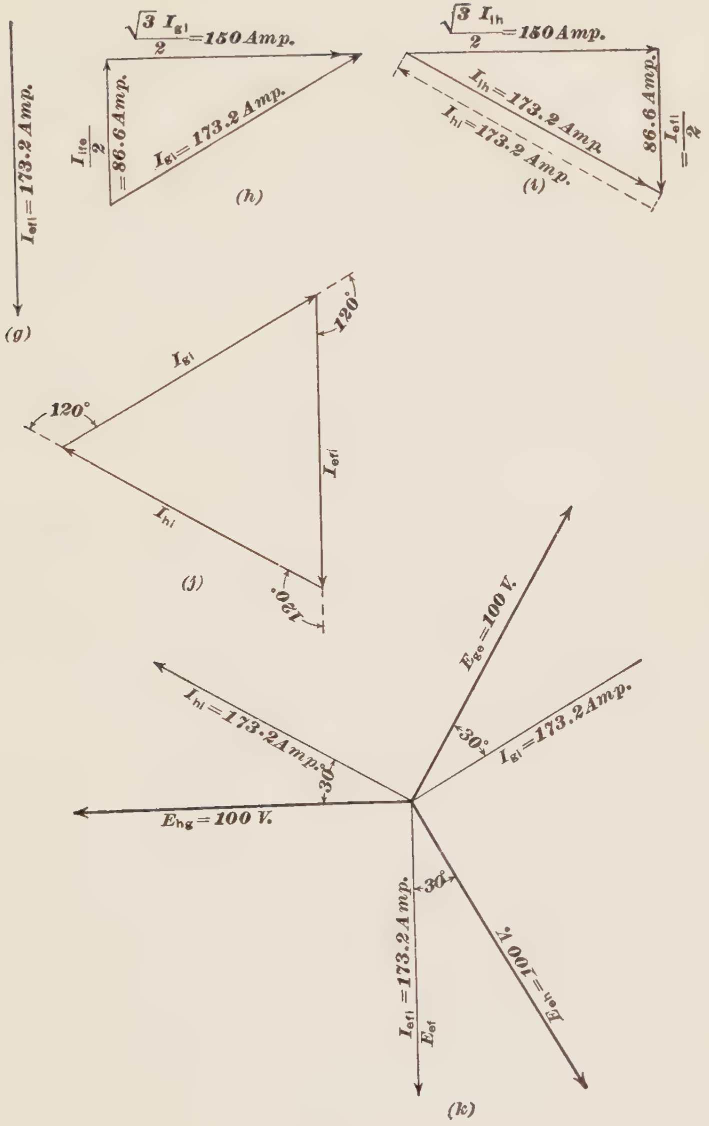
$I_{efl} = 173.2$ Amp.
(g)
$\frac{\sqrt{3}\ I_{gl}}{2} = 150$ Amp.
$\frac{I_{lfe}}{2} = 86.6$ Amp.
$I_{gl} = 173.2$ Amp.
(h)
$\frac{\sqrt{3}\ I_{lh}}{2} = 150$ Amp.
$I_{lh} = 173.2$ Amp.
$I_{hl} = 173.2$ Amp.
86.6 Amp.
$= \frac{I_{efl}}{2}$
(i)
120°
I_{gl}
120°
I_{efl}
I_{hl}
120°
(j)
$E_{ge} = 100$ V.
$I_{hl} = 173.2$ Amp.
30°
$I_{gl} = 173.2$ Amp.
30°
$E_{hg} = 100$ V.
30°
$I_{efl} = 173.2$ Amp.
E_{ef}
$E_{eh} = 100$ V.
(k)

Fig. 54 (g) to (k)

The three resultant vectors, each representing 100 volts, may be grouped as shown in Fig. 54 (*e*) and (*f*). The phase voltages of the three-phase system are each 100, but differ in phase by 120 time-degrees.

50. Assume unity power factor and a balanced load on the three-phase system, Fig. 53. The line wire connected to *e* must have the same current as that in the teaser coil *e f,* since the line wire and the coil are in series. For similar reasons, the current in the line wire connected to *g* has the same current as in the partial coil *g i,* and the line current in the wire connected to *h* has the same current as in partial coil *h i.* With the assumed load the line current is 173.2 amperes. Consider point *i,* then in accordance with Kirchhoff's first law,

$$I_{efi}+I_{gi}+I_{hi}=0$$

Vector I_{efi}, Fig. 54 (*g*), represents the line current 173.2 amperes in wire *e* and also the current in the teaser coil, Fig. 53. Its general direction is taken to accord with the direction of vector E_{ef}, Fig. 54 (*c*), since the system is assumed to have unity power factor and then the voltage and current in the teaser coil are in phase.

The total current in coil *g i,* Fig. 53, is the vectorial sum of a component current established in it by the inductive action of the primary coil *c d* of Phase *II* of the two-phase system and a component current from the teaser coil equal to one-half of the teaser current which comes through the circuit *f–e–Load Phase II–g–i–f.* These two component currents differ in phase by 90 time-degrees since they are established by voltages that differ in phase by 90 time-degrees. The total current in coil *g i* must be the same as the total current in the teaser coil when the loads are balanced.

In the vectorial equation for currents $I_{efi}+I_{gi}+I_{hi}=0$, the components of I_{gi} are used in determining the value and relative phase relation of the total current I_{gi}. Vector $\frac{I_{ife}}{2}$, Fig. 54 (*h*), which is equal to $\frac{I_{gi}}{2}$, represents the component current from the teaser coil. The current is $\frac{173.2}{2}=86.6$ amperes. The other component current, which is 150 amperes, is represented by the vector $\frac{\sqrt{3}\,I_{gi}}{2}$. The term $\frac{\sqrt{3}}{2}=.866$ expresses the value of the cosine of the angle formed between the vector I_{gi} and the vector representing the component current that is 90 time-degrees out of phase with the teaser-current vector $\frac{I_{ife}}{2}$. The vector $\frac{I_{ife}}{2}$ is the part of the total current I_{gi} that is established by the transformer action of coil *a b*, Fig. 53. The current in each primary coil *a b* and *c d* is 150 amperes. The direction of the component current $\frac{\sqrt{3}\,I_{gi}}{2}$ is taken as being from *g* to *i*. The resultant vector, Fig. 54 (*h*), is

$$I_{gi}=\sqrt{\left(\frac{I_{gi}}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\,I_{gi}\right)^2}=\sqrt{I^2_{gi}}=173.2 \text{ amperes}$$

In the current equation, the term I_{hi} is used. In considering the total current in the partial coil *i h*, Fig. 53, first determine the vector for the total current from *i* to *h* and then consider the current as reversed in order to obtain the term I_{hi}. Consider the teaser-component current as coming into coil *i h* at *i* through the circuit *e–f–i–h–Load Phase I–e*. Vector $\frac{I_{efi}}{2}$, Fig. 54 (*i*), is drawn in a vertical direction with its arrowhead pointing downwards. The vector for the component current due to the transformer action of primary coil *c d*, Fig. 53, has the same direction as the vector for the corresponding component current, Fig. 54 (*h*). The resultant vector is I_{ih} and the vector for the current from *h* to *i*, Fig. 53, is I_{hi}, Fig. 54 (*i*).

View (*j*) is a vector diagram for the current equation $I_{efi}+I_{gi}+I_{hi}=0$. The resultant of the vector diagram is zero, since the vectorial addition of the component vectors forms a closed figure.

51. View (*k*) is a combination of views (*f*) and (*j*) with the vectors for voltages and currents drawn to a common origin. In Fig. 54 (*c*), the vector E_{ef} lags behind vector E_{eh} by 30 time-degrees. The current I_{efi} is in phase with voltage E_{ef}, then in view (*k*) vector I_{efi} coincides in position with vector E_{ef}. The vectors for the other currents are drawn at angles representing 120 time-degrees. With unity power factor and balanced loads, the phase relations of the voltages between the line wires and the currents in the line wires are indicated in view (*k*). The line currents are taken in the direction from the loads to the transformers. If desired, the current may be assumed to be in the opposite directions and the current vectors, view (*k*), drawn 180° from their present position and the subscripts on them reversed.

The electrical energy of the two-phase circuit has been transformed into electrical energy of the three-phase circuit through the action of the transformers connected in the manner shown. The losses in this action are neglected in order to simplify the explanation. The power in the two-phase circuit is $(100\times150)+(100\times150)=30{,}000$ watts. The power in the three-phase circuit is $1.732\times100\times173.2=30{,}000$ watts. The calculation of power in two-phase and three-phase circuits is treated in the following articles.

CALCULATION OF POWER

52. Fundamental Principles.—Whatever may be the number of phases or their interconnection, the power of the balanced polyphase system is always the product of the power in each phase multiplied by the number of phases. The power in each phase is the product of the current, the voltage, and the power factor. When the separate phases have the same power factor, this power factor applies to the whole system. When the power factors of the separate phases are nearly alike, an average may be considered as the power factor of the system. When the power factors of the separate phases differ greatly, as with unbalanced loads, no power factor applies to the whole system, and the power should be determined for each of the phases separately, as for a single-phase circuit (see

Art. **20**); the power of the separate phases added together then gives the power of the system.

53. Power in Two-Phase Circuits.—In a balanced two-phase system, either four wire or three wire,

Let P = total power, in watts;
E = phase voltage;
I = phase current, in amperes;
$\cos\phi$ = power factor.

Then, $$P = 2\,E\,I\cos\phi$$

EXAMPLE.—The voltage across each phase of a two-phase, three-wire circuit is 220, the phase current is 100 amperes, and the power factor is .86. What is the power in kilowatts?

SOLUTION.—According to the formula,

$P = 2 \times 220 \times 100 \times .86 = 37{,}840$ watts, or 37.84 K. W. Ans.

54. Power in Three-Phase Circuits.—In a balanced three-phase star-connected circuit, the line current I equals the phase winding current i, and the line voltage E equals 1.732 times the phase winding voltage e (see Arts. **40** and **41**). The power in each phase is

$$p = e\,i\cos\phi \qquad (1)$$

Substituting for e its equivalent $E \div 1.732$ and for i its equivalent I, the power per phase is $\frac{E\,I\cos\phi}{1.732}$. Similarly, in a balanced three-phase, delta-connected circuit, the line voltage E equals the phase winding voltage e, and the line current I is 1.732 times the phase winding current i (see Arts. **43** and **44**). The power in each phase is $e\,i\cos\phi = \frac{E\,I\cos\phi}{1.732}$, as before. In either case, the total power P is the product of the power in each phase by the number of phases, or

$$P = \frac{3\,E\,I\cos\phi}{1.732} = 1.732\,E\,I\cos\phi \qquad (2)$$

EXAMPLE 1.—What power, in kilowatts, is represented by a line current of 150 amperes at unity power factor in a balanced three-phase 6,600-volt circuit?

Solution.—According to formula **2**,

$$P=\frac{1.732\times 6{,}600\times 150\times 1}{1{,}000}=1{,}714.7 \text{ K. W. Ans.}$$

Example 2.—Find the total power in an unbalanced three-phase circuit loaded as follows: Phase 1, 120 volts, 100 amperes, non-inductive; phase 2, 100 volts, 230 amperes, 80 per cent. power factor; phase 3, 110 volts, 85 amperes, 77 per cent. power factor.

Solution.—Applying formula **1**, $p_1=120\times 100\times 1\div 1{,}000=12$ K. W.; $p_2=100\times 230\times .8\div 1{,}000=18.4$ K. W.; and $p_3=110\times 85\times .77\div 1{,}000=7.2$ K. W. Therefore,

$$P=p_1+p_2+p_3=12+18.4+7.2=37.6 \text{ K. W. Ans.}$$